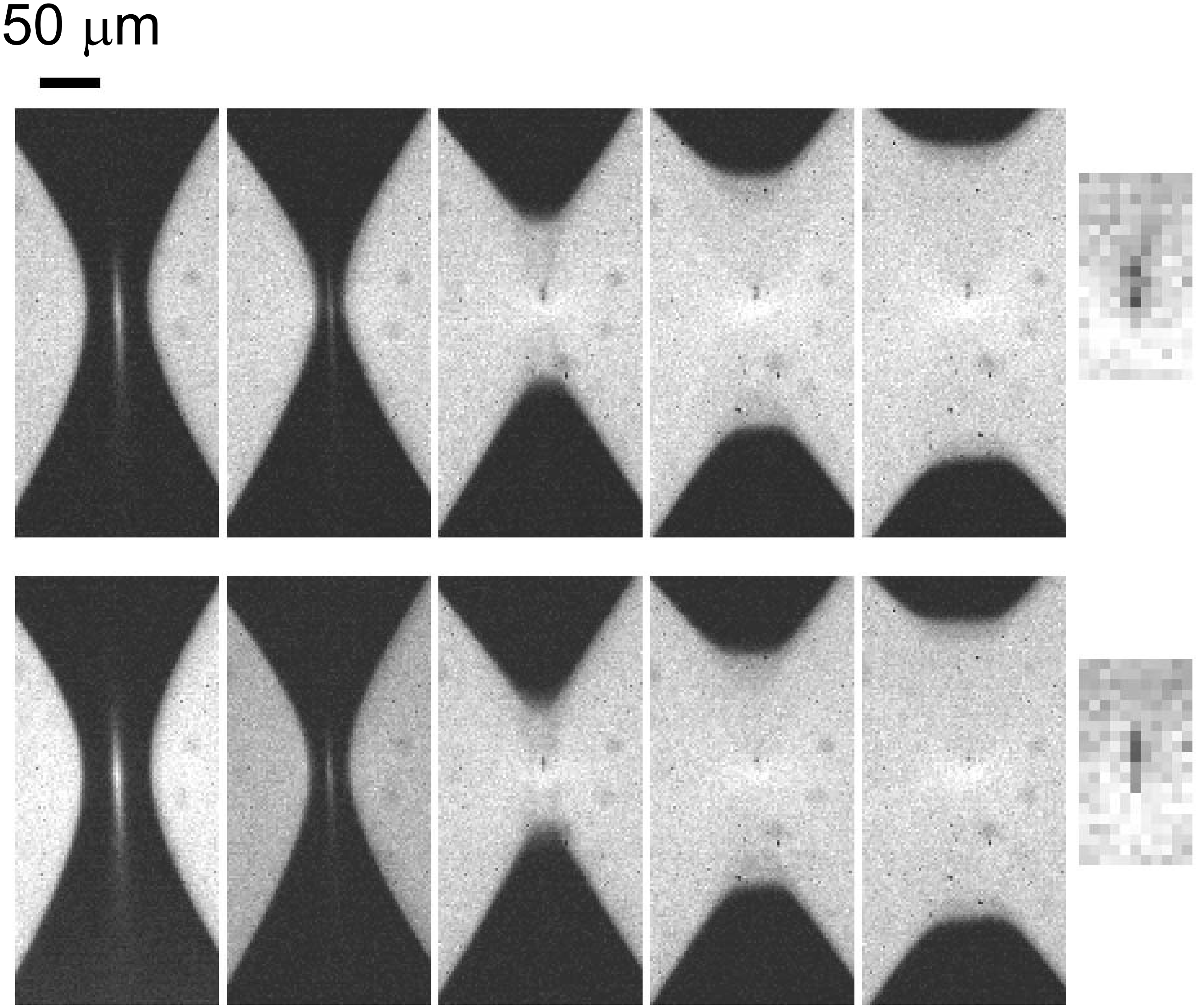
8 times viscosity of water



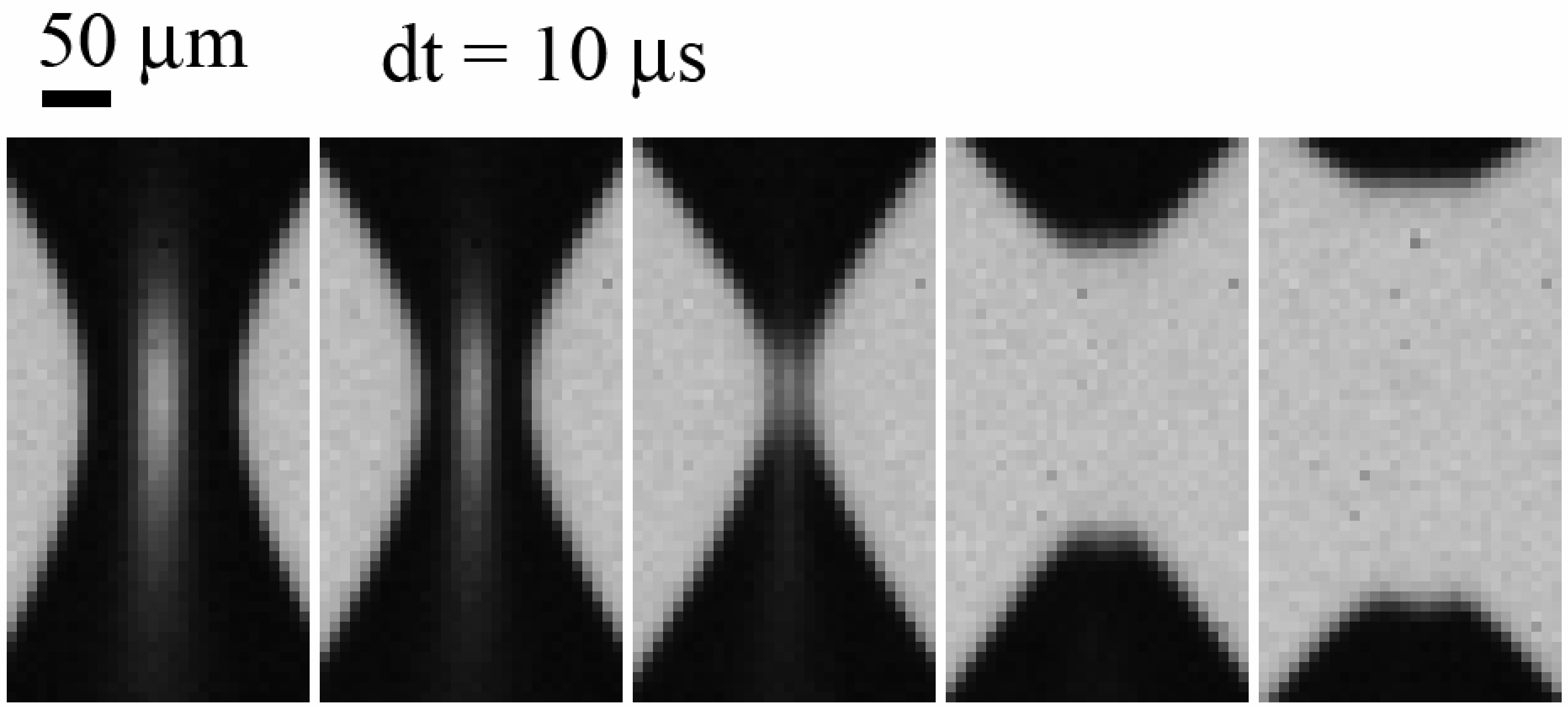
dt = 5 μs

Neck Rupture?

Burton, Waldrep & Taborek
Phys. Rev. Lett., ***94***,
Paper no. 184502 (2005)

