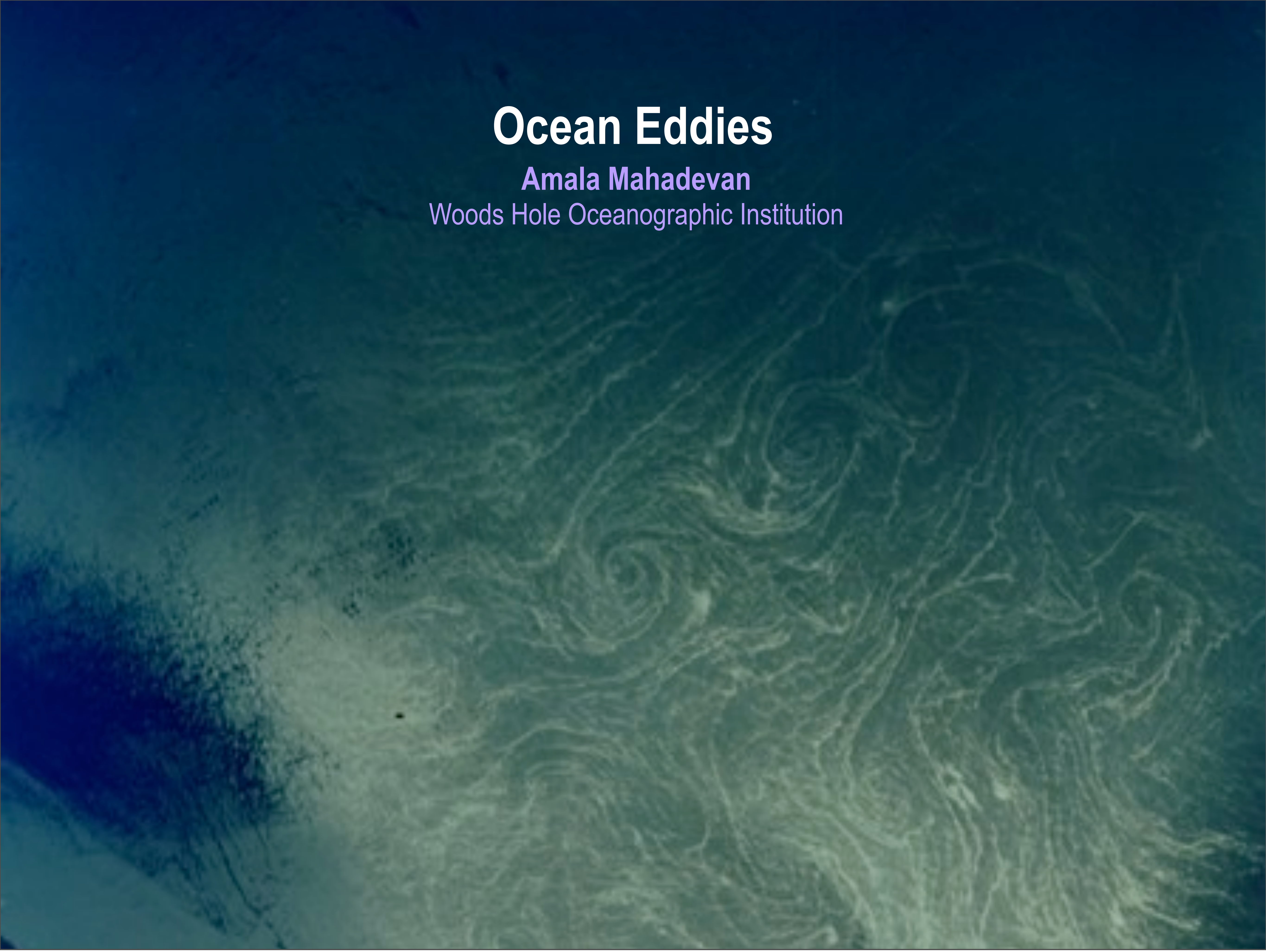
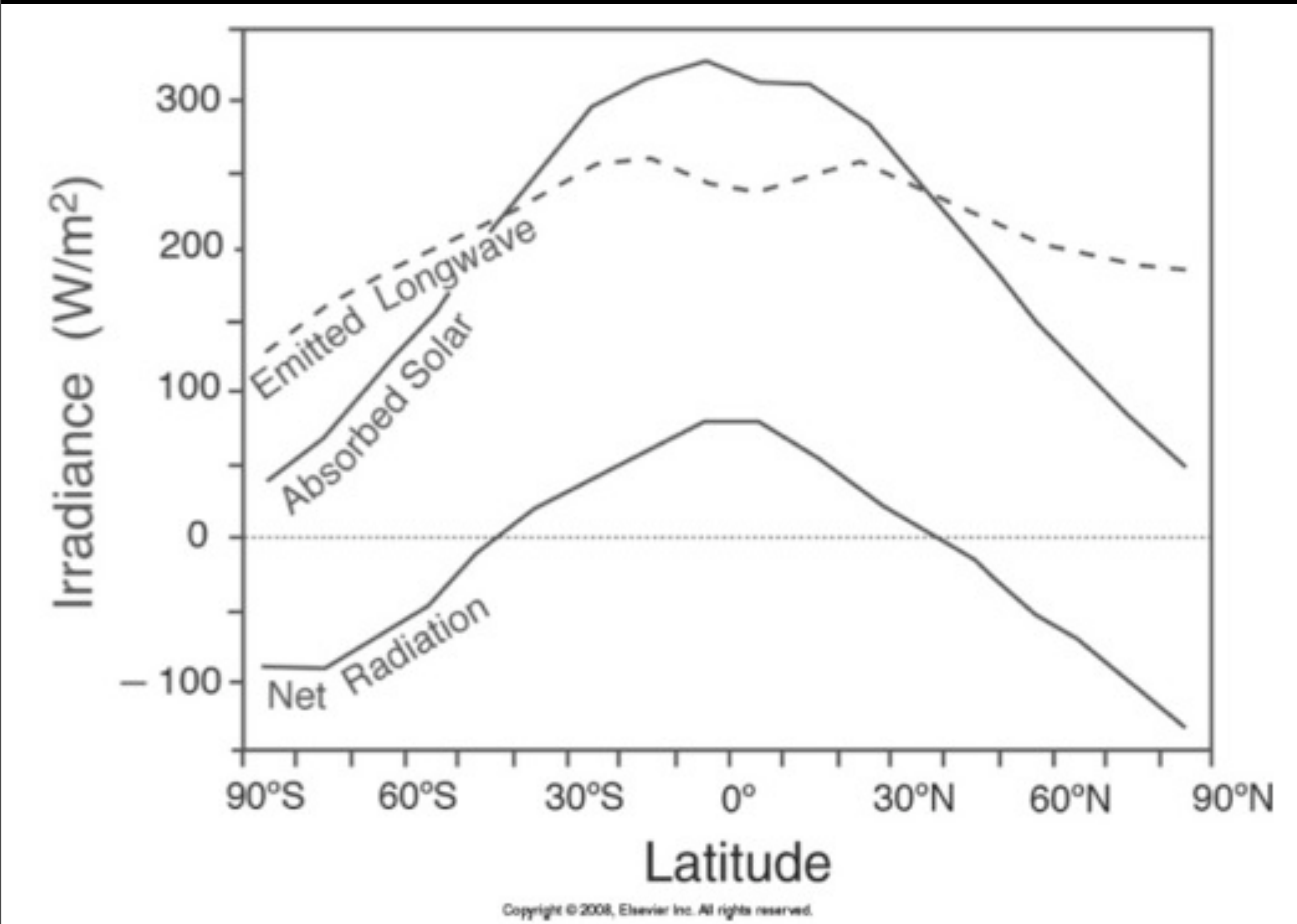
Ocean Eddies