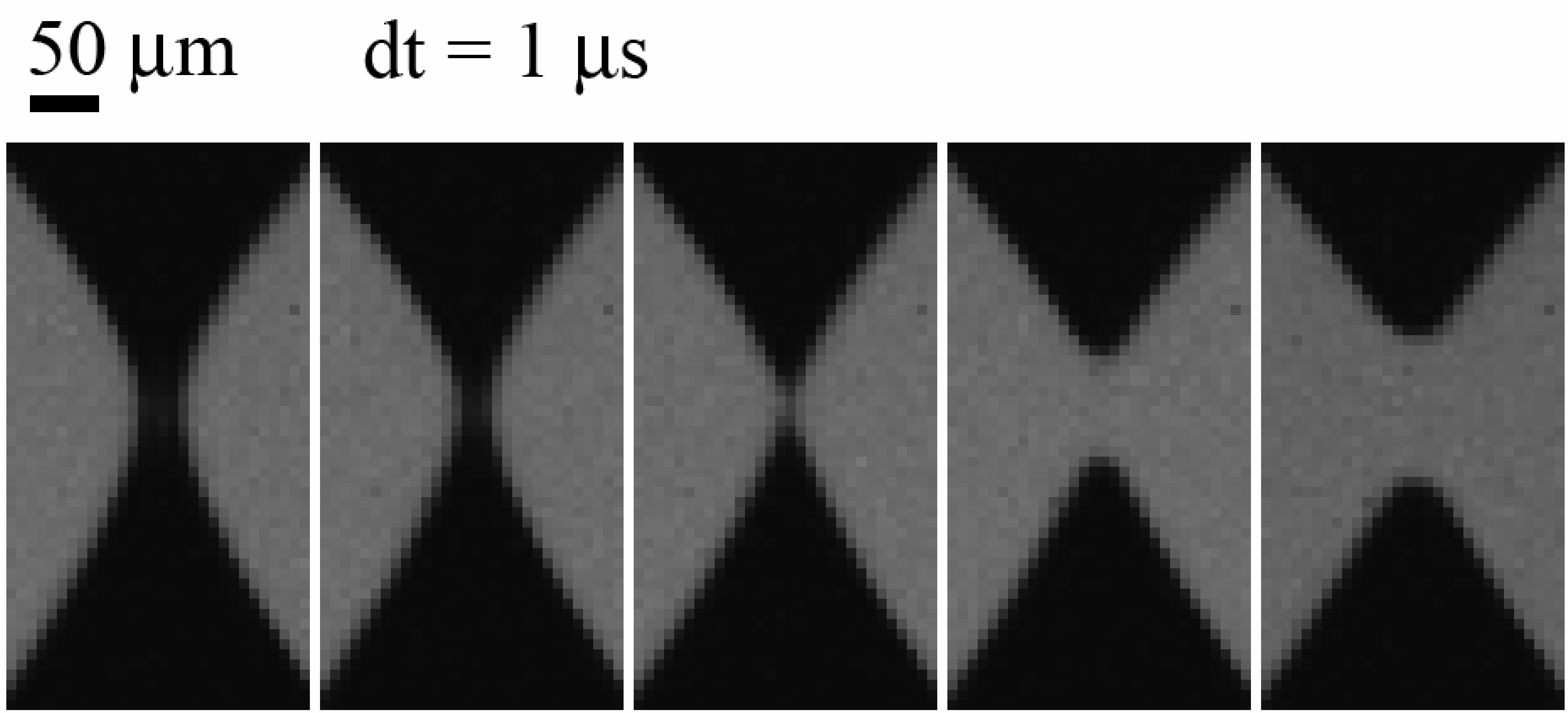
100,000 fps

Helium



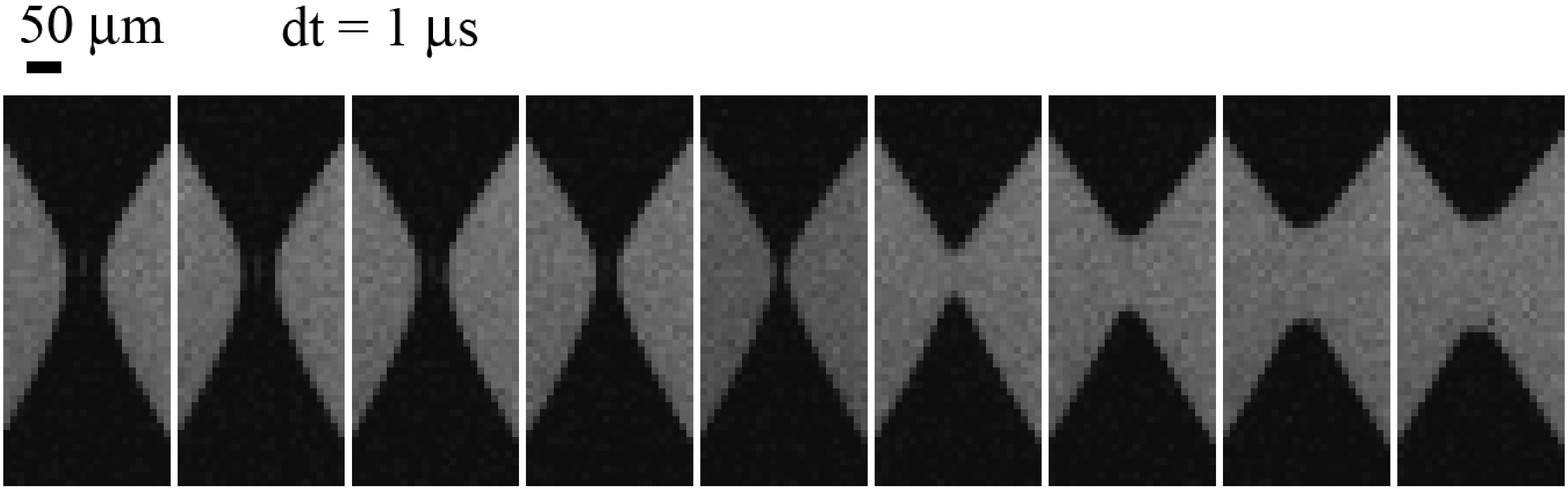
100 000 fps

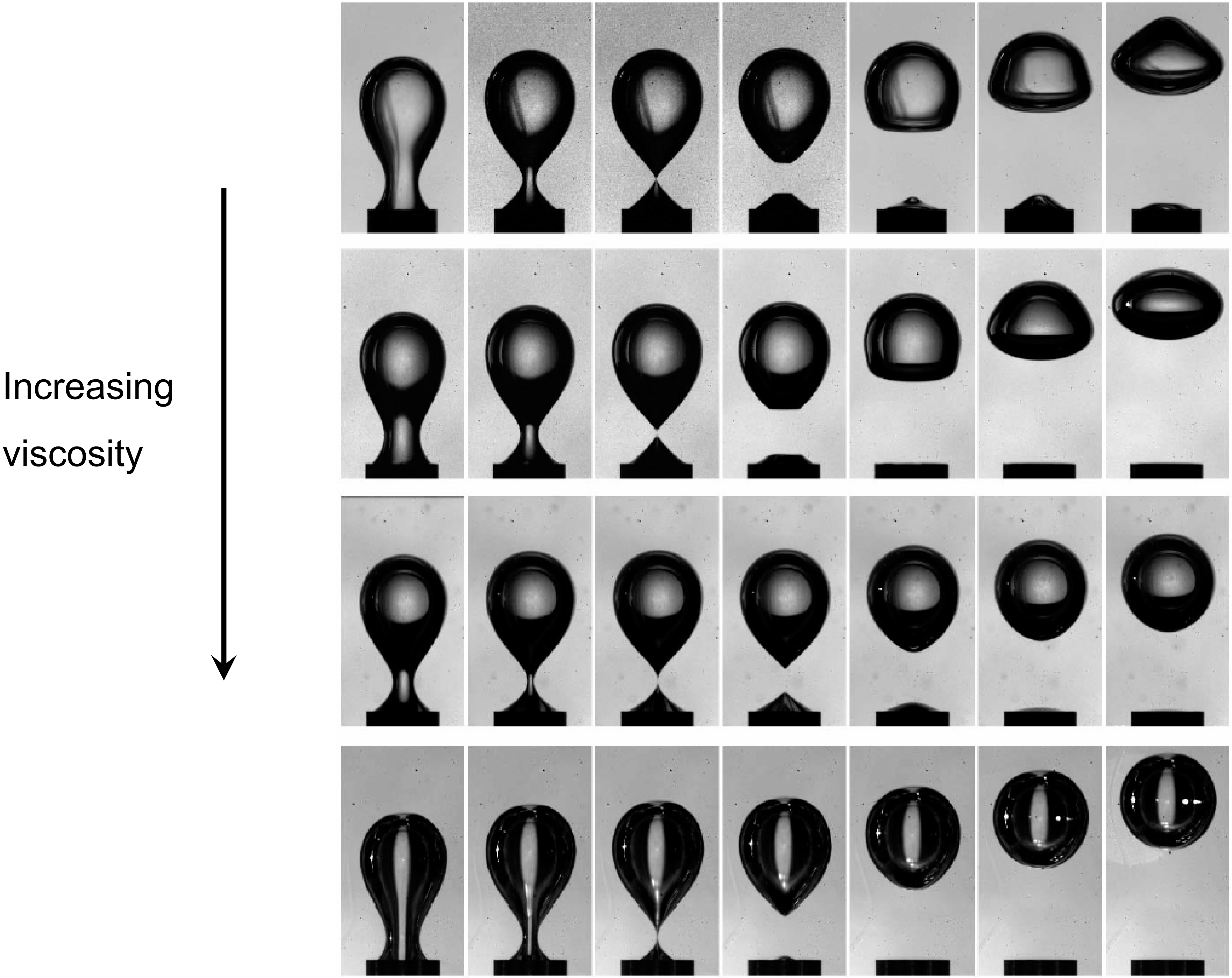
Helium in water



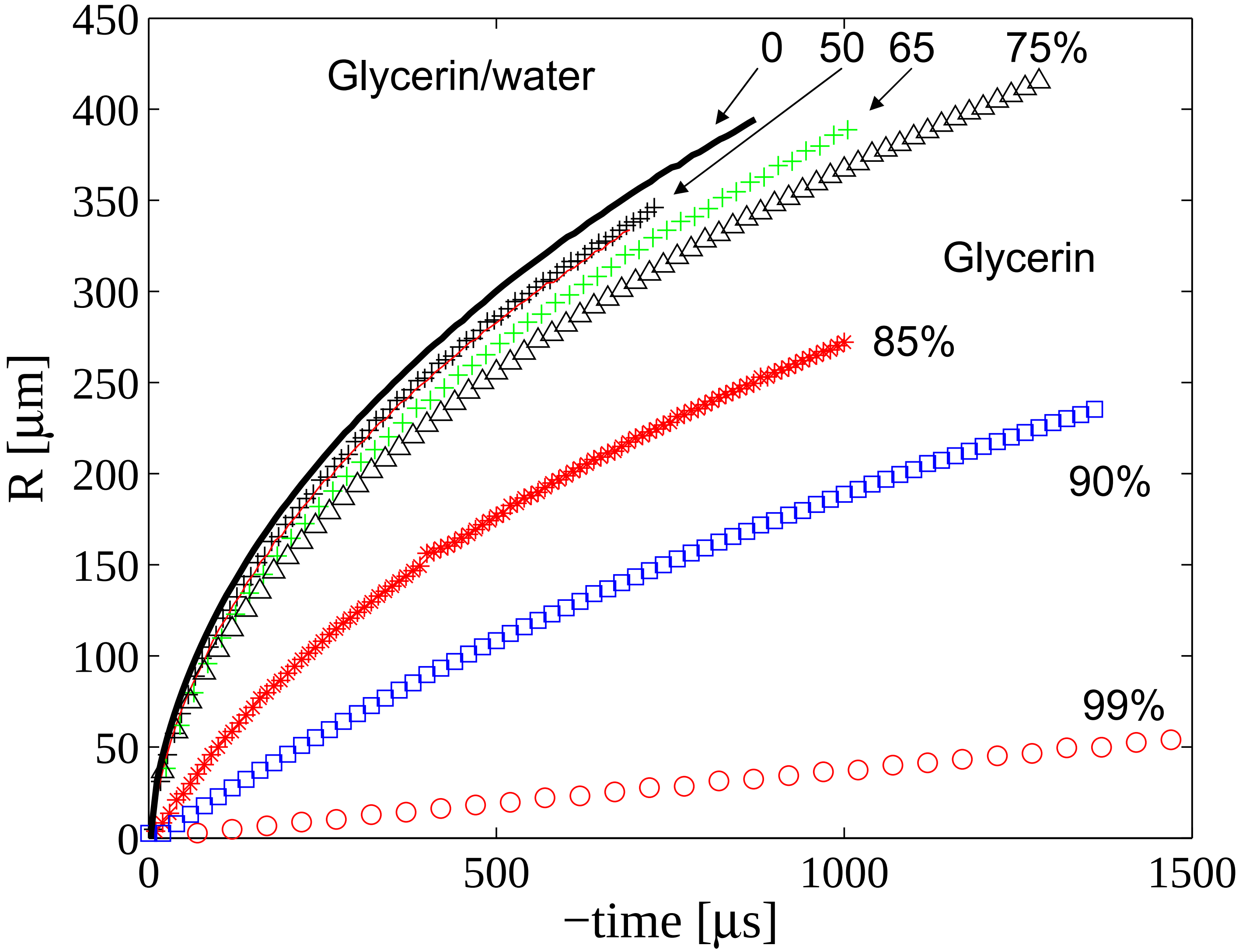
1 000 000 fps

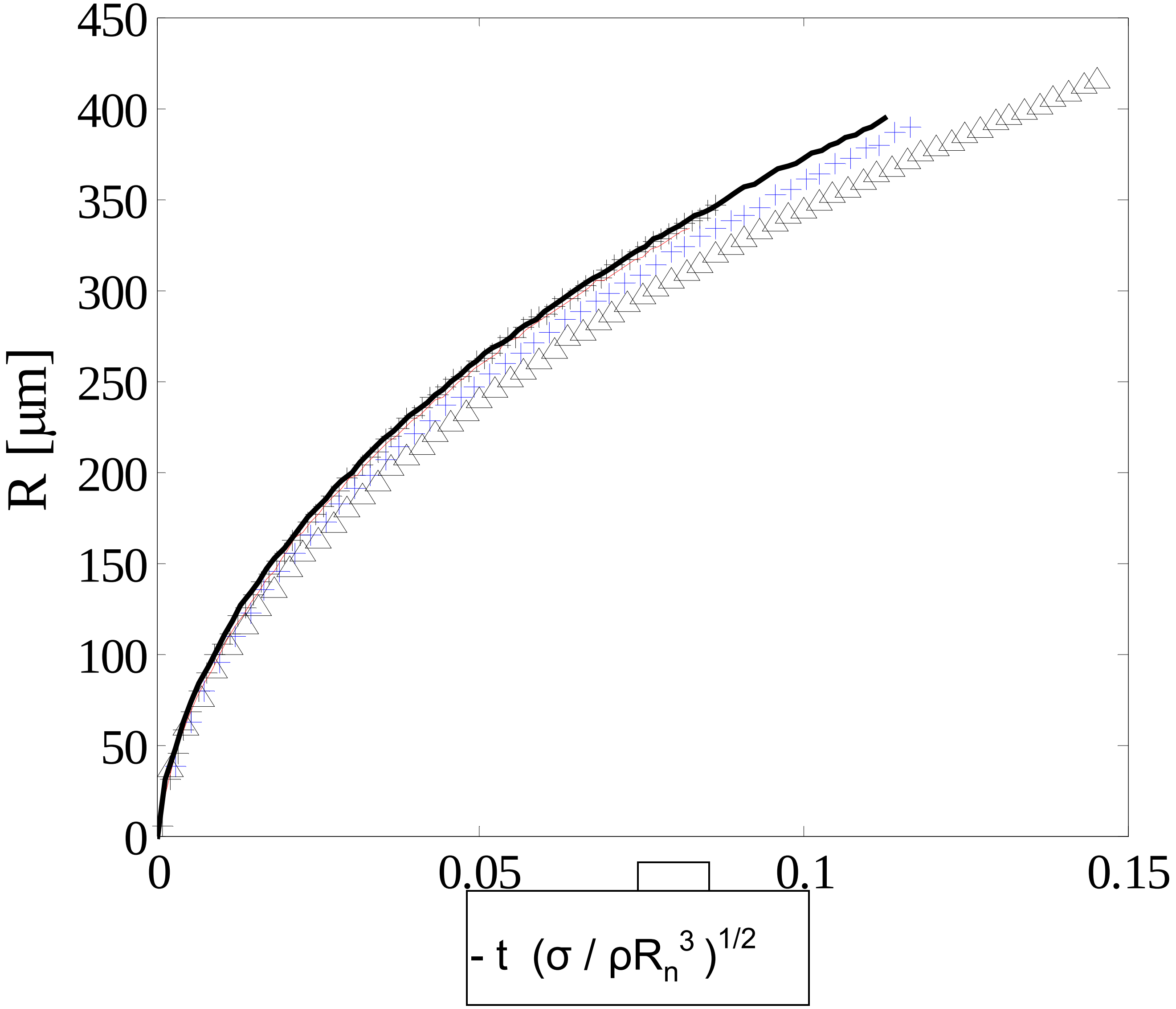
Air in water




Increasing
viscosity

Higher liquid viscosity

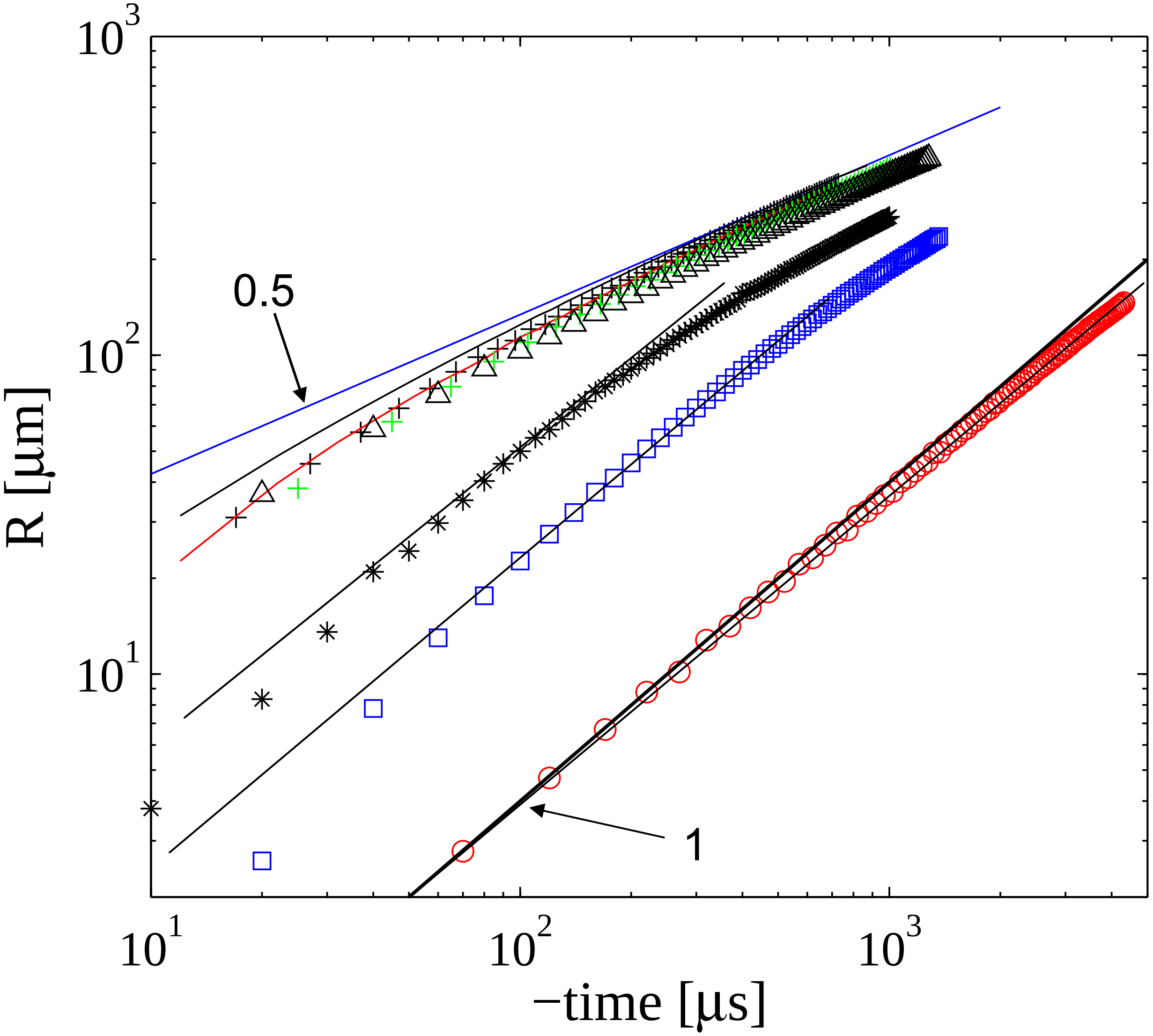




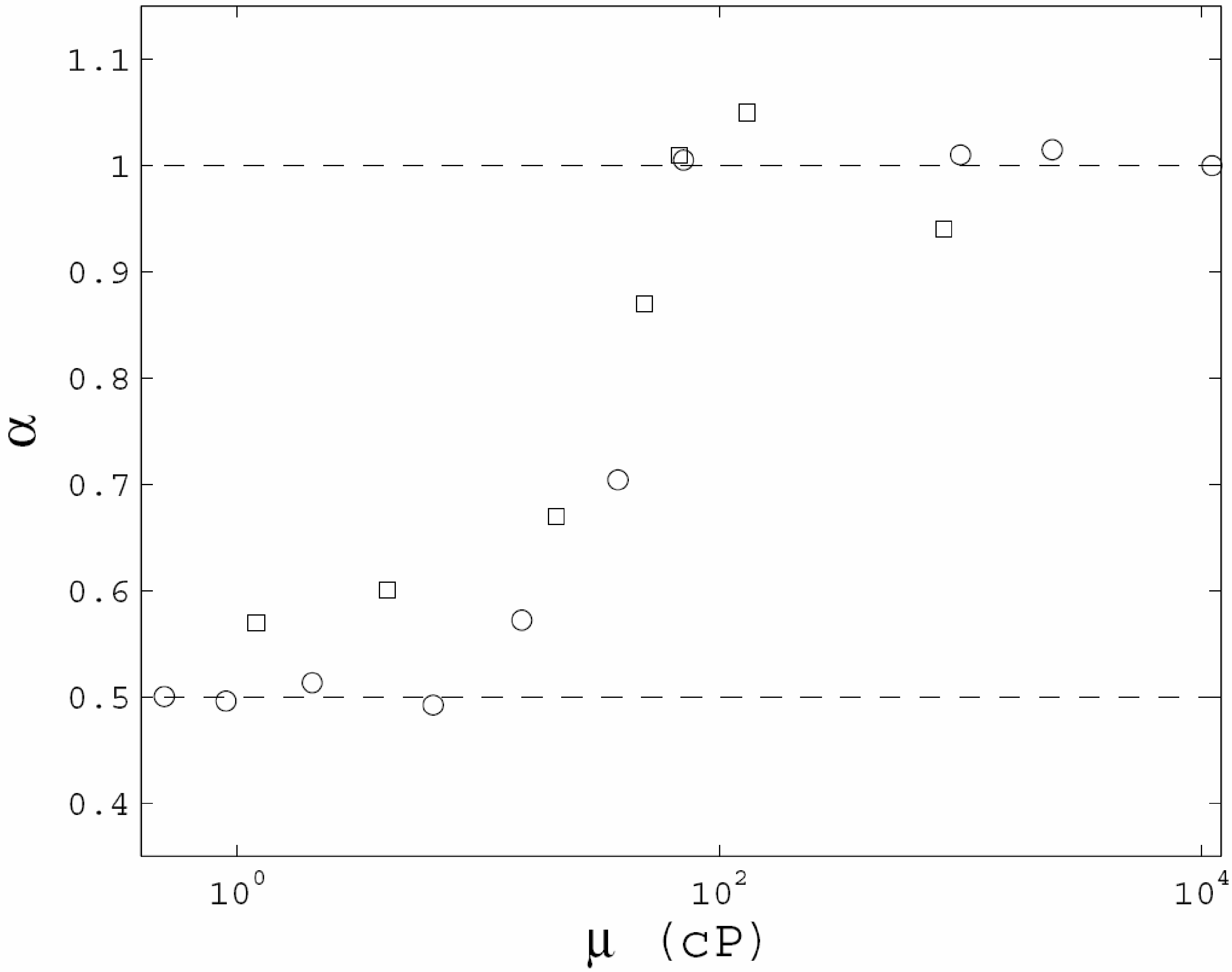
Deviates at

μ ≈ 10 water

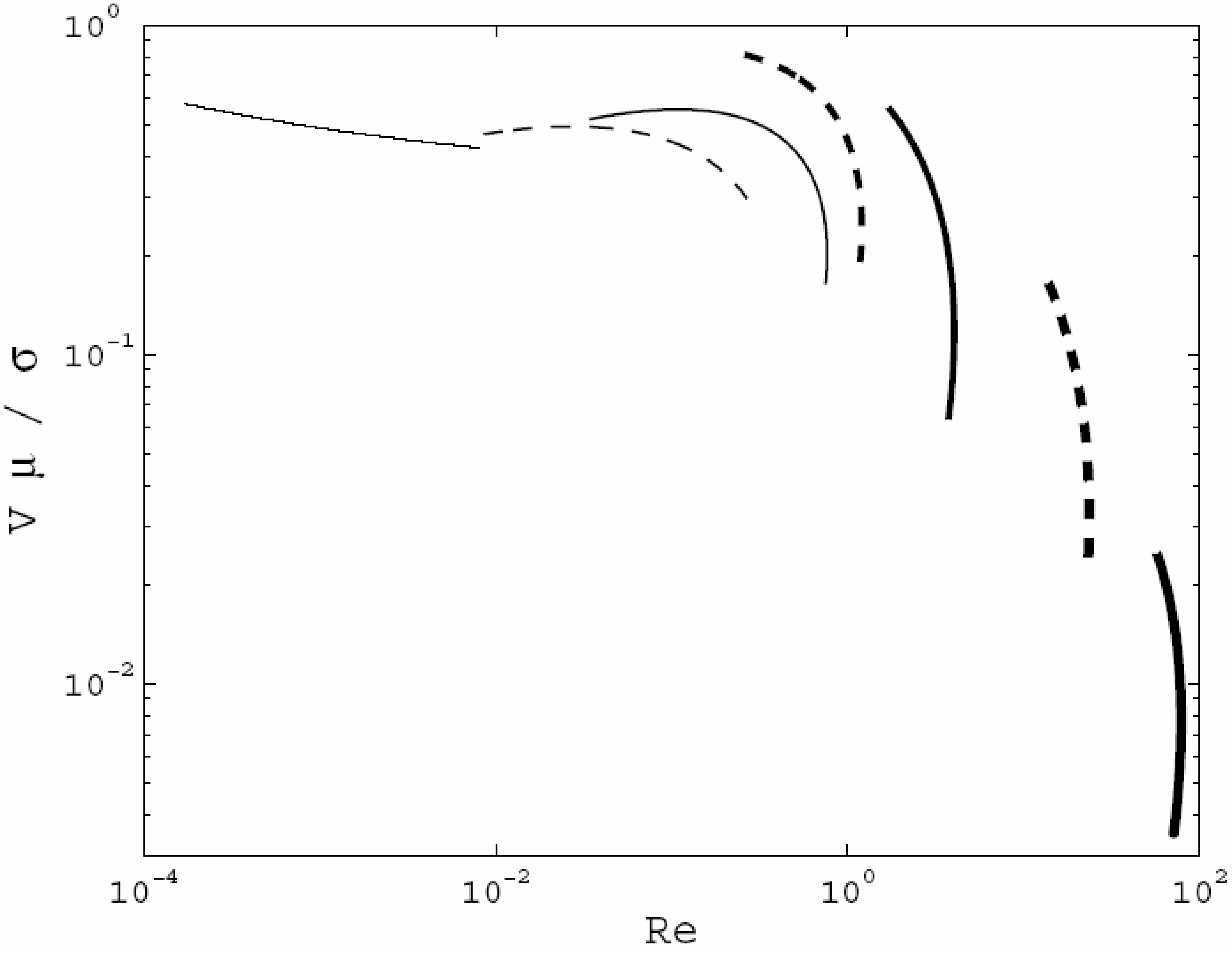
Power-laws ?

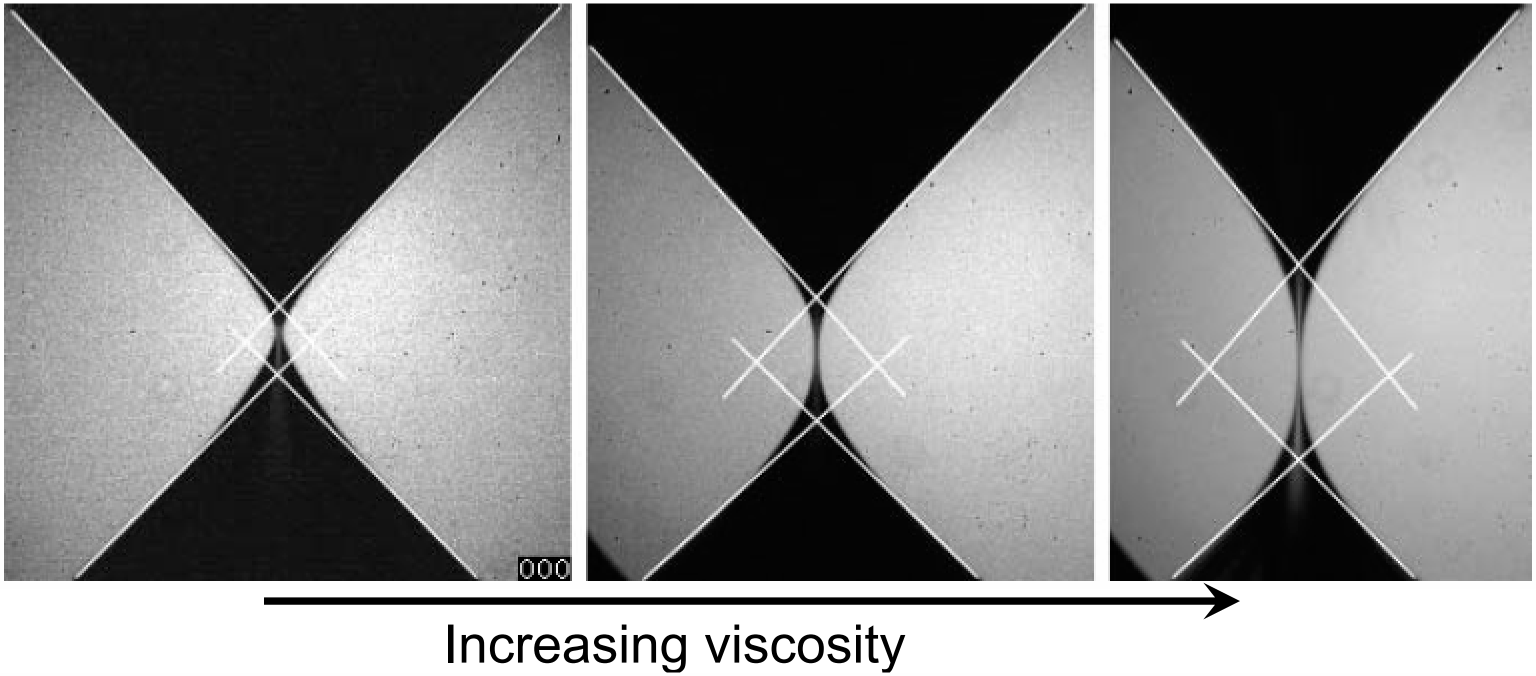


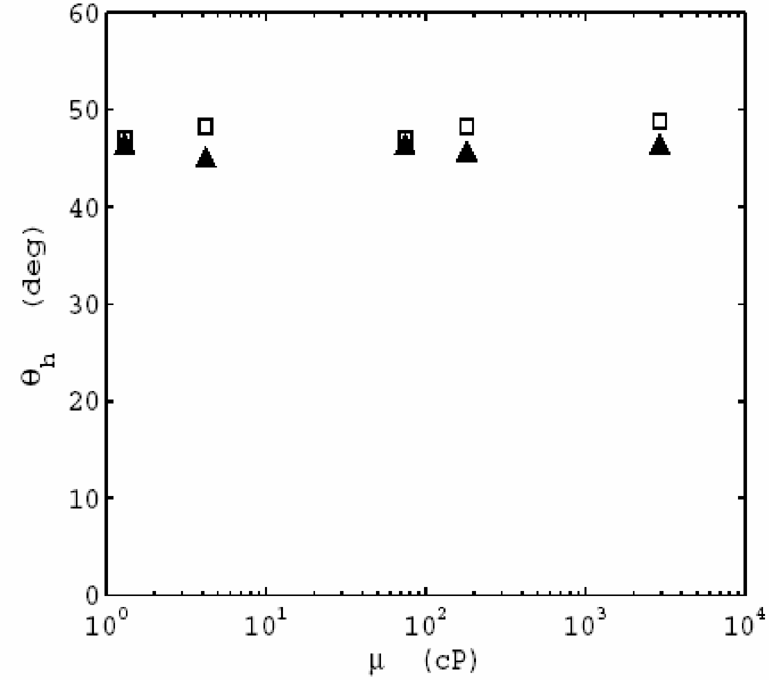
Power-law exponent,   Compared to Burton et al. (2005)

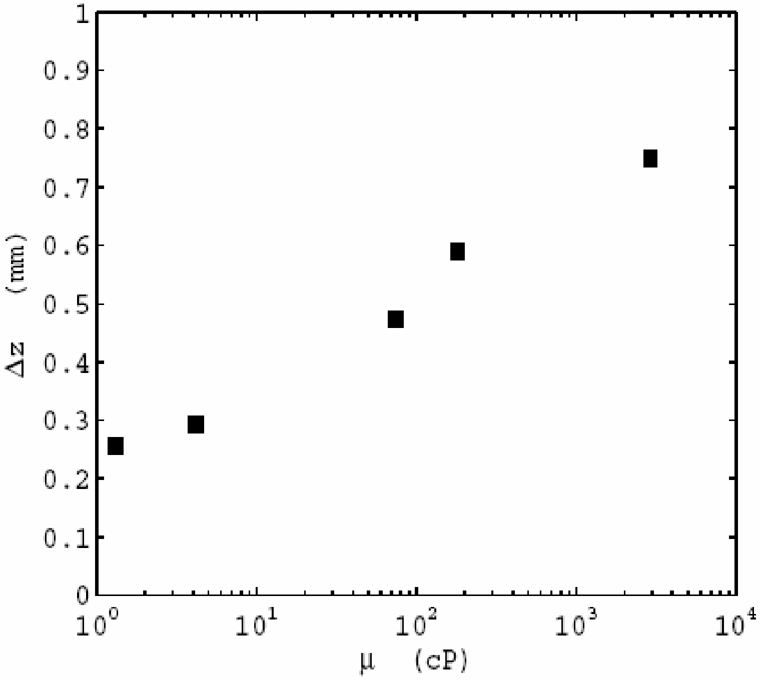


“Local” importance of viscous forces

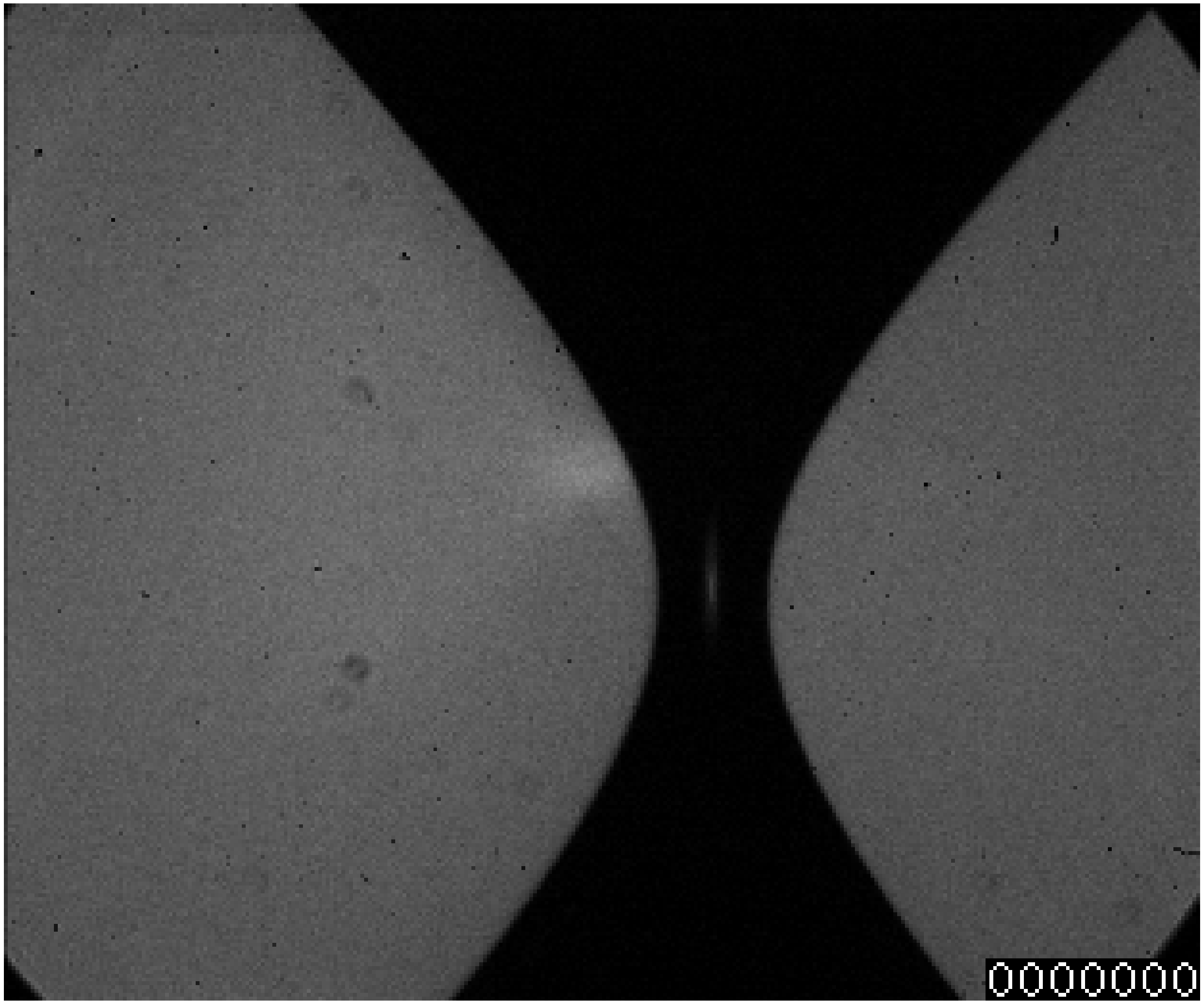



000
Increasing viscosity


60
50
40
30
20
10
0
θh (deg)
10^0
10^1
10^2
10^3
10^4
μ (cP)


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Δz (mm)
10^0
10^1
10^2
10^3
10^4
μ (cP)