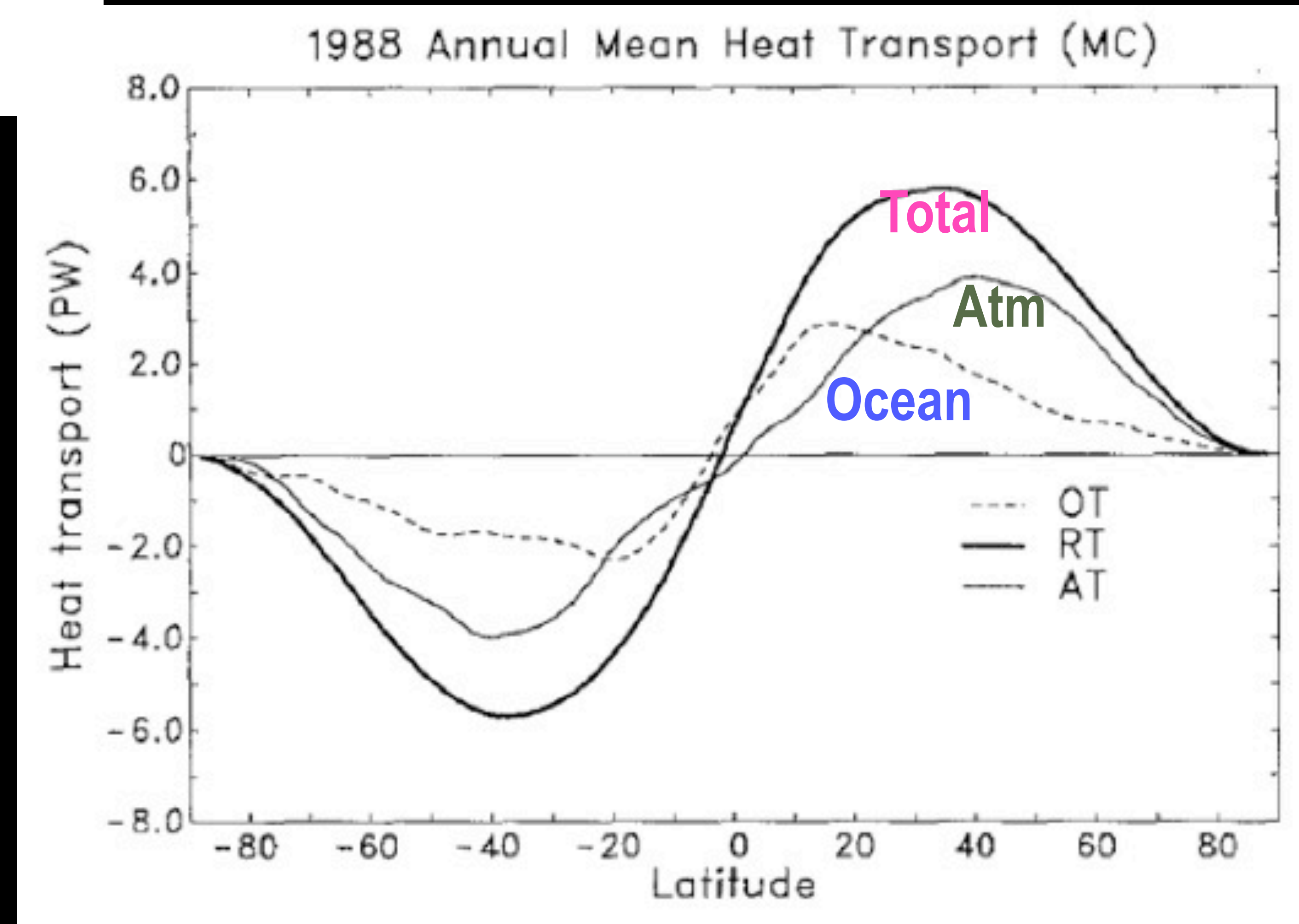
Amala Mahadevan
Woods Hole Oceanographic Institution





Solar radiation Incoming/Outgoing

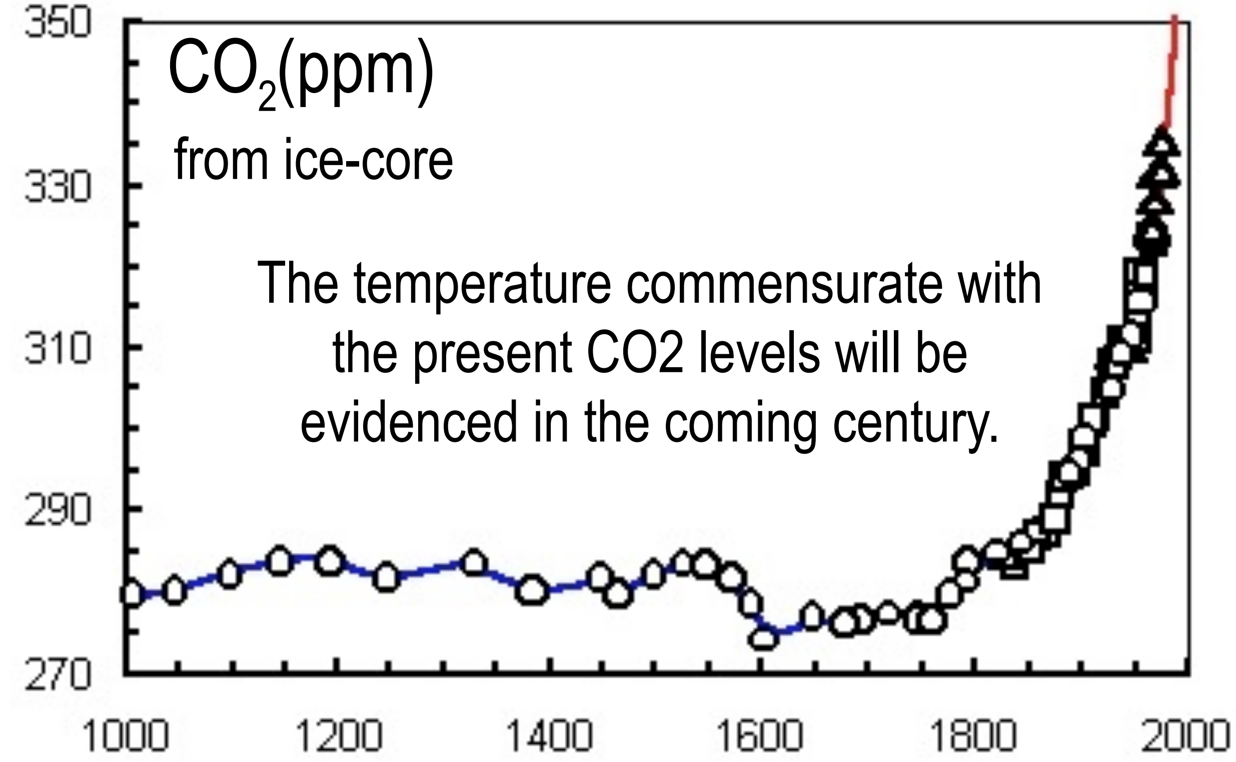
measured at top of atmosphere



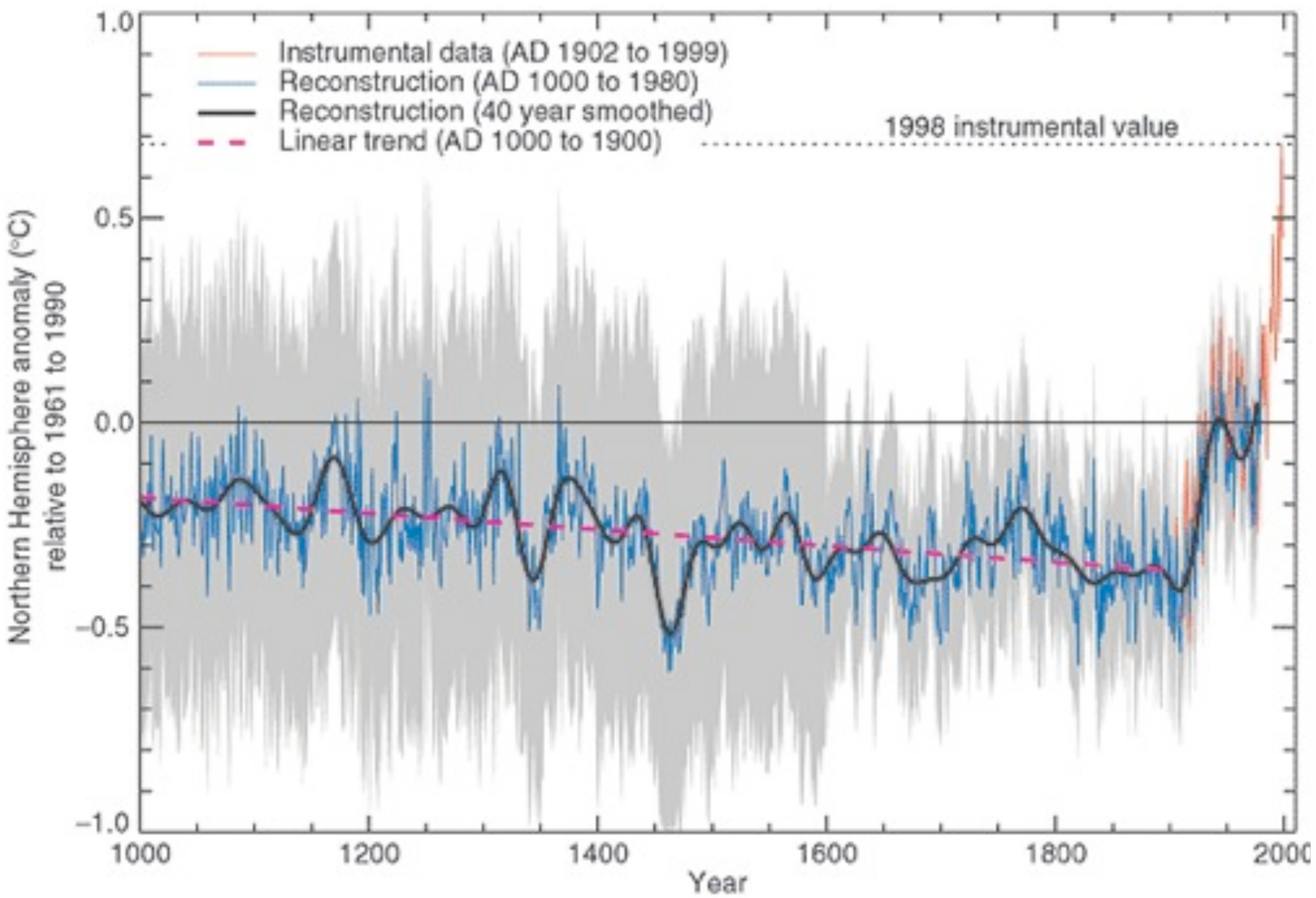
Mean Meridional Heat Transport

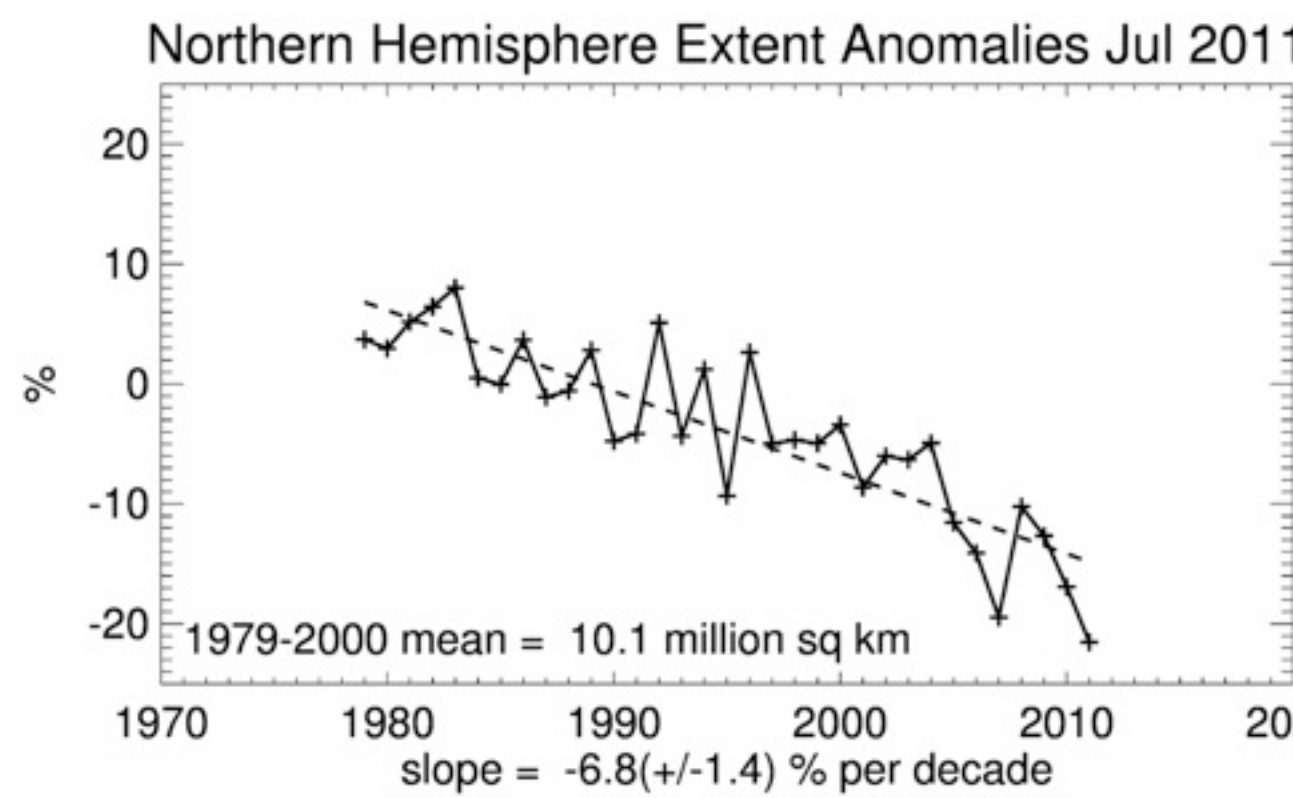
Trenberth and Solomon (1994)

We are presently experiencing in one of the largest and most abrupt perturbations to climate and ecology



Temperature





Millennial Northern Hemisphere (NH) temperature recor and instrumental data (red) from AD 1000 to 1999, adapted from Mann et al. (1999).

IPCC report, 2001

Global Temperature (Hansen et al., 2006)