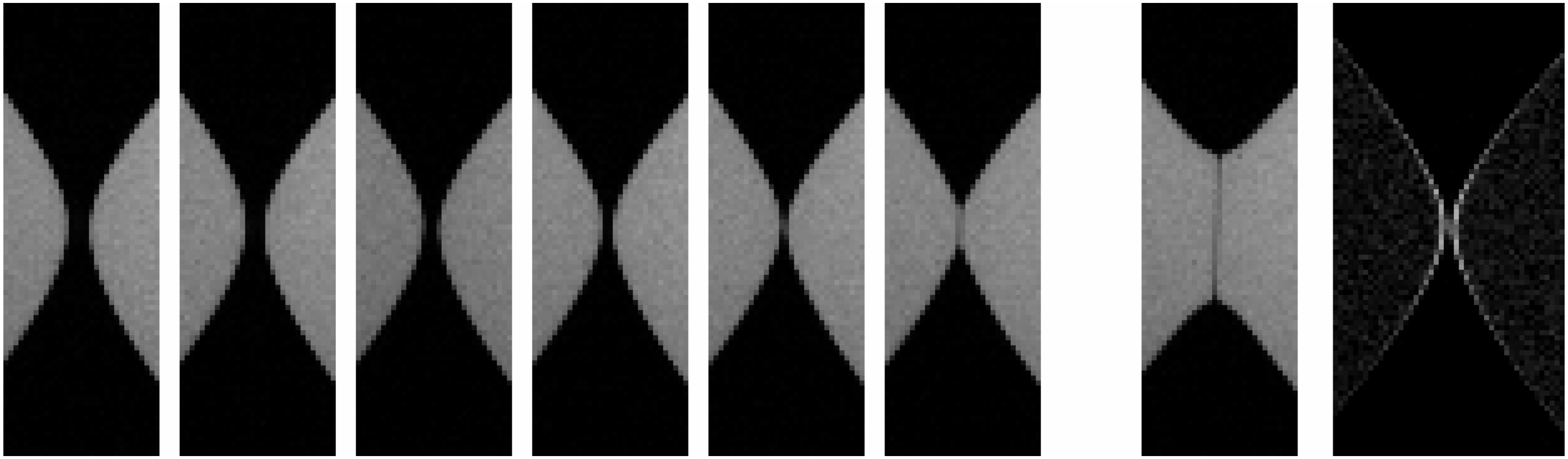
Higher viscosity



100,000 fps

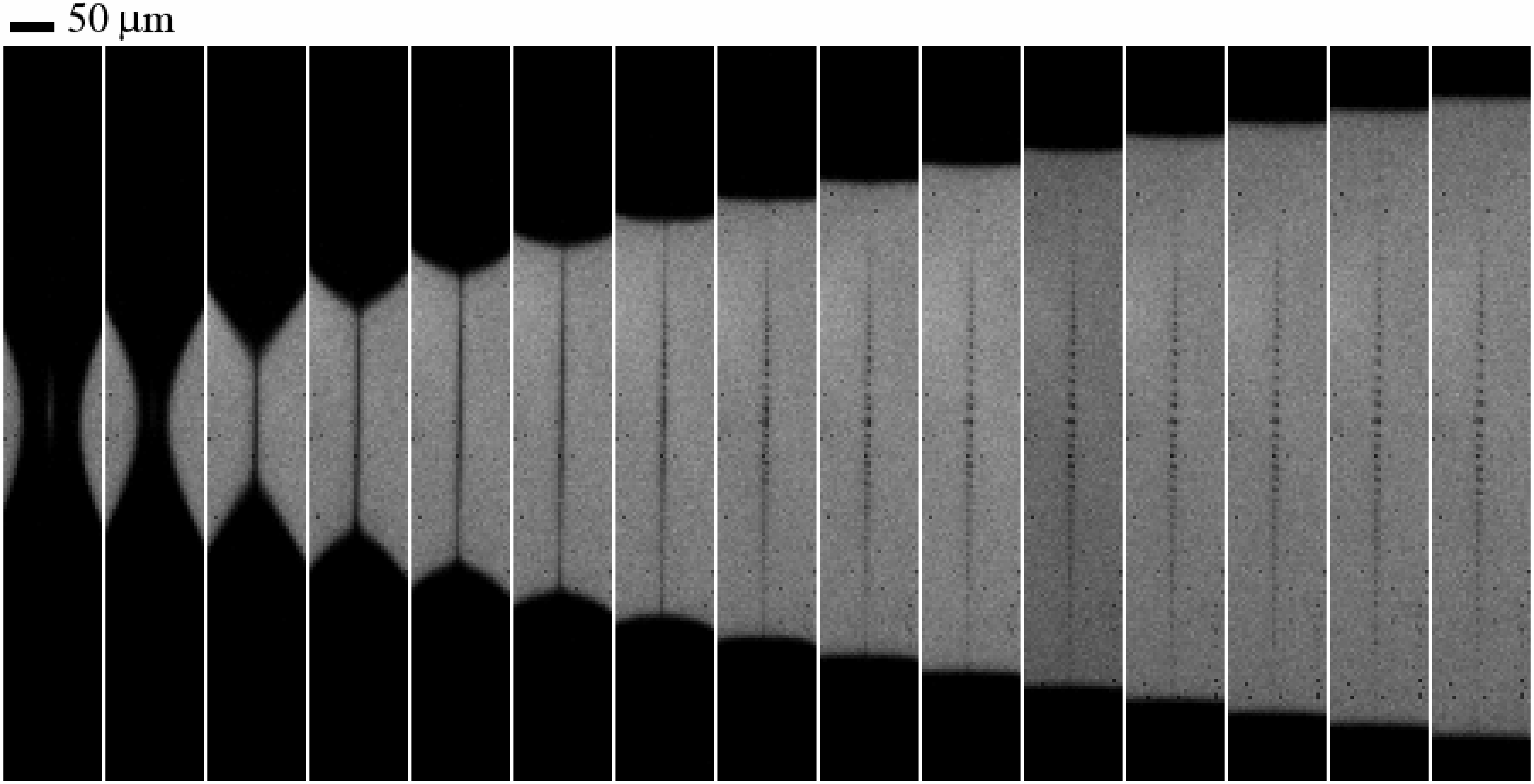
Higher viscosity

75% glycerin, $\mu = 20\ \mu_w$



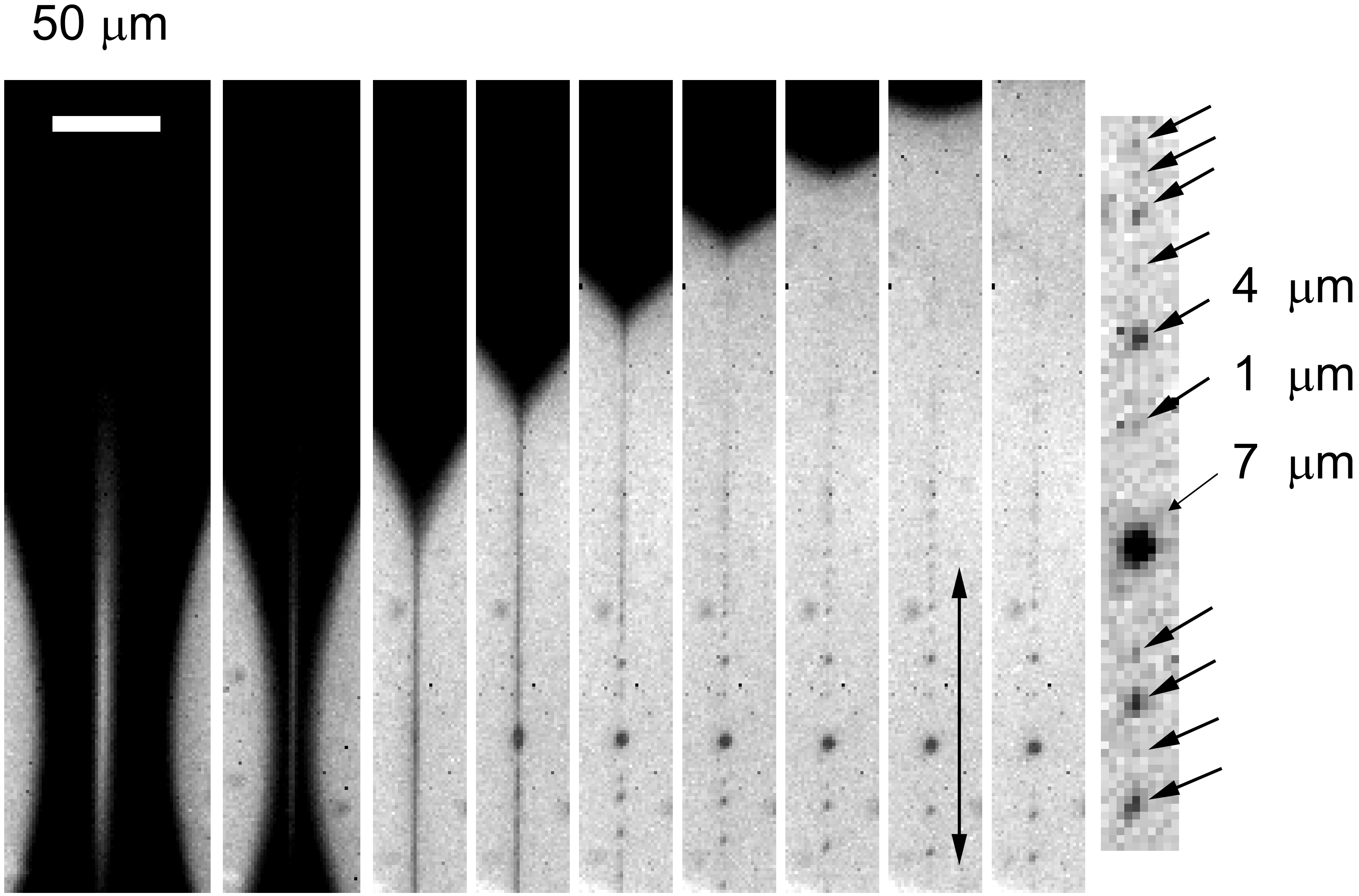
dt = 2 µs

dt = 4 µs



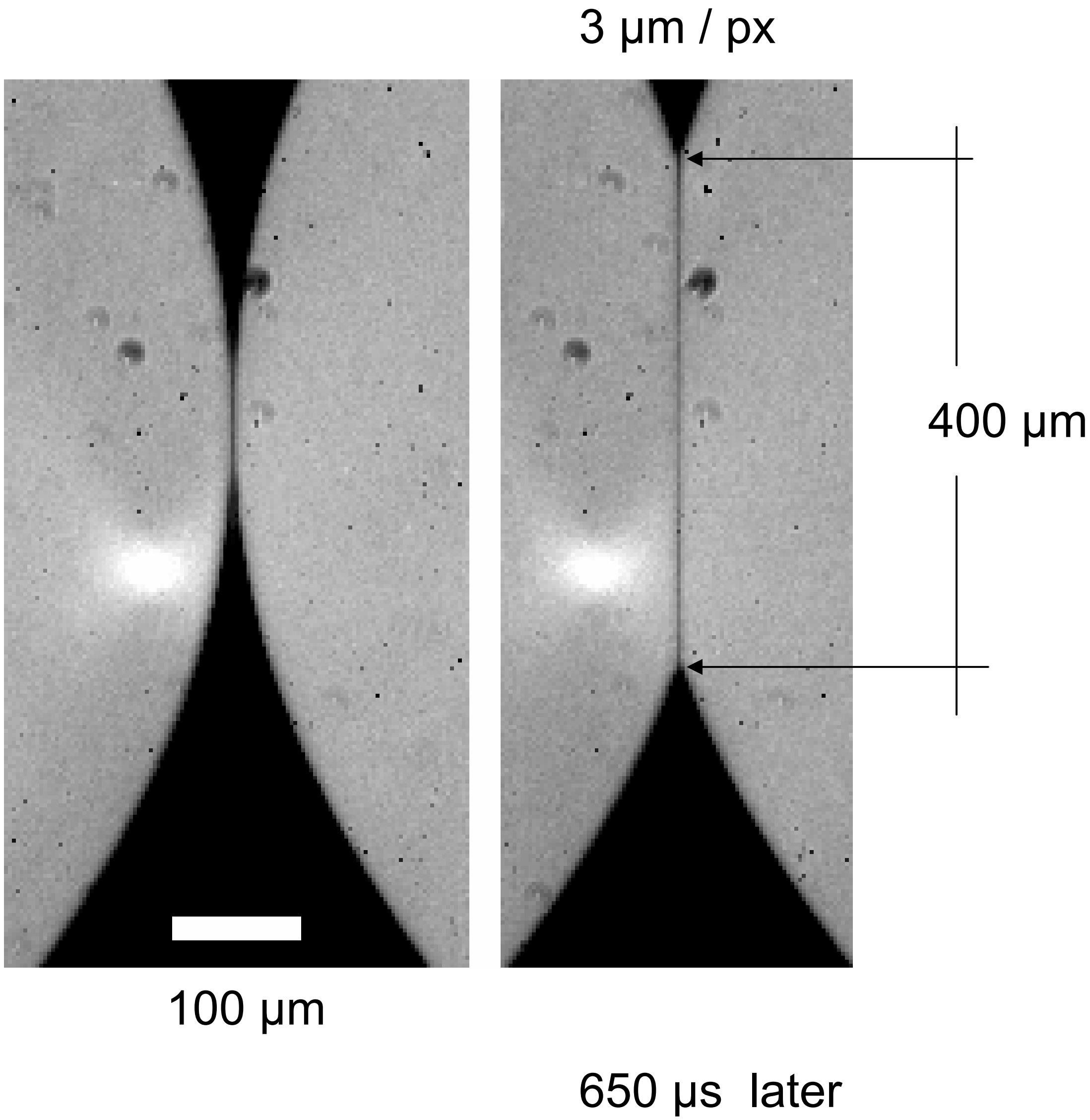
dt = 10 μs

50 times viscosity of water



99% glycerin

μ = 0.80 Pa s



Summary on bubble pinch-off

- Most results consistent with *Burton et al. (2005)*
- Exponent > 1/2. For water = 0.54 - 0.60
- No ‘Rupture’ of air tube in water at 50 µm
- For viscous liquid find ~ 3 µm air tube
- No power laws for intermediate viscosities

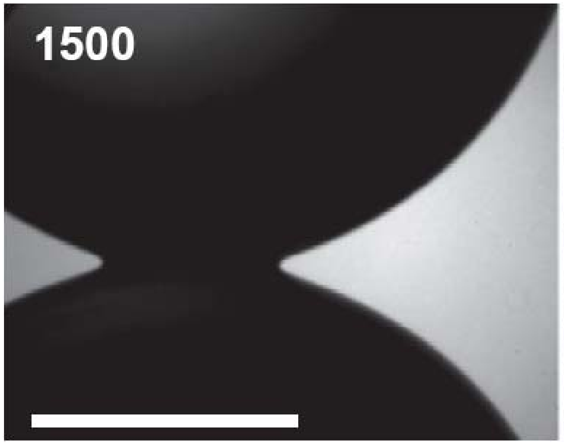
Outline

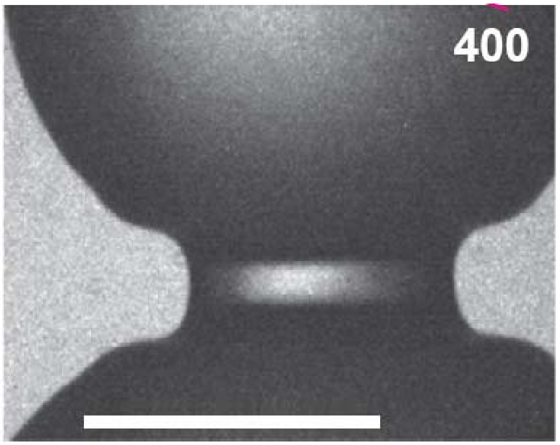
- **1. How fast do free-surface flows move?**
  why 1,000,000 fps?
- **2. Imaging and high-speed camera types**
  high-speed CCD video cameras
- **3. The pinch-off of a drop or a bubble from a nozzle**
  Different dynamics!
- **4. Coalescence of two drops or bubbles**
  or Miscible drops

4. Coalescence of two drops or two bubbles

Viscous

Liquids

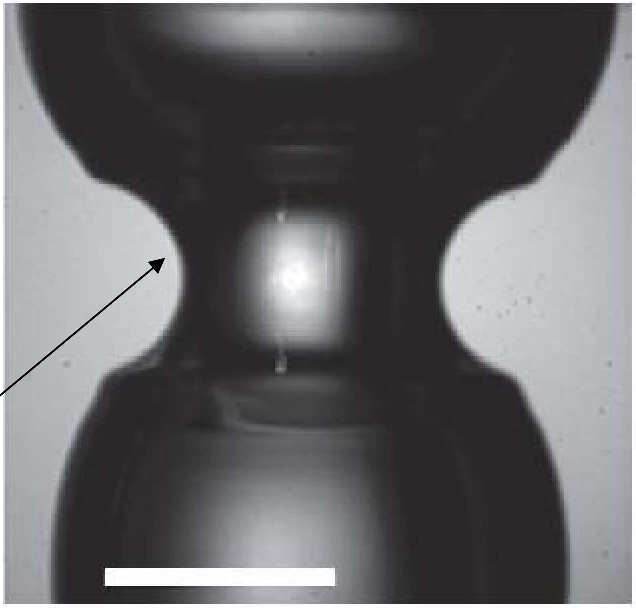




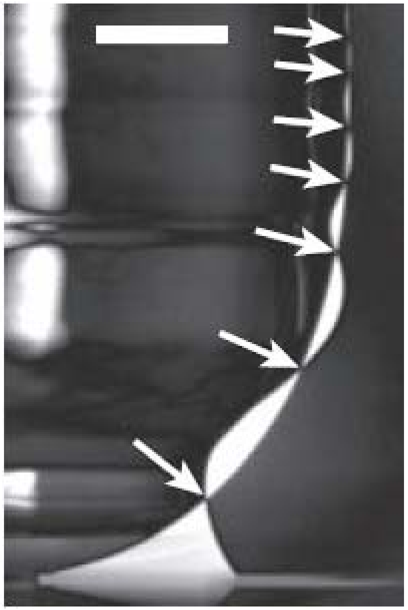
Water

Drops

Bubbles



Perfect arc



Marangoni Waves

Water/Etanol

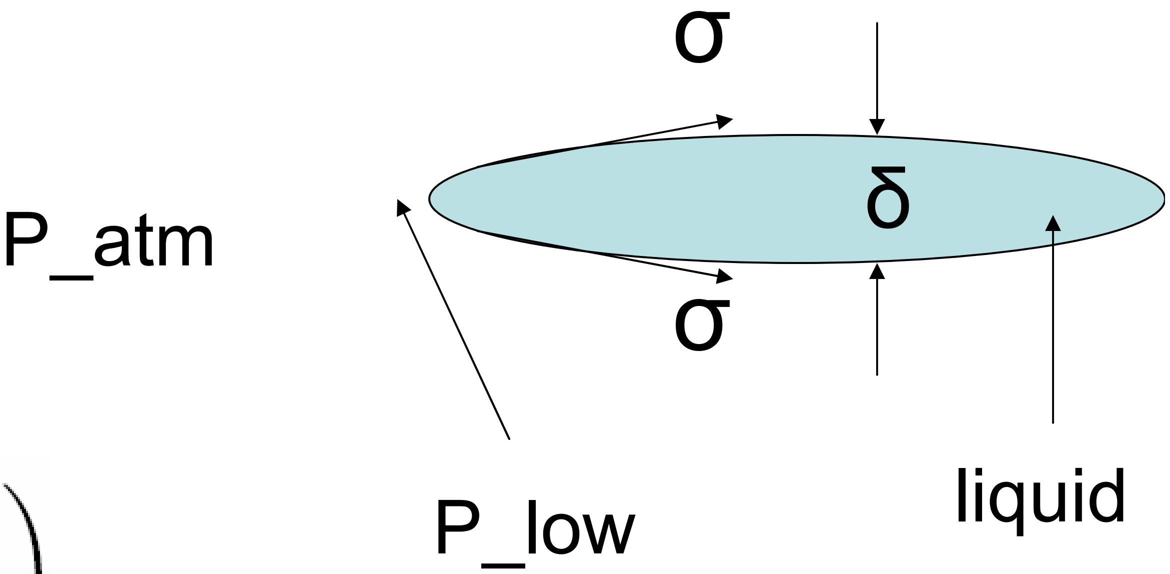
Image difference

What determines the speed of the coalescence?

Simple Capillary-Inertial Motions
Contraction of an air disc

- High surface curvature produces capillary pressure, Young-Laplace
- Inertia or surface tension resists this motion

Surface tension-
Capillary pressure



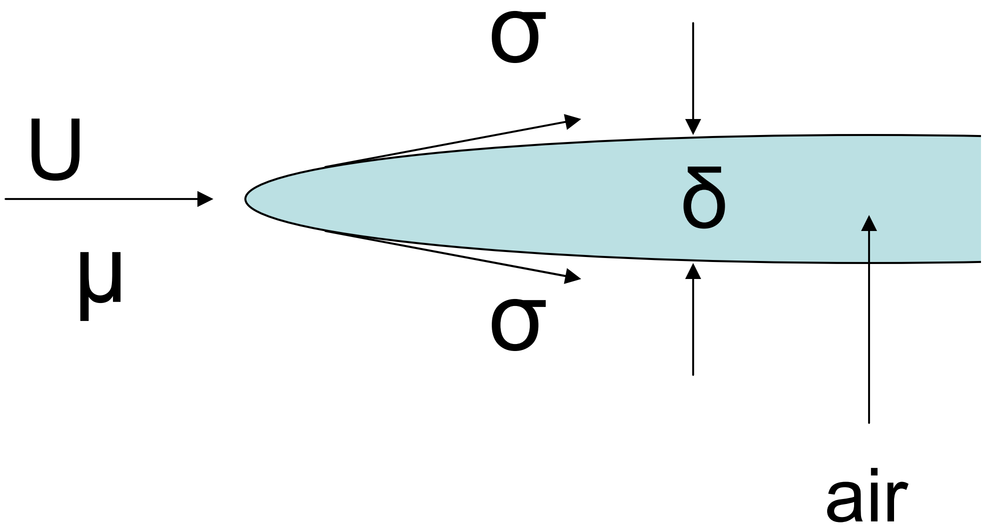
Young-Laplace law

$$\Delta p = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

1 micron bubble in water

$$\frac{2\sigma}{R} = \frac{2 \times 0.073}{0.5 \times 10^{-6}} = 2.9 \times 10^5 N/m^2 \simeq 3 \; P_{atm}$$

Surface tension-viscous balance

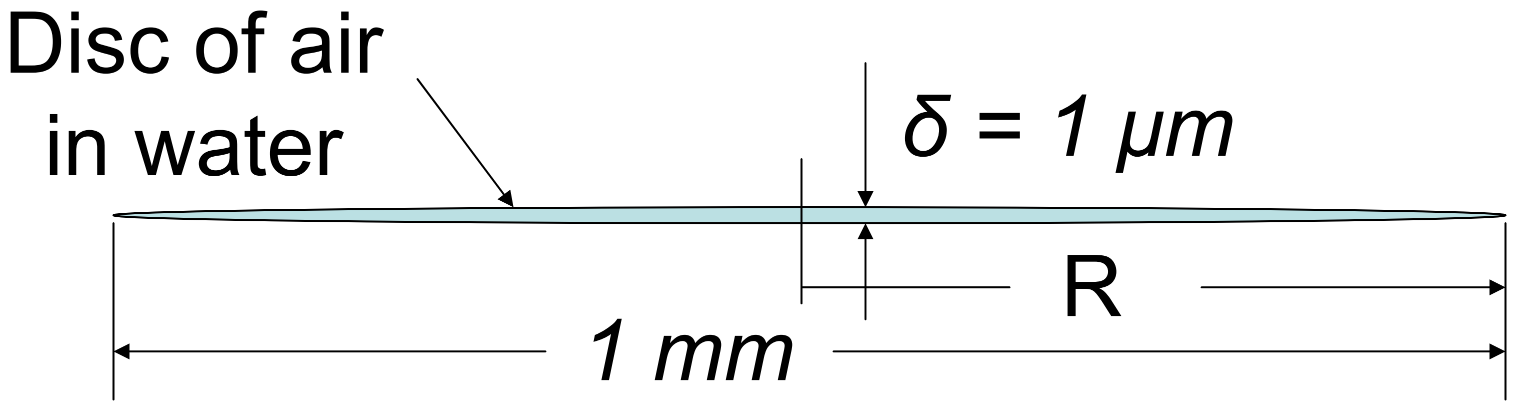


- Viscous force $\sim \mu\, U / \delta$
- Surface tension $\sim\ \sigma / \delta$
- Become equal when $\mu\, U / \delta = \sigma / \delta$

$$U = \sigma / \mu \quad or \quad Ca = U\mu/\sigma = 1$$

- For water $\sigma = 0.073\ N/m$
- Dynamic viscosity $\mu = 0.001\ Pa\ s$

➔ $\underline{U = 73\ m/s!}$

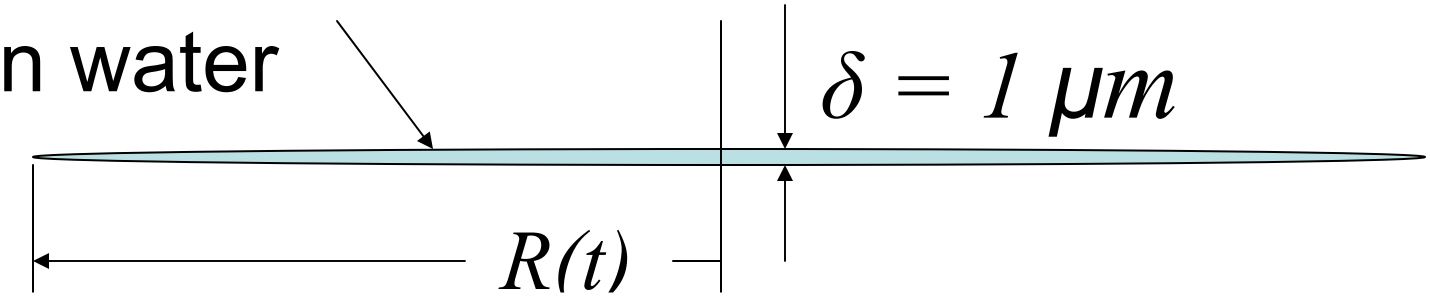


- Including inertia, dynamic pressure ~ ρU^2
- Capillary pressure ~ σ / δ
- In balance $U \sim C\, (\sigma / \rho\delta)^{1/2}$
- Disc thickness: $\delta\, \pi R^2 = Vol. \quad \Rightarrow \quad \delta = Vol. / \pi R^2$

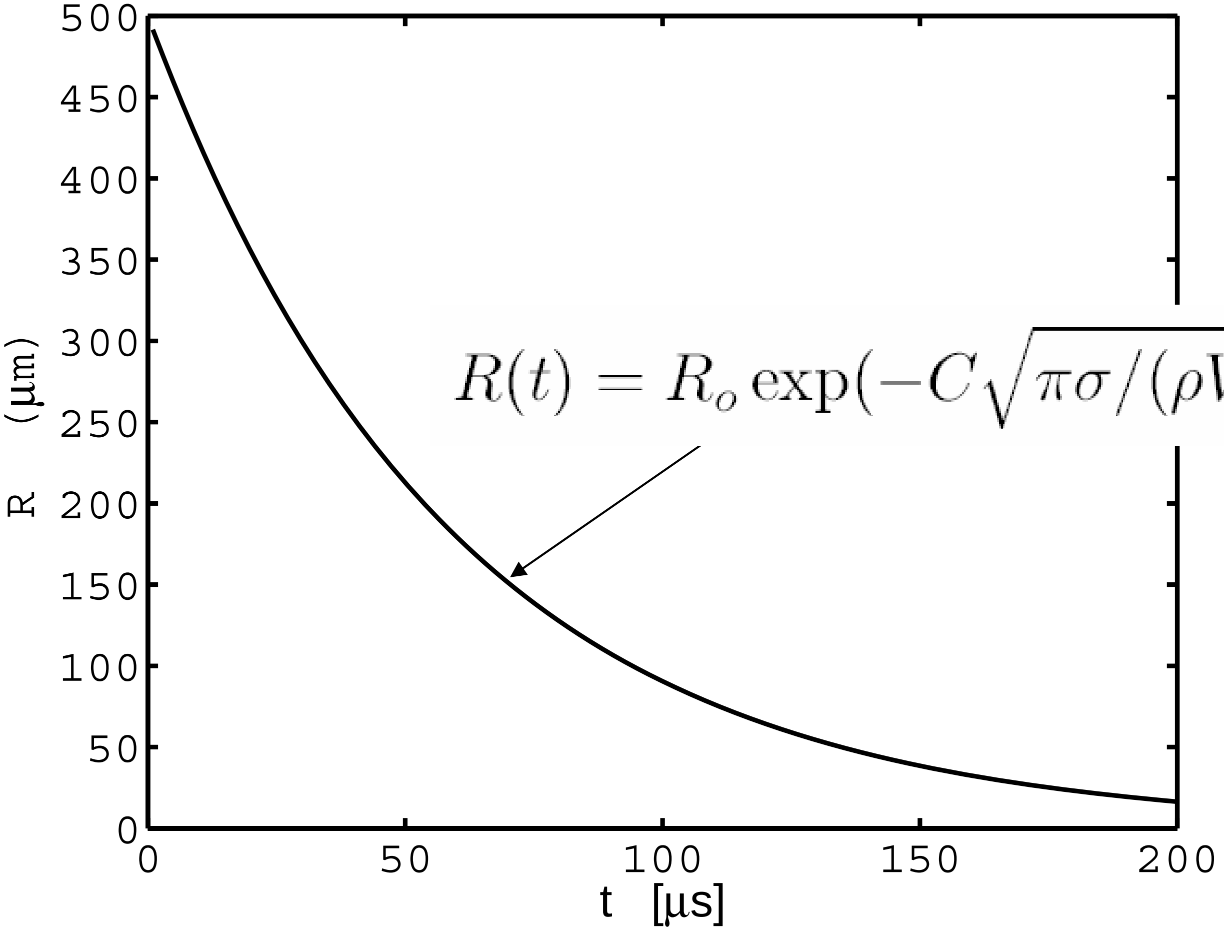
$$U = -dR/dt = (\sigma / \rho\delta\,)^{1/2} \quad = (\sigma\, \pi R^2 / \rho\, Vol.)^{1/2}$$

$$-dR/dt = (\sigma\, \pi / \rho\, Vol.)^{1/2}\ \ R$$

$$\underline{R = R_o\, exp[-(\sigma\, \pi / \rho\, Vol.)^{1/2}\ ]\ \ t}$$



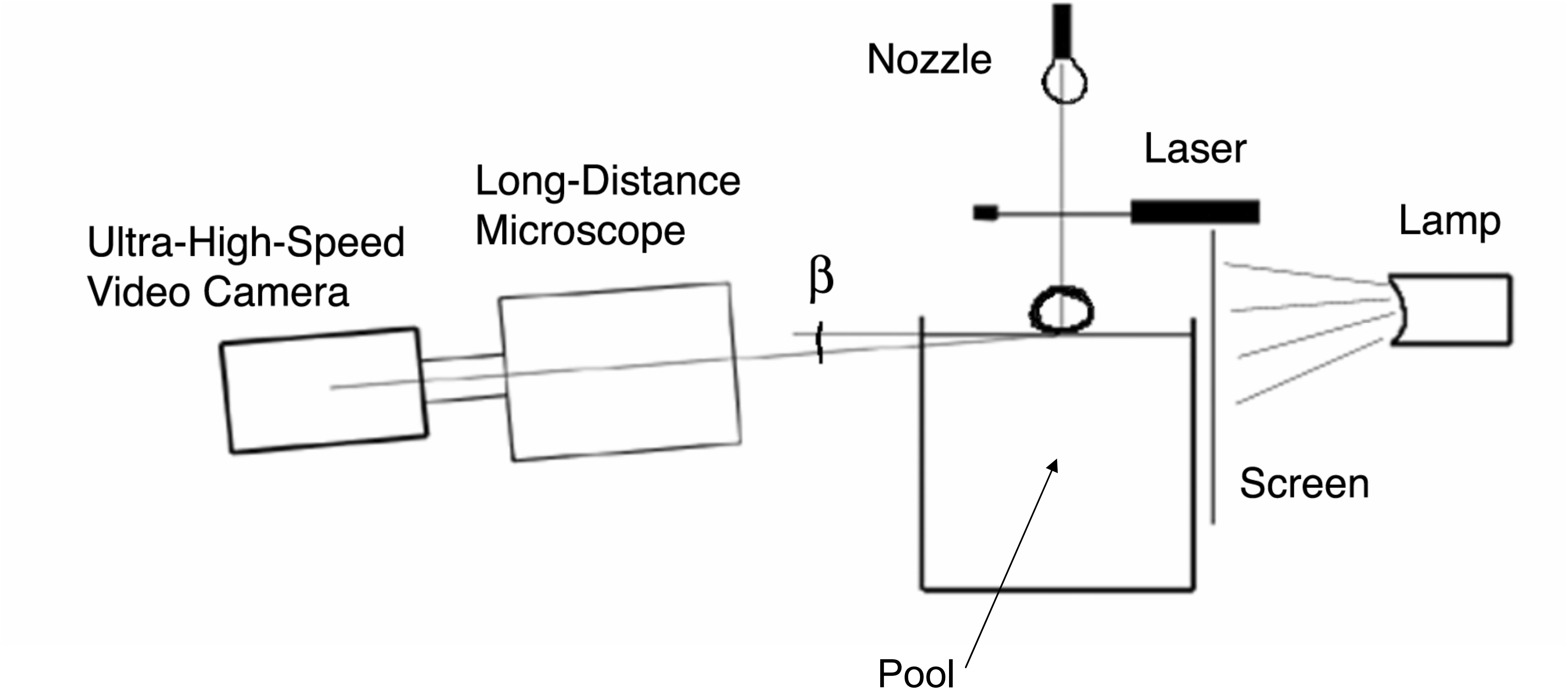
Disc of air
in water

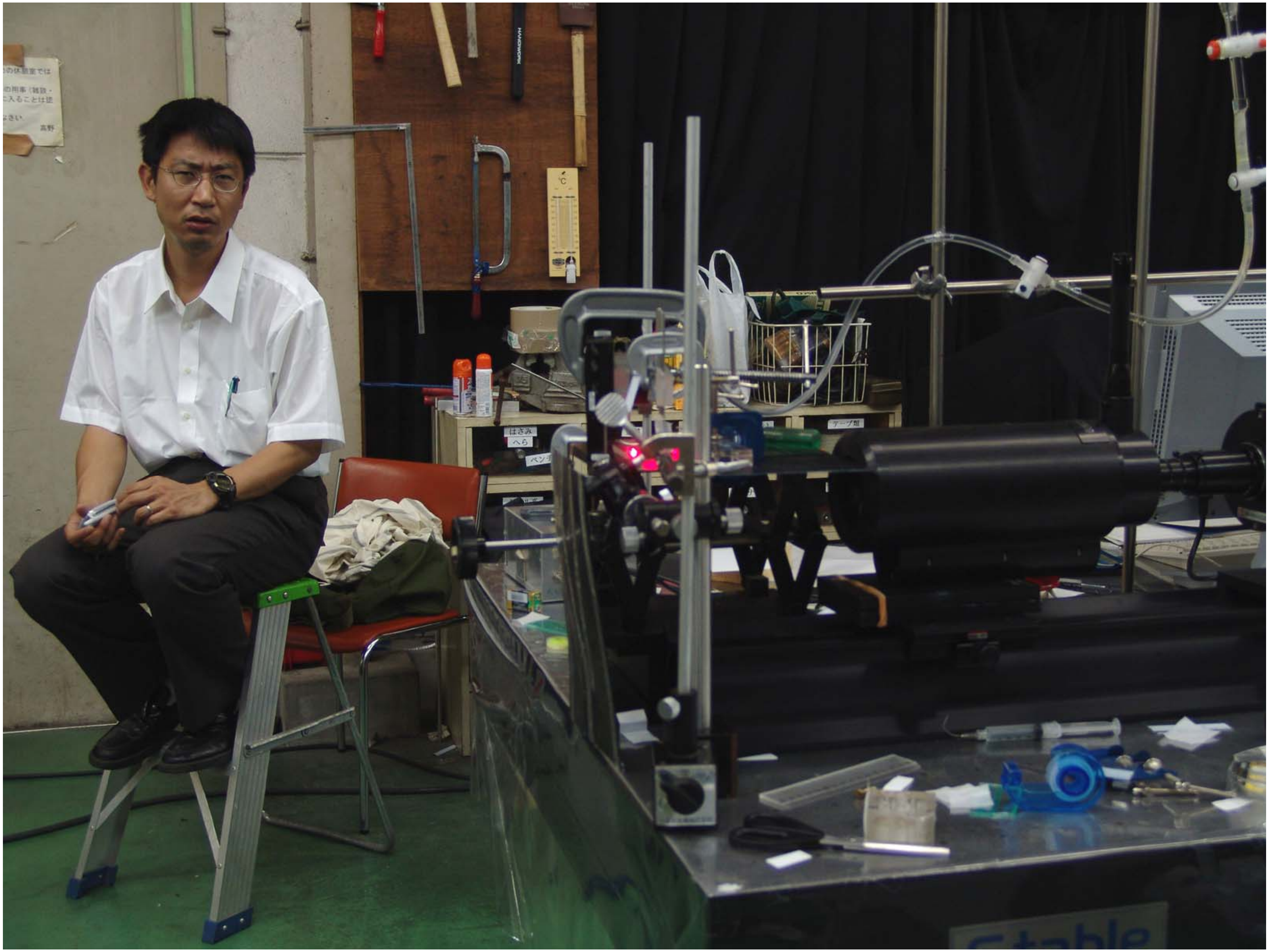


$$R(t) = R_o \exp(-C\sqrt{\pi\sigma/(\rho Vol.)}\ t)$$

⇒ 50 000 fps

Experimental Setup

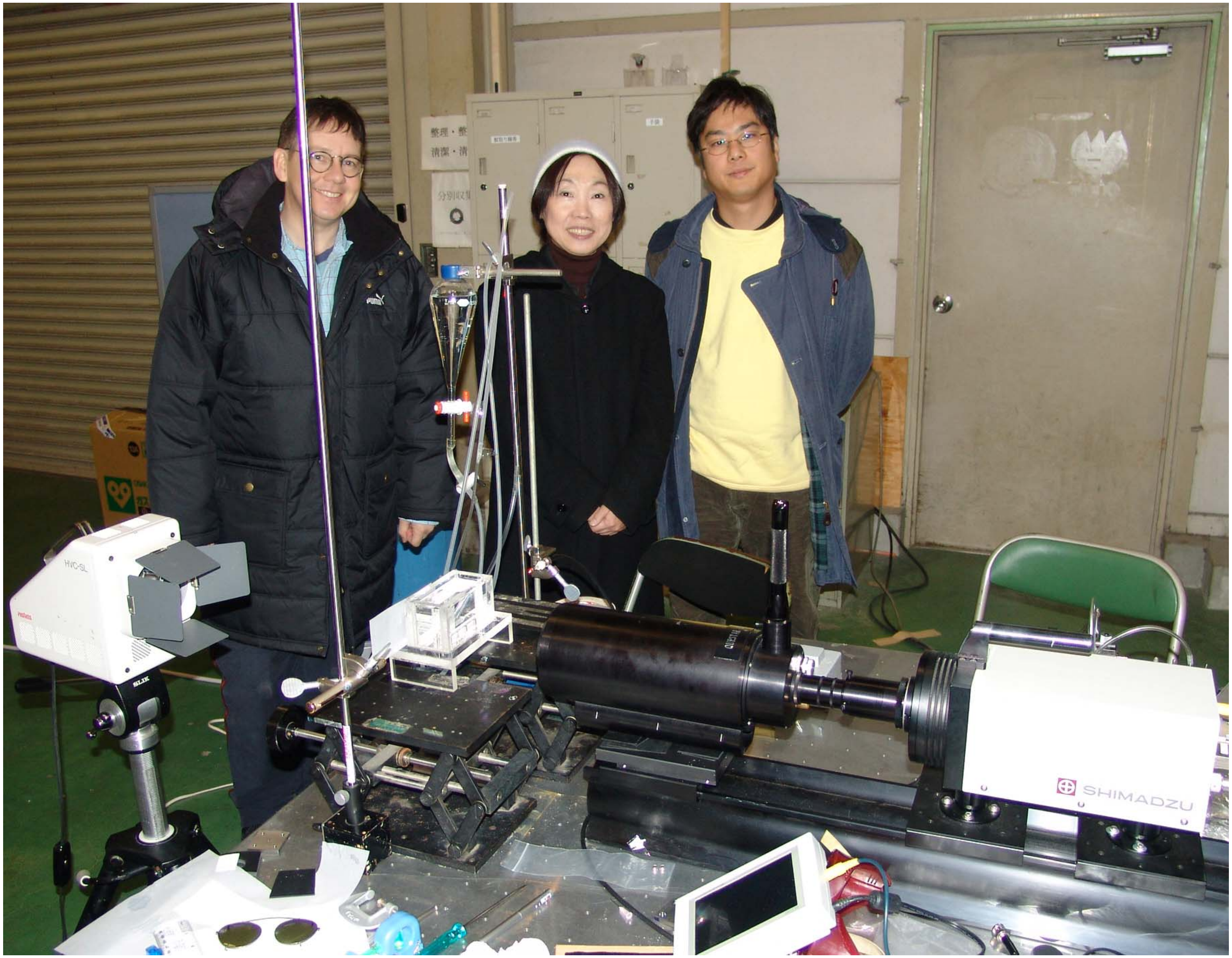



はさみ
へら
テープ類
Stable

Guchi

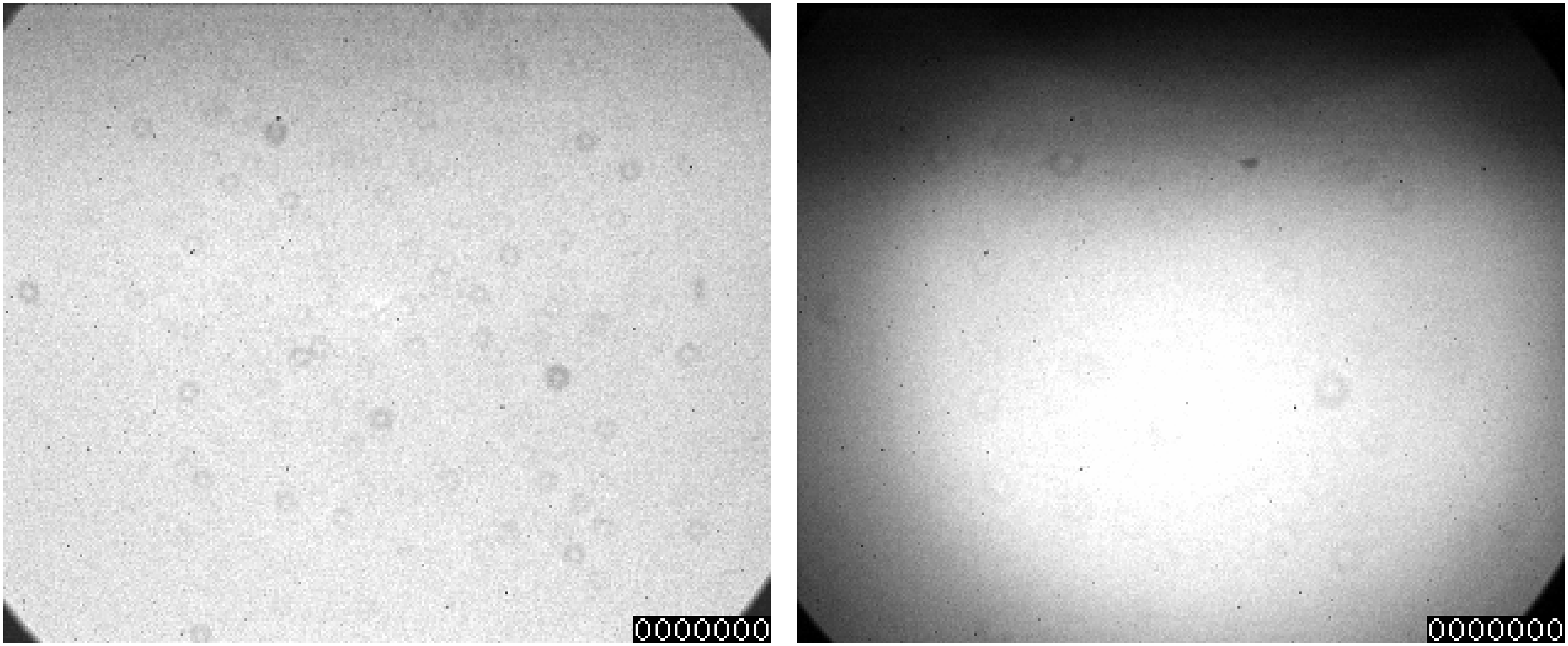
Masa

Osaka, July, 35°C

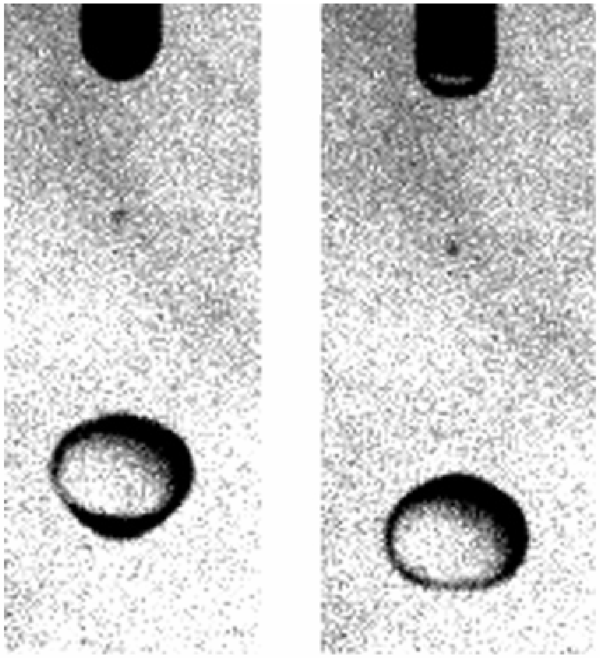


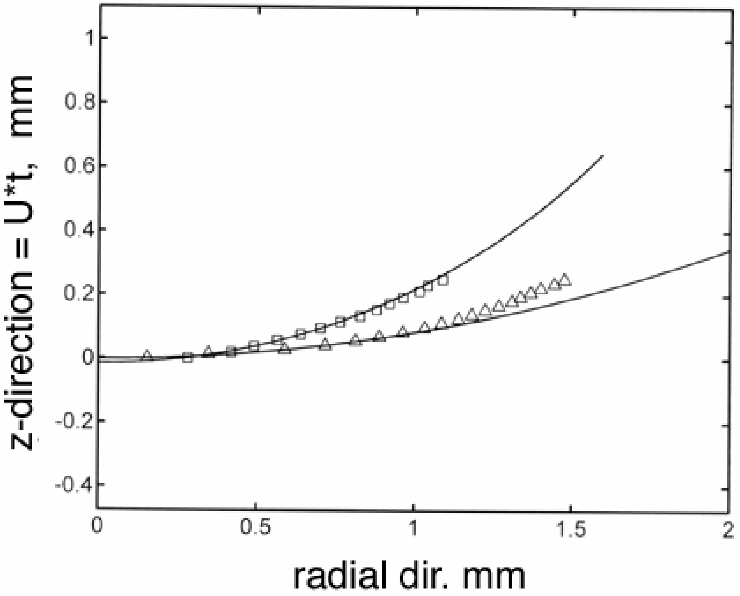
Civil Engineering, Kinki University, Osaka, Japan, 5° C