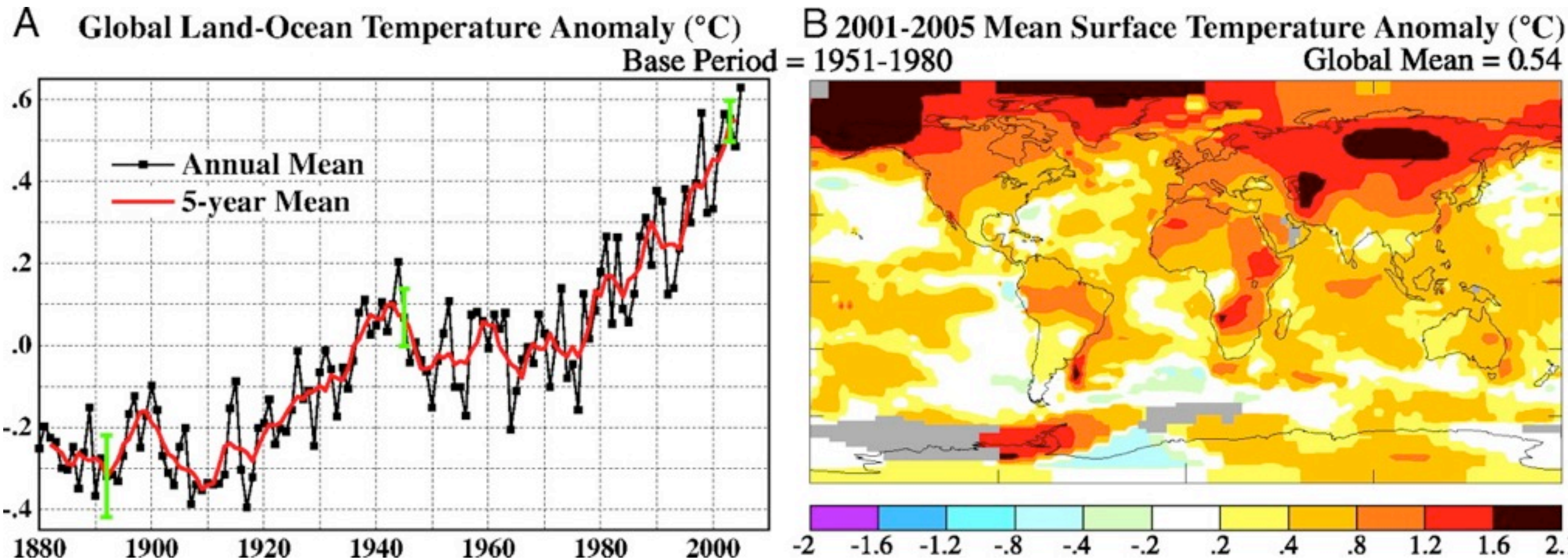


Fig. 1. Surface temperature anomalies relative to 1951–1980 from surface air measurements at meteorological stations and ship and satellite SST measurements. (A) Global annual mean anomalies. (B) Temperature anomaly for the first half decade of the 21st century.

What generates the complex pattern of flow?

Density variations

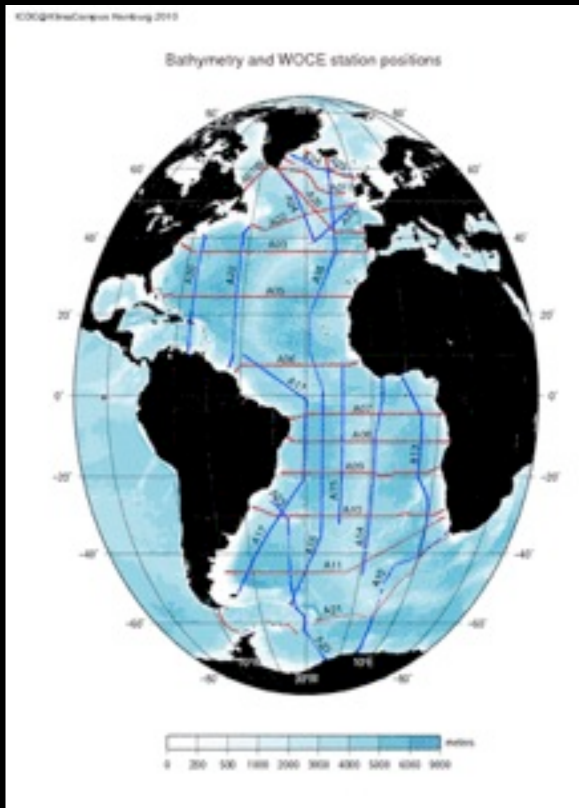
Effects of rotation
large scale motion- geostrophy

Mechanical forcing by wind

Fronts Lateral density variation

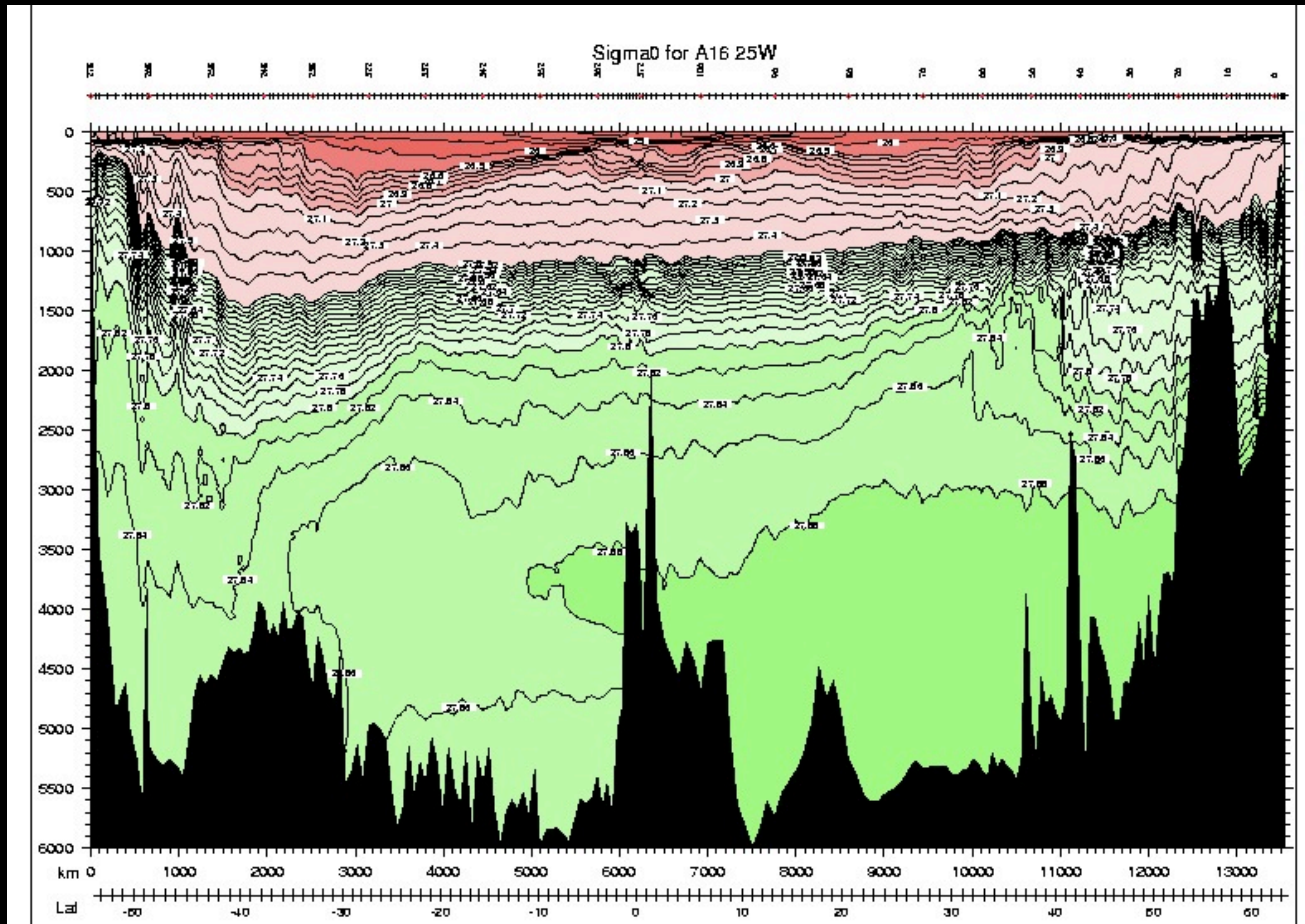
Baroclinic instability - eddy formation, mixing