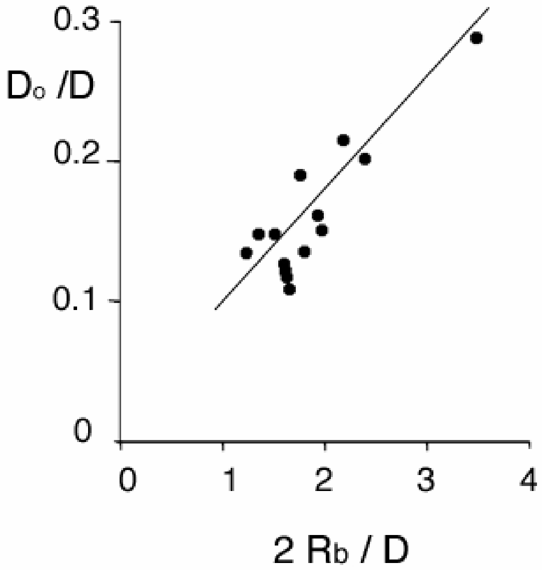
Air entrapment

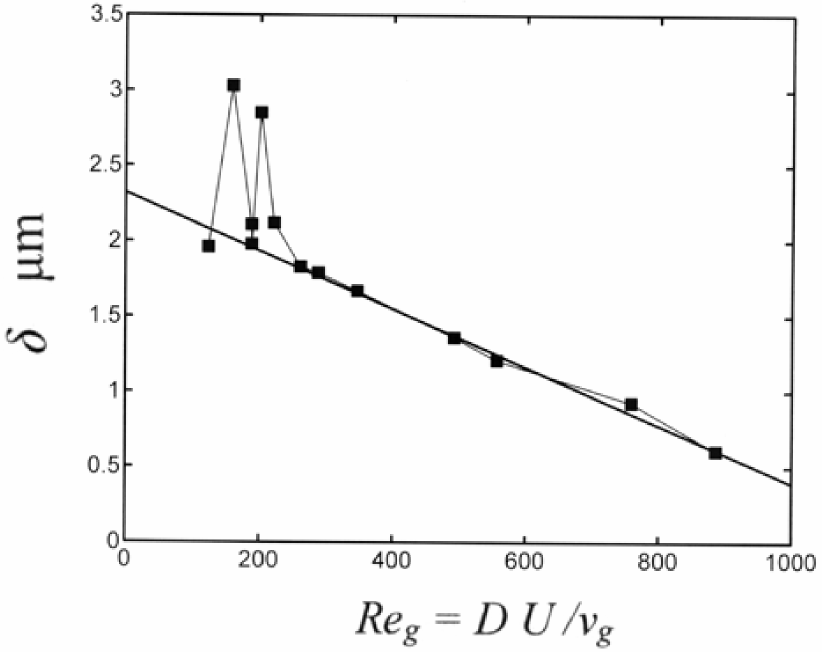


Bubbles 30 – 50 µm

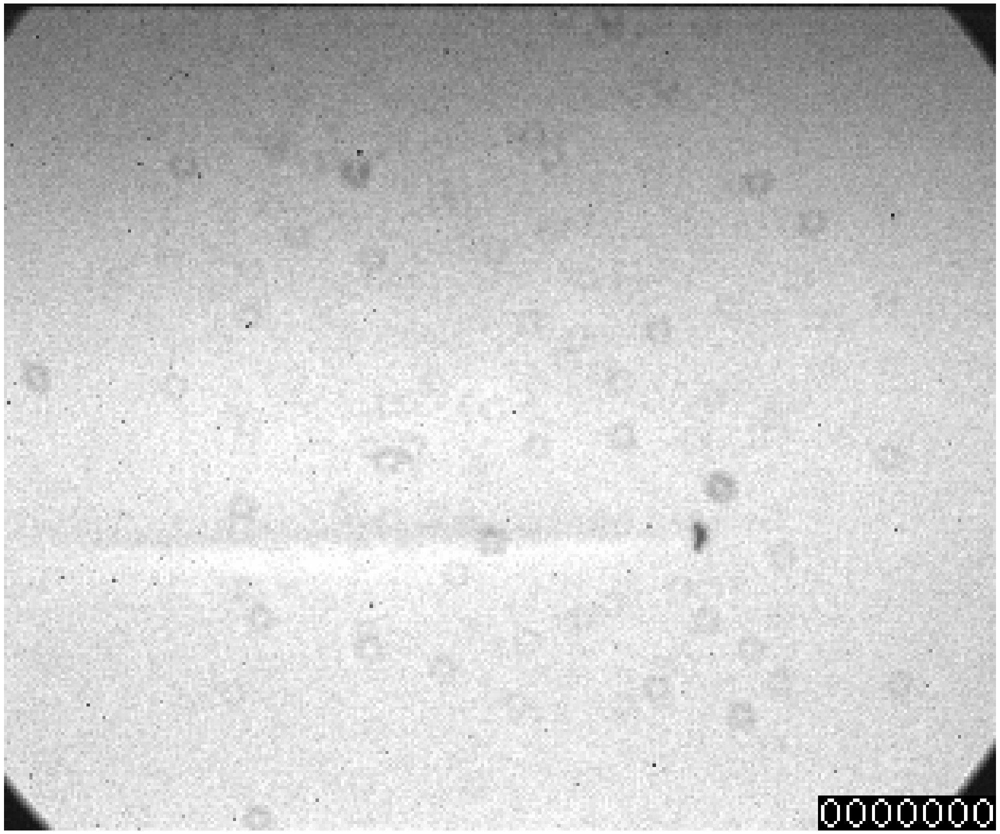


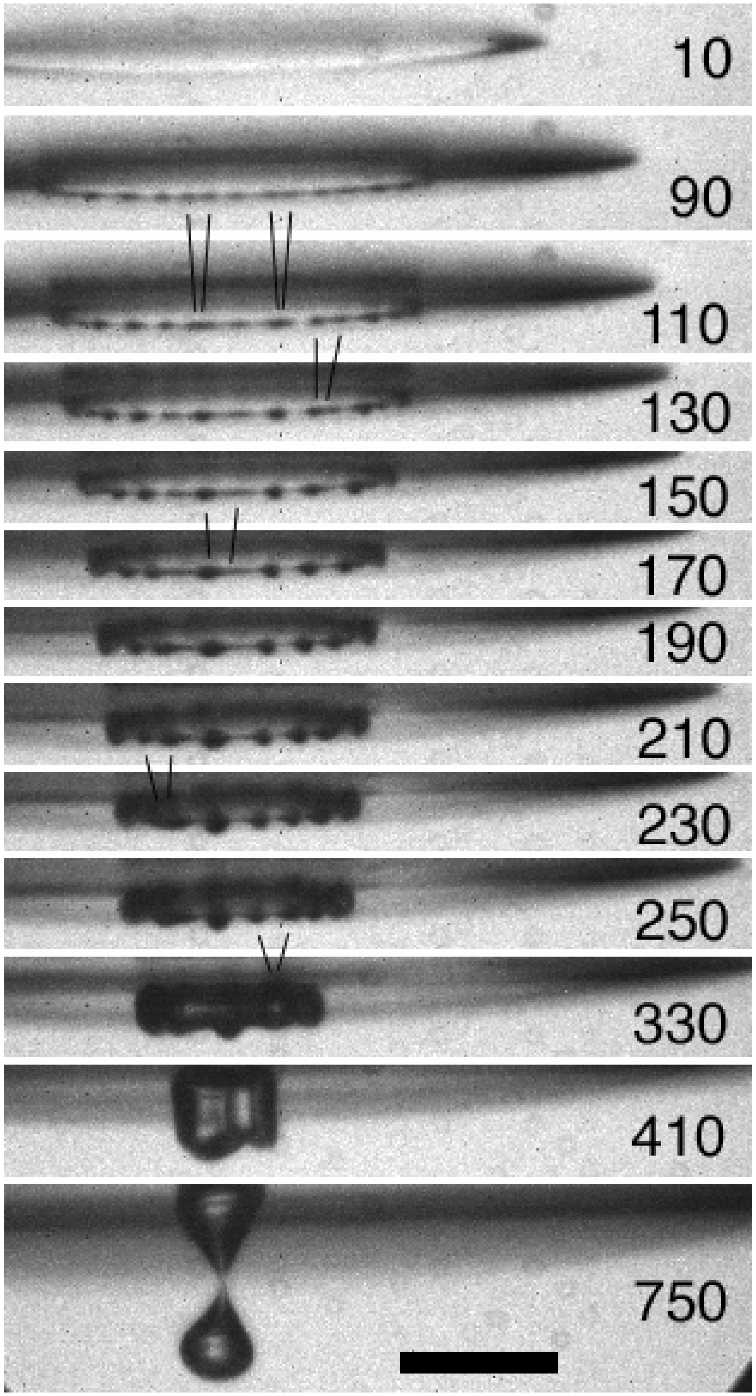

z-direction = U*t, mm
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
0.5
1
1.5
2
radial dir. mm


0.3
D_o/D
0.2
0.1
0
0
1
2
3
4
$2R_b/D$


3.5
3
2.5
2
1.5
1
0.5
0
δ μm
0
200
400
600
800
1000
$Re_g = D\,U/\nu_g$

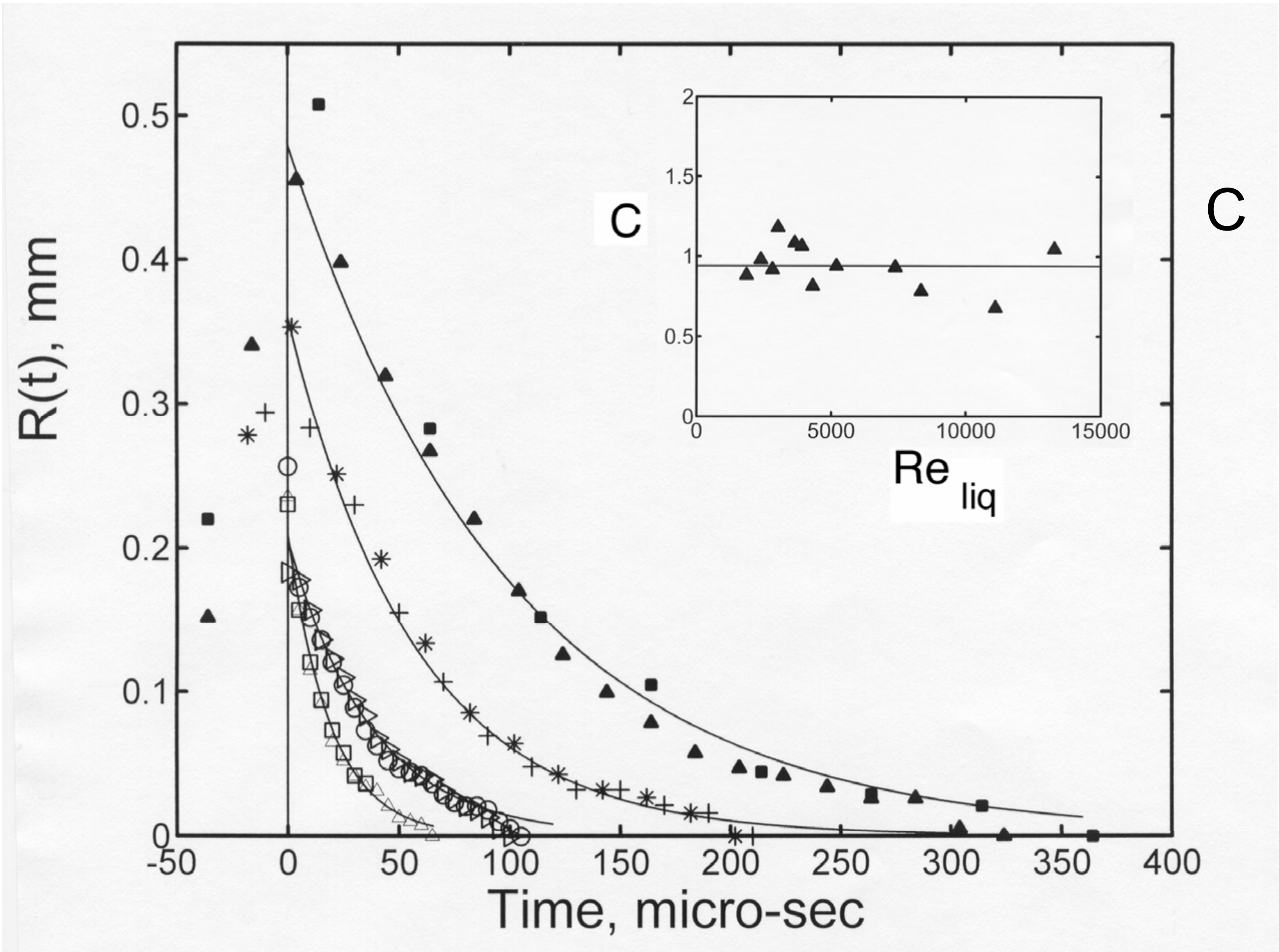
Thoroddsen, Etoh & Takehara,
***JFM*, 478 (2003)**





µs →

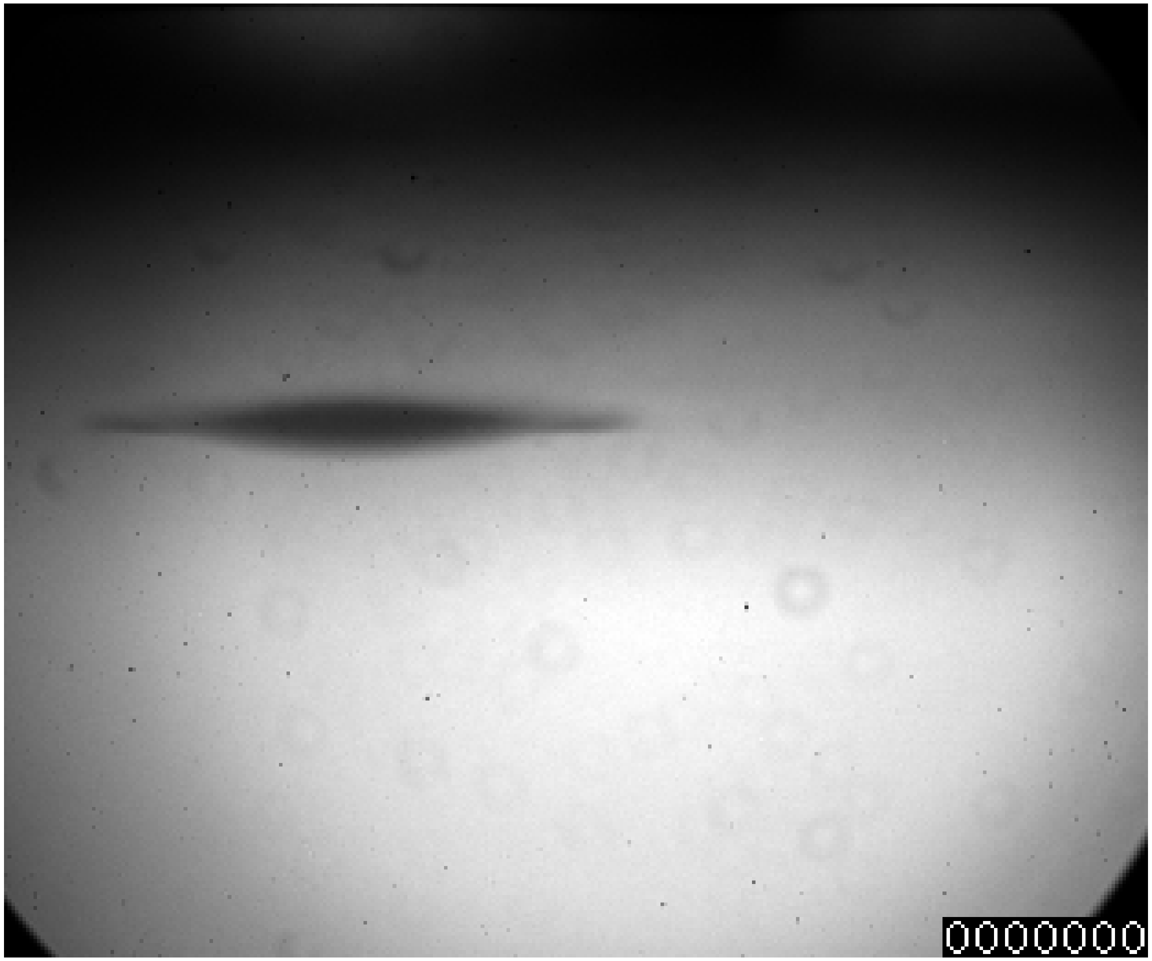
500 µm →



C ≈ 0.94

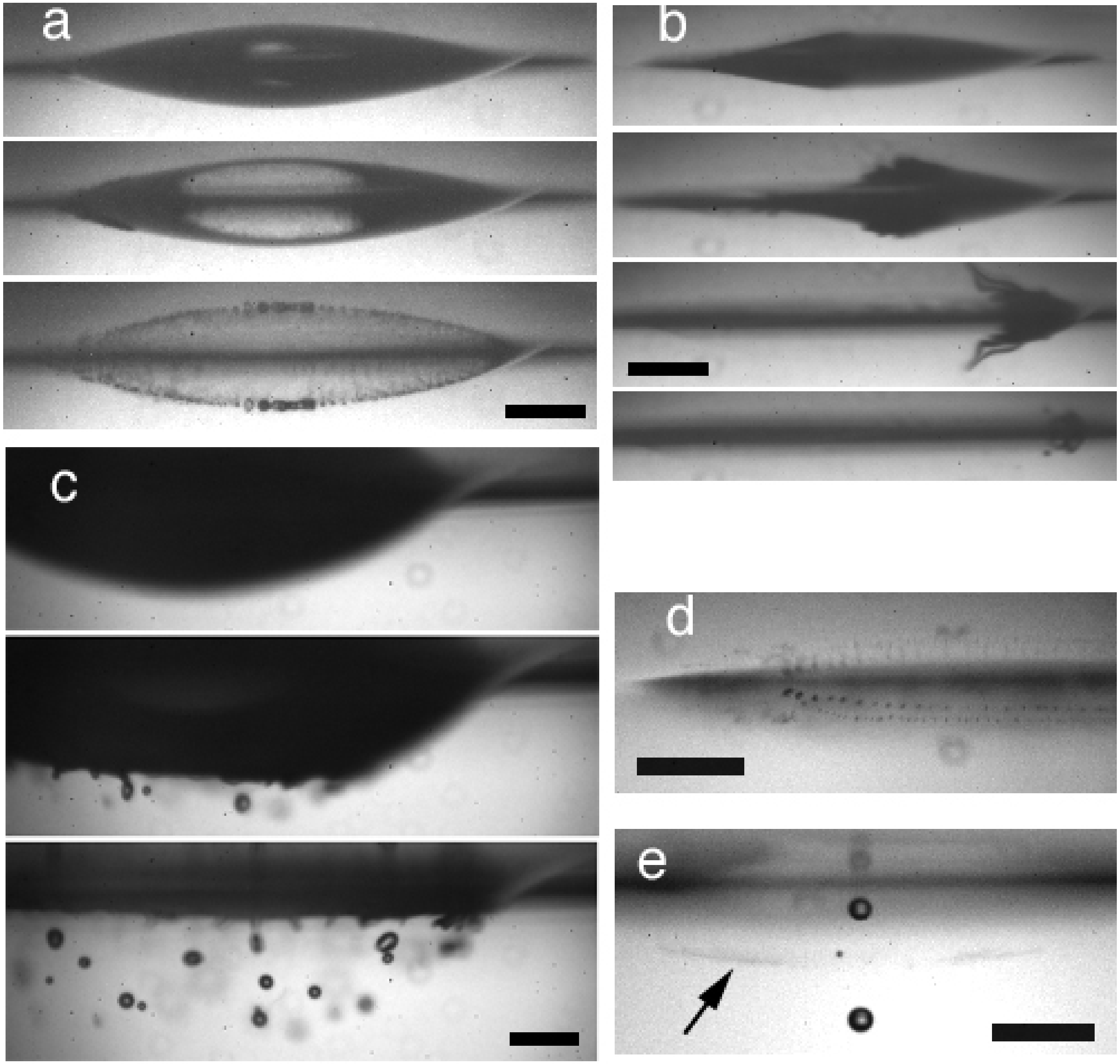
$$R(t) = R_o \exp(-C\sqrt{\pi\sigma/(\rho Vol.)}\ t)$$

Thin film under drop



Low Re / We

Bars
0.5 mm

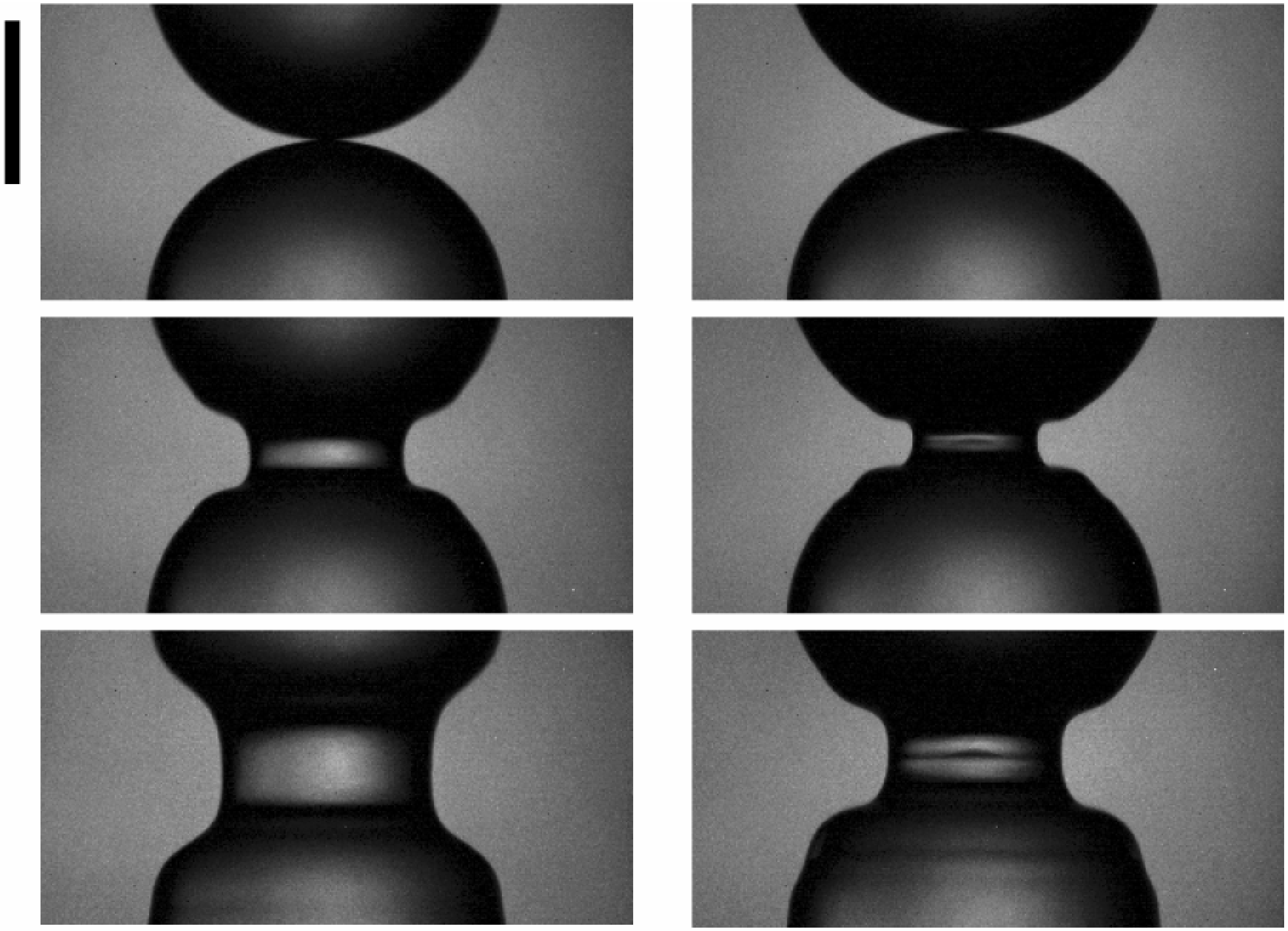


Coalescence of Miscible Drops

- Water drop coalescing with
  an ethanol drop (lower σ)

What controls the speed of coalescence?

Thoroddsen, Qian, Etoh & Takehara, (2007)
The initial coalescence of miscible drops,
Phys. Fluids, **19**, 072110



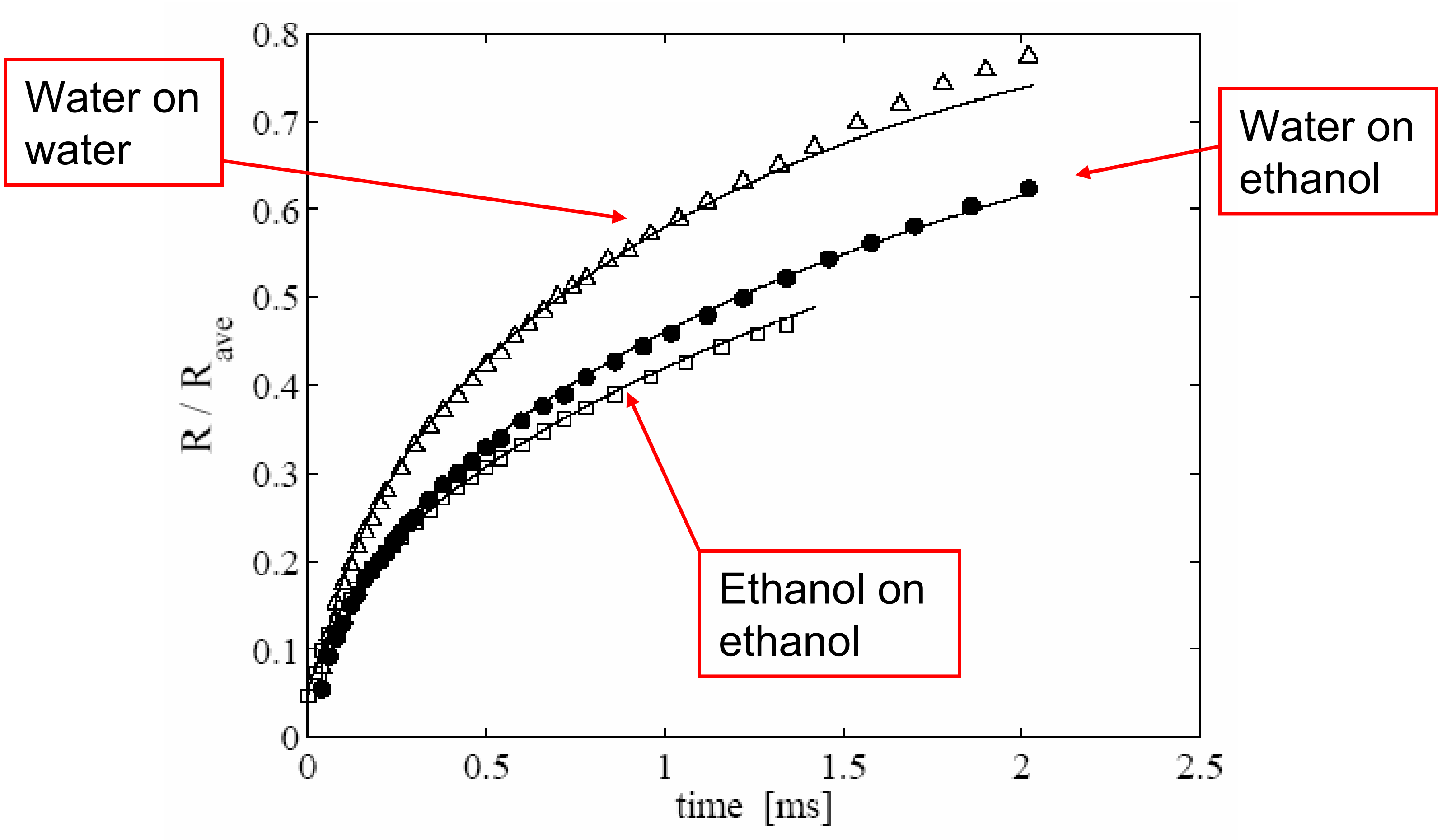
Water on water

Ethanol on top

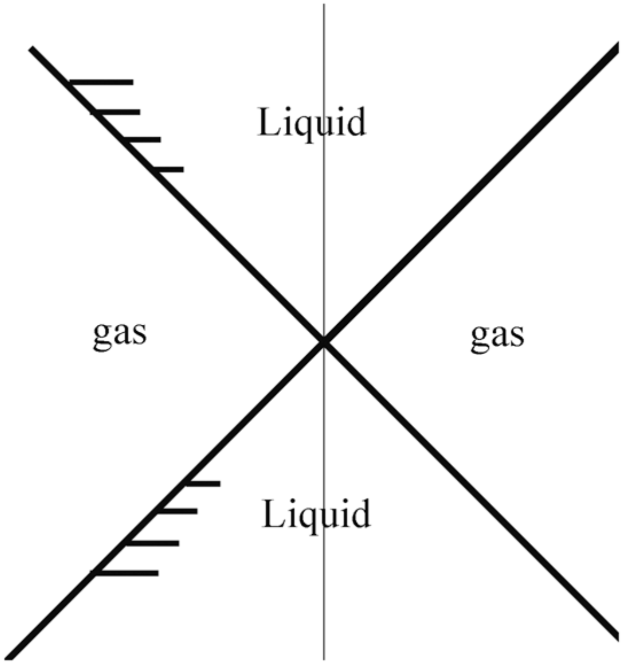
Water on bottom

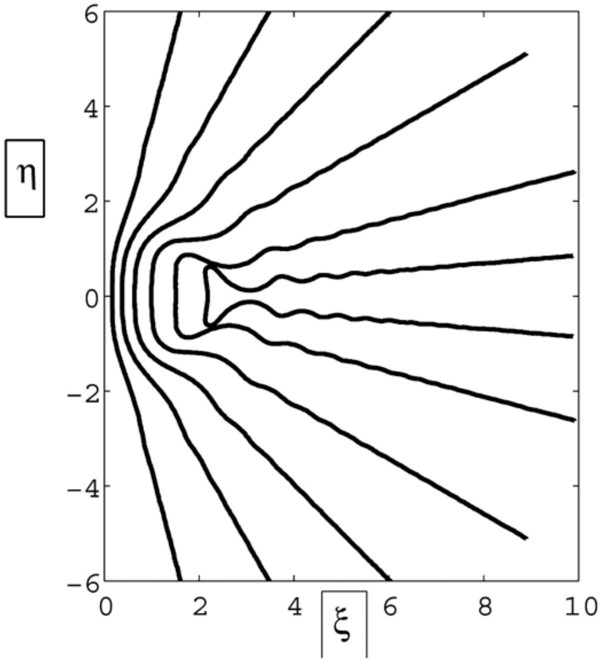
Speed of coalescence

Is determined by the lower surface tension



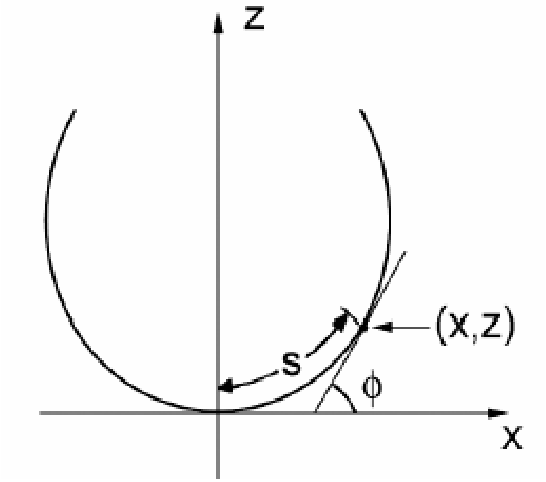
Selfsimilar Marangoni waves





Wedges or Cones

Self-similar geometry



Water drop coalescing with an ethanol pool

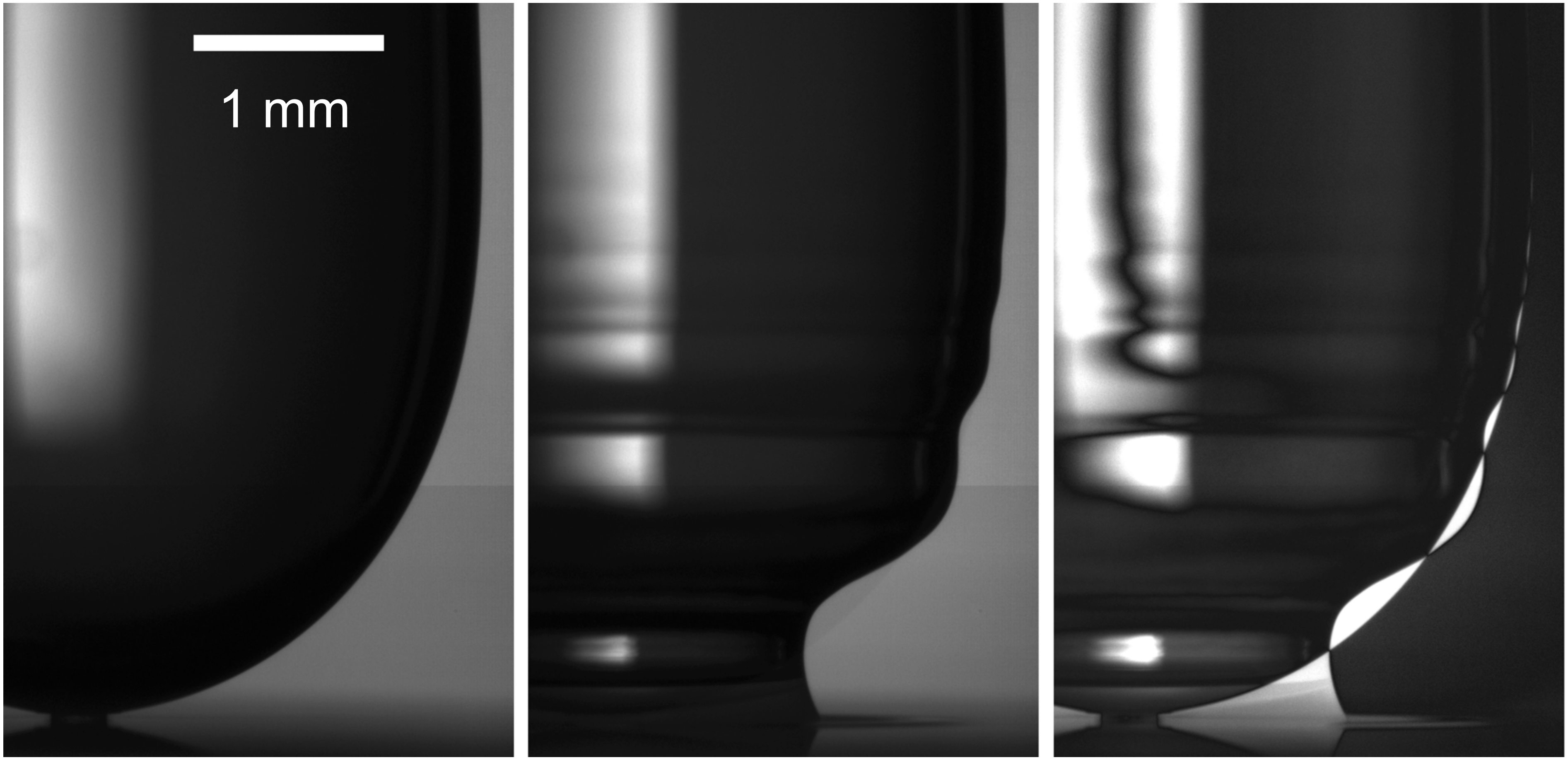
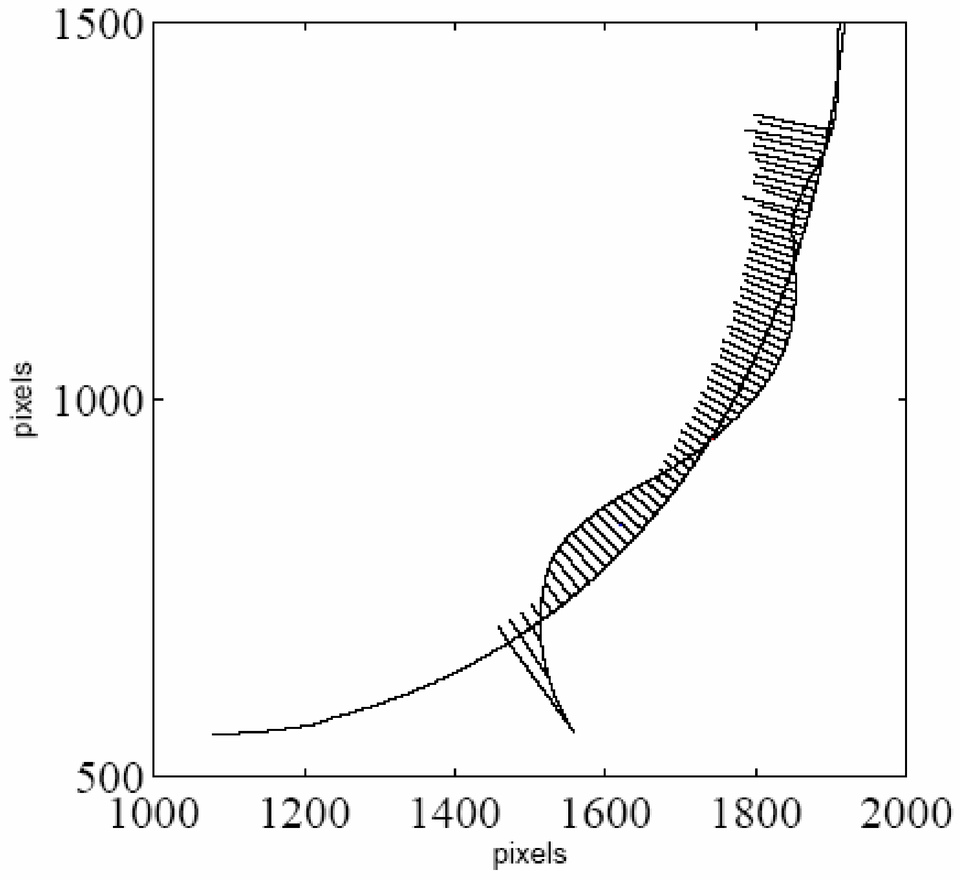
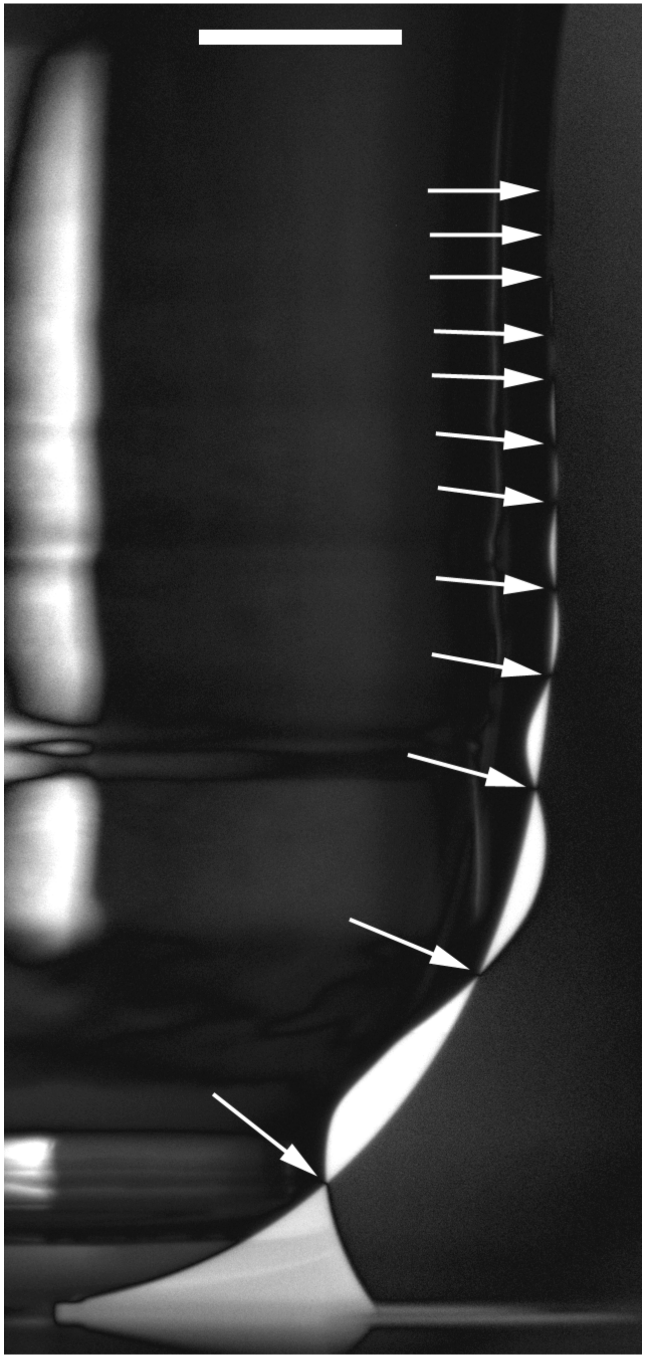