Vertical motion



WOCE - Atlantic section potential density

$$\frac{D\rho}{Dt} = S_\rho$$



$$N^2 = -\frac{g}{\rho}\frac{\partial \rho}{\partial z}$$

Buoyancy frequency

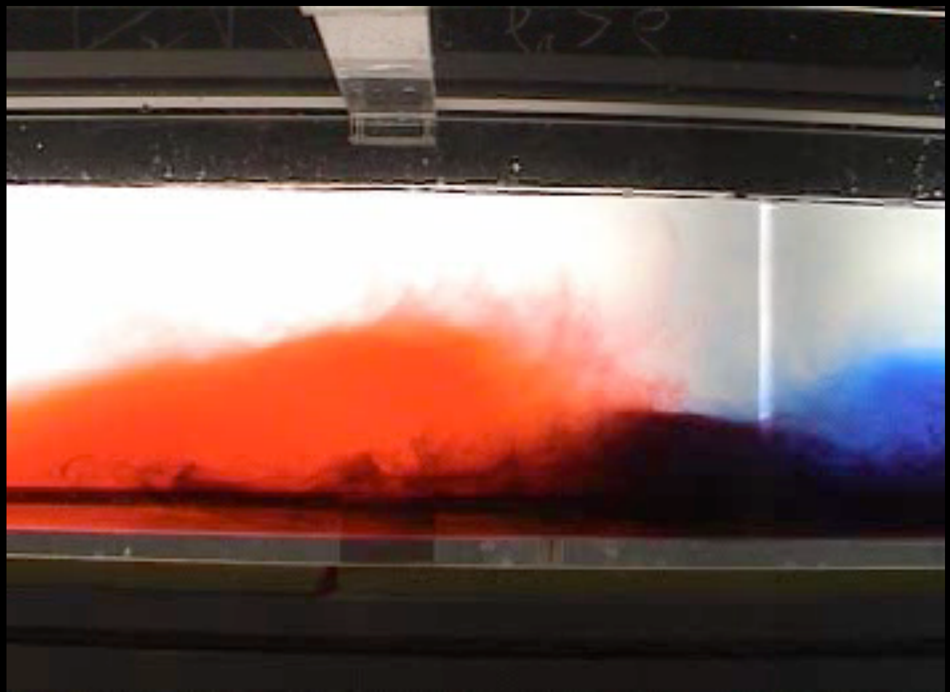
1024-1029 kg/m3

Internal gravity waves

Breaking leads to diapycnal mixing

Dense fluid collapse

with rotation

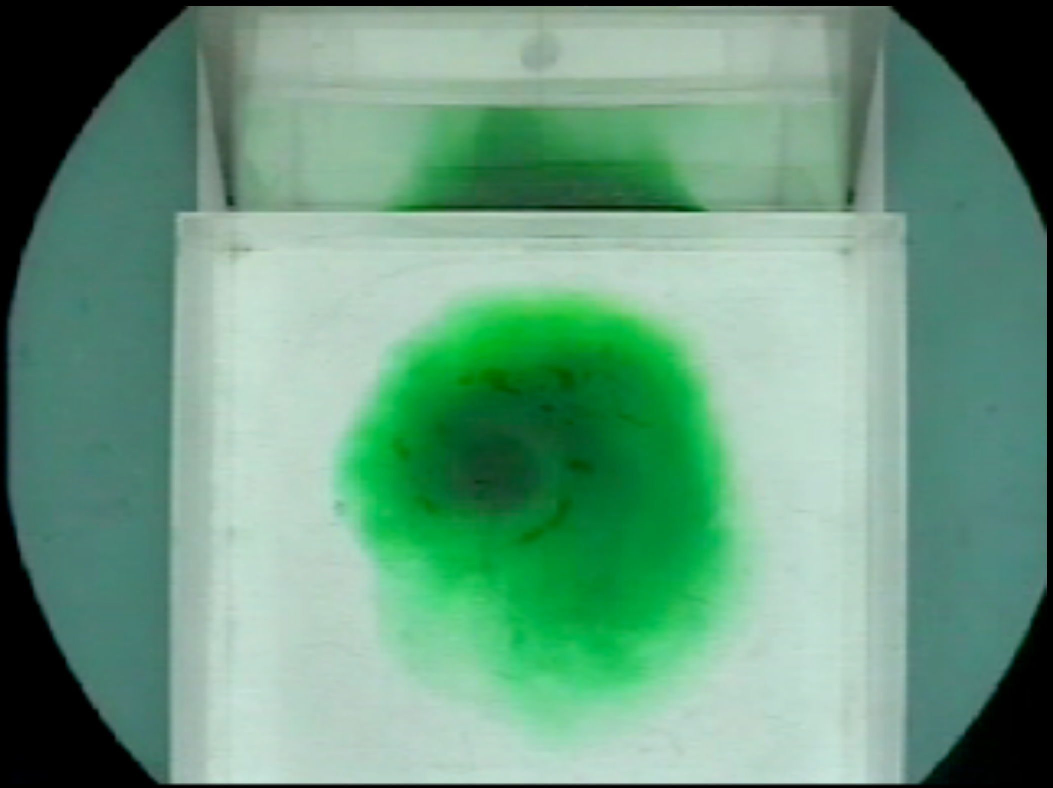


R. Goler, Meteoro. Inst, Munich

MIT Synoptic Meteero. Lab

Dense fluid collapse

with rotation



MIT Synoptic Meteero. Lab

Equations of motion in a rotating frame

$$\left(\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}\right)_I = \left(\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}\right)_R + \boldsymbol{\Omega} \times \boldsymbol{r}$$