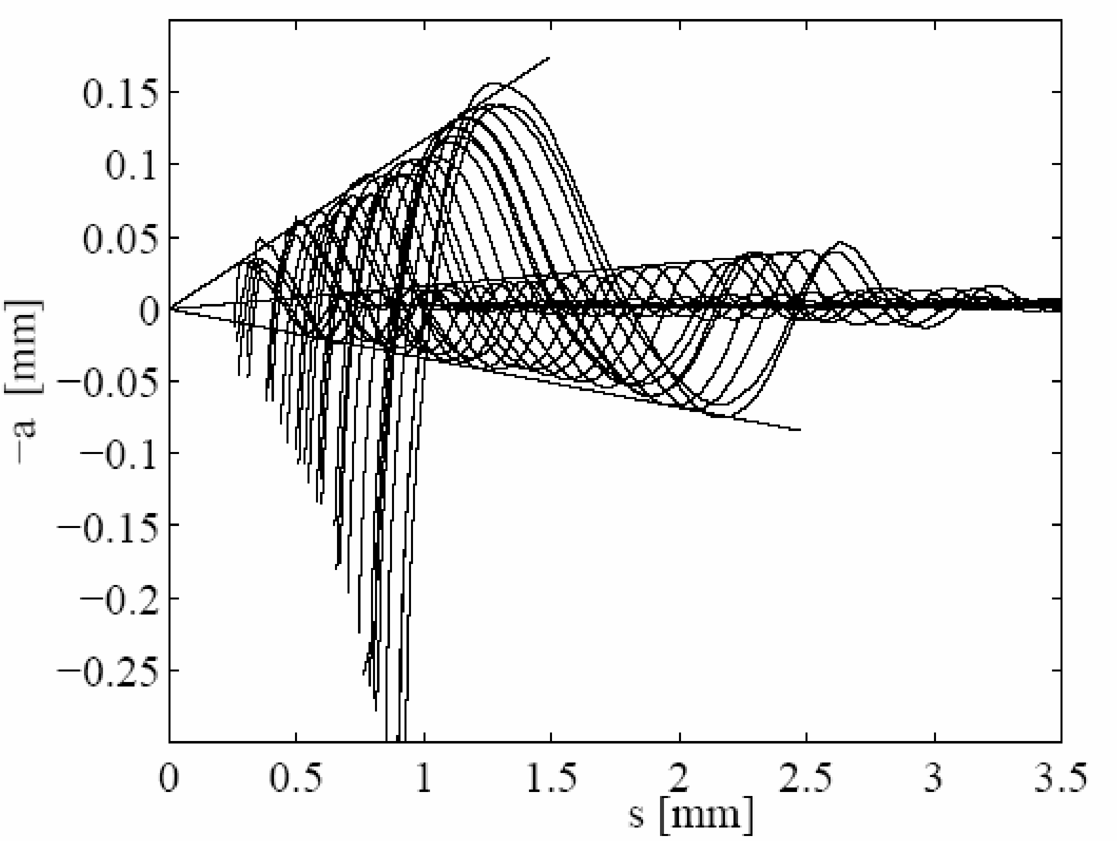
Image difference

Extracting

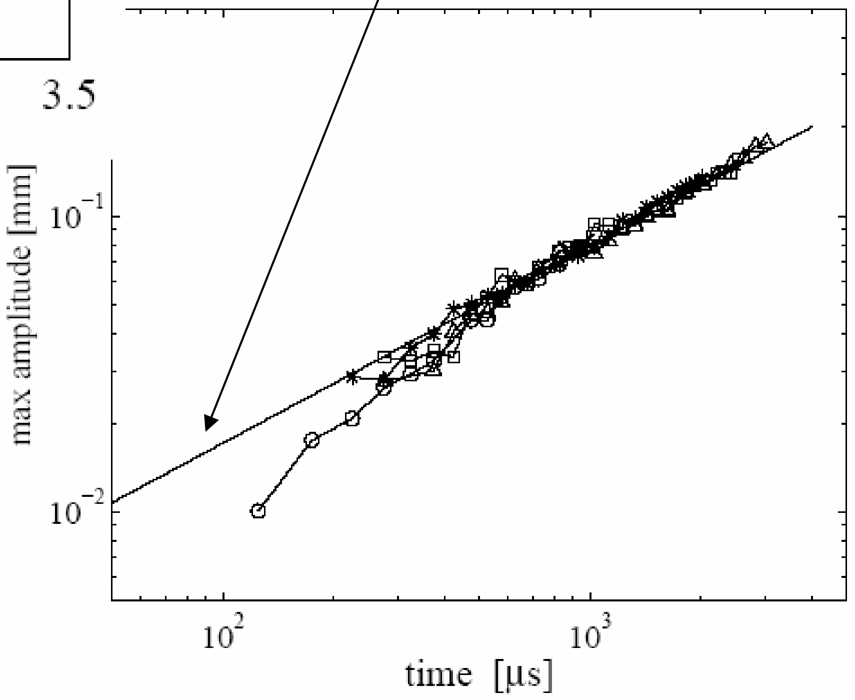
Wave amplitudes



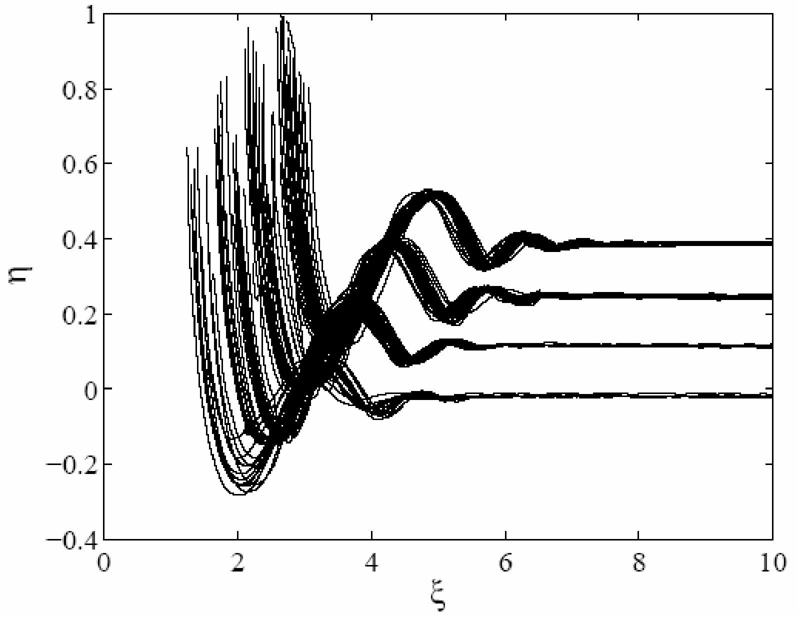


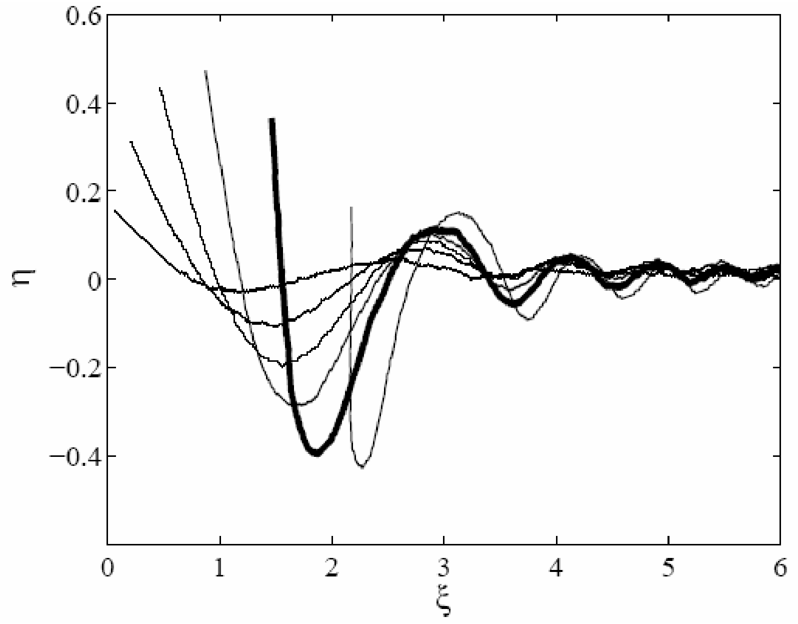

0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
−0.25
−a [mm]
0
0.5
1
1.5
2
2.5
3
3.5
s [mm]

$R \sim t^{2/3}$

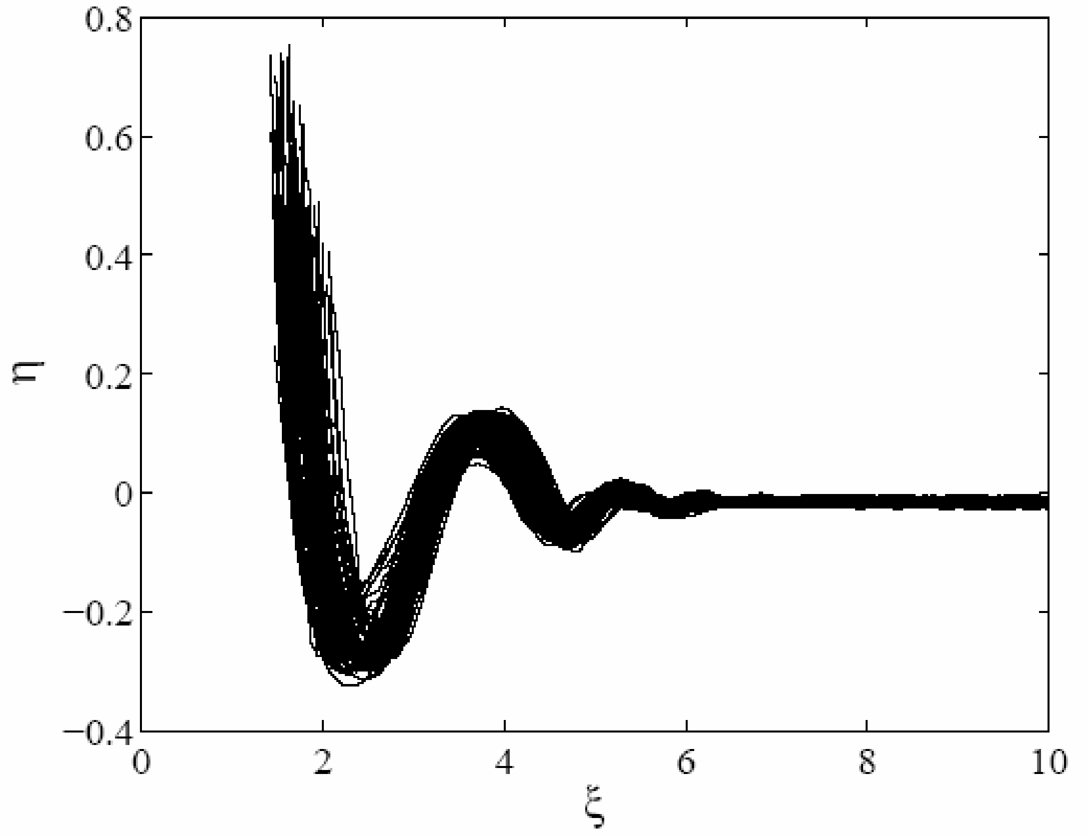

10^{-1}
10^{-2}
max amplitude [mm]
10^{2}
10^{3}
time [µs]

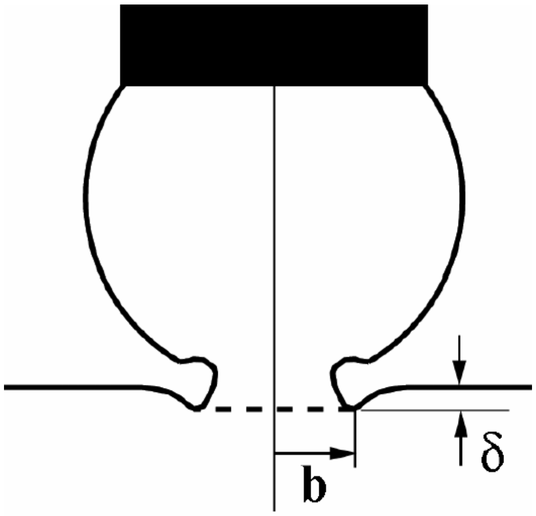
Self-similar Shapes

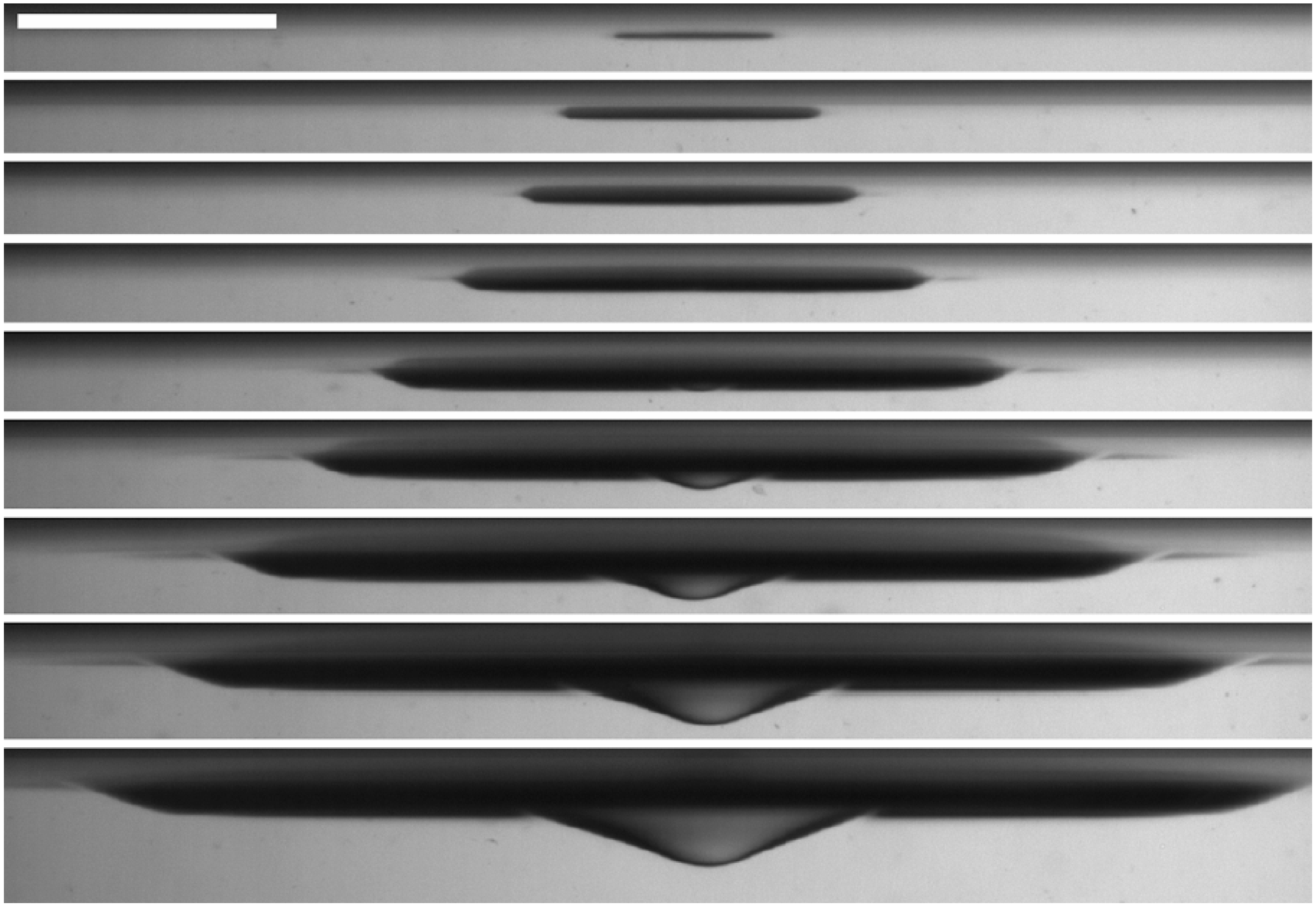




Keller & Miksis (1983)







Water cone

Entering ethanol pool

Overview and Conclusions

- Ultra-high-speed imaging required for small drops/bubbles
- Time-resolved imaging of pinch-off and coalescence
- Interesting capillary-inertial dynamics below to 1 µm length scale
- Possible dynamics below optical limit?