$$\boldsymbol{v}_I = \boldsymbol{v}_R + \boldsymbol{\Omega} \times \boldsymbol{r}.$$

$$\left(\frac{\mathrm{d}\boldsymbol{v}_R}{\mathrm{d}t}\right)_R = \left(\frac{\mathrm{d}\boldsymbol{v}_I}{\mathrm{d}t}\right)_I - 2\boldsymbol{\Omega} \times \boldsymbol{v}_R - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r})$$

Coriolis accn

Centripetal accn (modifies gravitational potential)

$$\frac{D\mathbf{u}}{Dt} + \rho^{-1}\nabla p + 2\Omega \times \mathbf{u} + \nabla\phi = \nabla(\nu\nabla u)$$

$\frac{U^2}{L}$ $\frac{P}{\rho L}$ ΩU g $\frac{\nu U}{L^2}$

Effects of rotation
We view the earth in the rotating frame

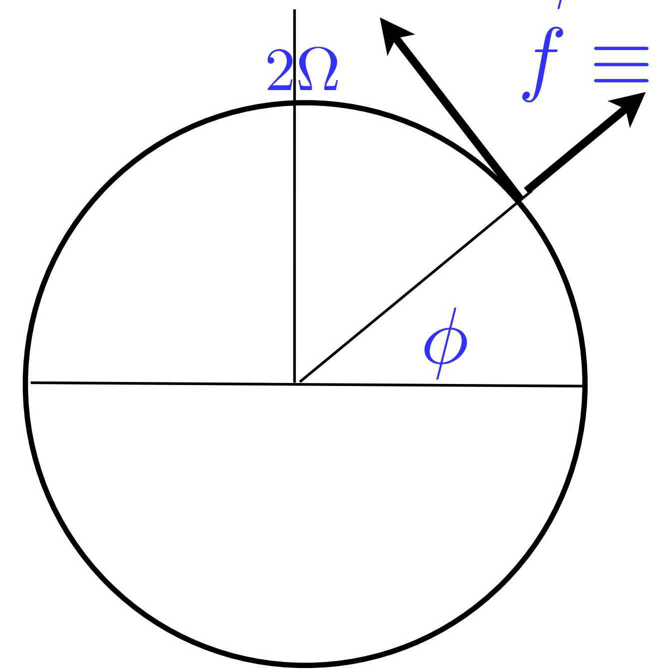
Effects of rotation
We view the earth in the rotating frame

Horizontal Large Scale Dynamics

$$2\Omega \times \mathbf{u} = (-fv + bw,\ fu,\ bu)$$

$$b \equiv 2\Omega \cos\phi$$

$$f \equiv 2\Omega \sin\phi$$



$$Du/Dt + \left(\frac{P}{\rho U^2}\right) p_x - \left(\frac{\Omega L}{U}\right) fv = \left(\frac{\nu}{UL}\right) \nabla^2 u$$

$$For\ \ U = 0.1\ m/s,\ L = 10^5\ m,\ \Omega = 10^{-4}/s$$

Rossby number $R_o = \dfrac{U}{\Omega L} << 1$

$$Re = UL/\nu \sim 10^{10}$$

$$Du/Dt + Ro^{-1}(p_x - fv + Ro\delta\ bw) = F^x$$

$$Dv/Dt + Ro^{-1}(p_y + fu) = F^y$$

Geostrophy

$$fv = p_x, \quad fu = -p_y$$

$$u_x + v_y + w_z = 0$$

Incompressibility

$$\left(\frac{U}{L}\right)(u_x + v_y) + \left(\frac{W}{D}\right) w_z = 0$$

$$\delta = D/L << 1$$

$$Ro = \frac{U}{\Omega L}$$

In the Vertical

$$W \sim Ro\, \delta\, U$$

$$w_z = -Ro^{-1}(u_x + v_y)$$

$$\frac{Dw}{Dt} + \frac{1}{Ro^2 \delta}\left(\frac{1}{\rho} p_z + g - \delta b u\right) = 0$$

Hydrostatic balance

$$\frac{\partial p}{\partial z} = -\rho g$$

Hydrostatic Pressure gradient

$$p_x = g h_x + r_x$$

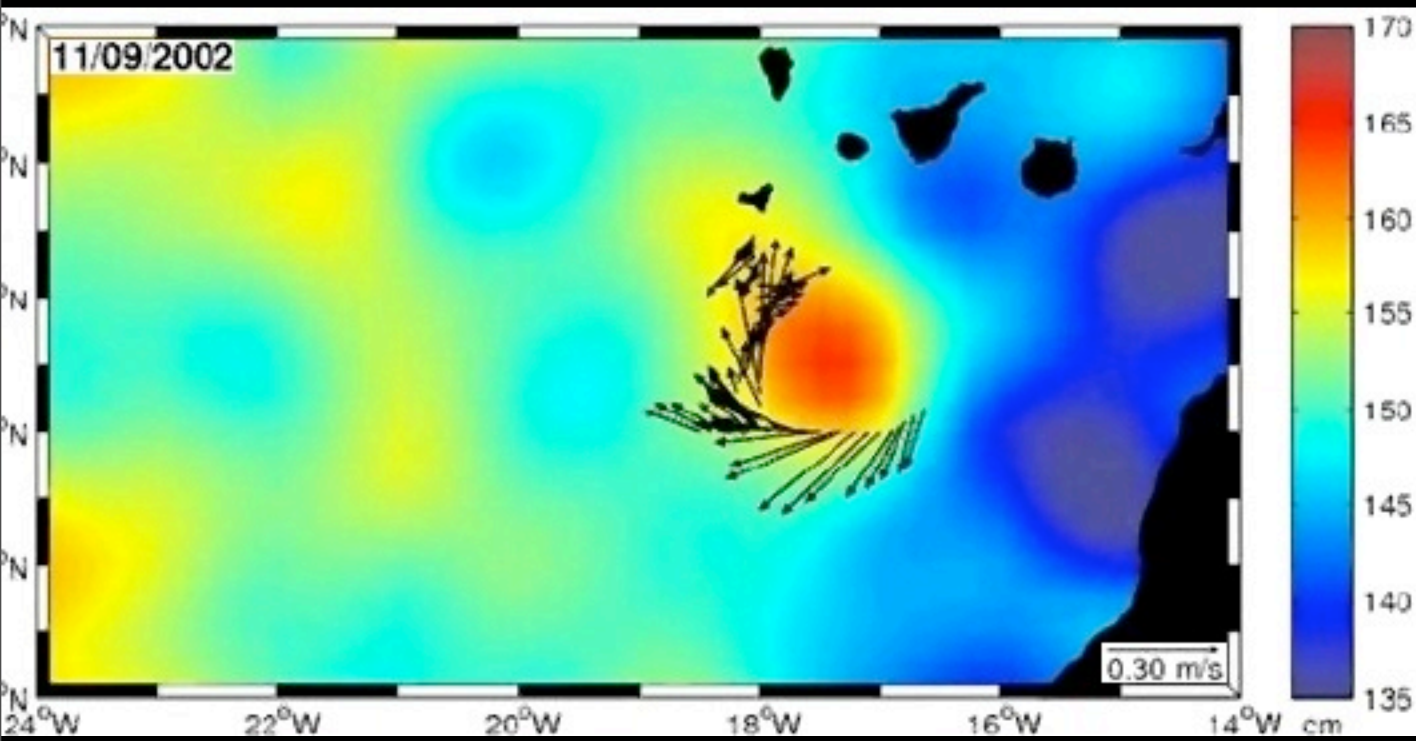
$$r_x = \frac{\partial}{\partial x} \int_z^h \rho dz$$

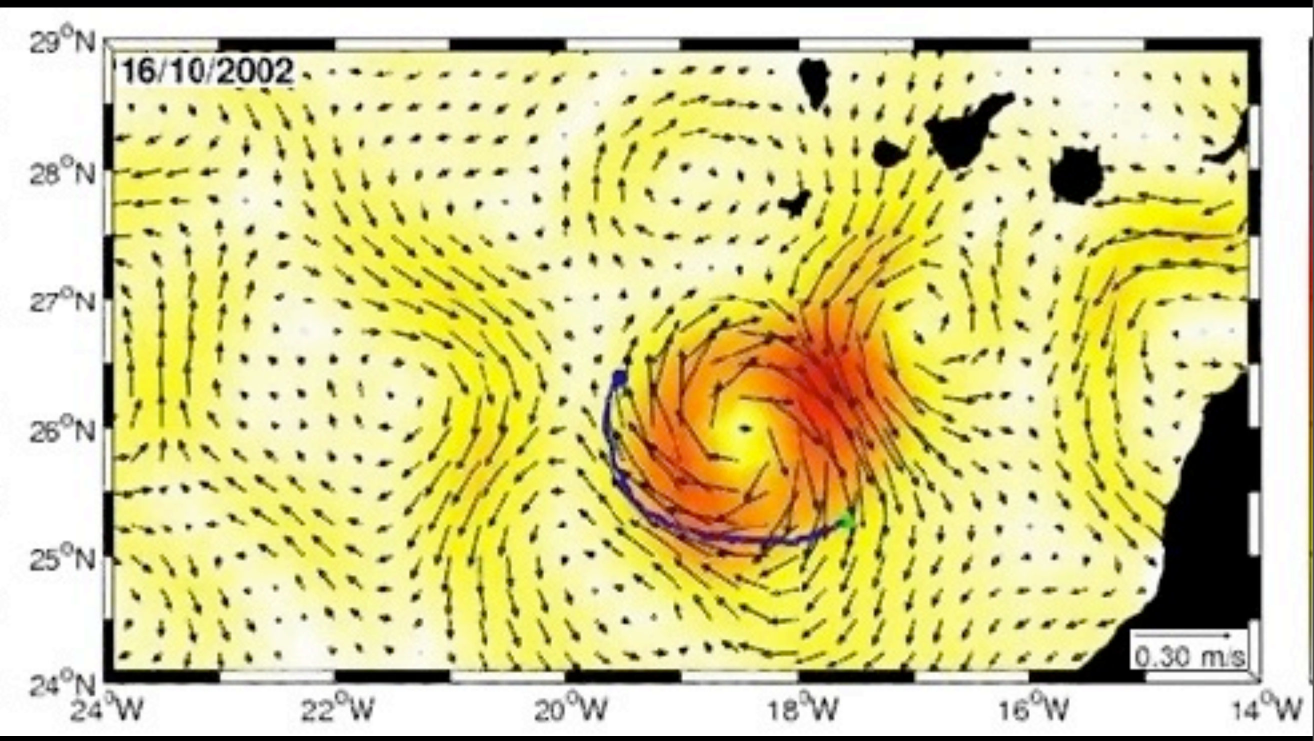
At the sea surface

$$v_g = \frac{1}{f} g h_x, \quad u_g = -\frac{1}{f} g h_y$$

Sea surface height

Geostrophic surface velocity





In the Boundary Layer

Ekman transport

$$2\Omega \times \mathbf{u} = \frac{\partial}{\partial z}\left(K_v \frac{\partial u}{\partial z}\right) \qquad K_v \frac{\partial u}{\partial z} = \tau/\rho$$

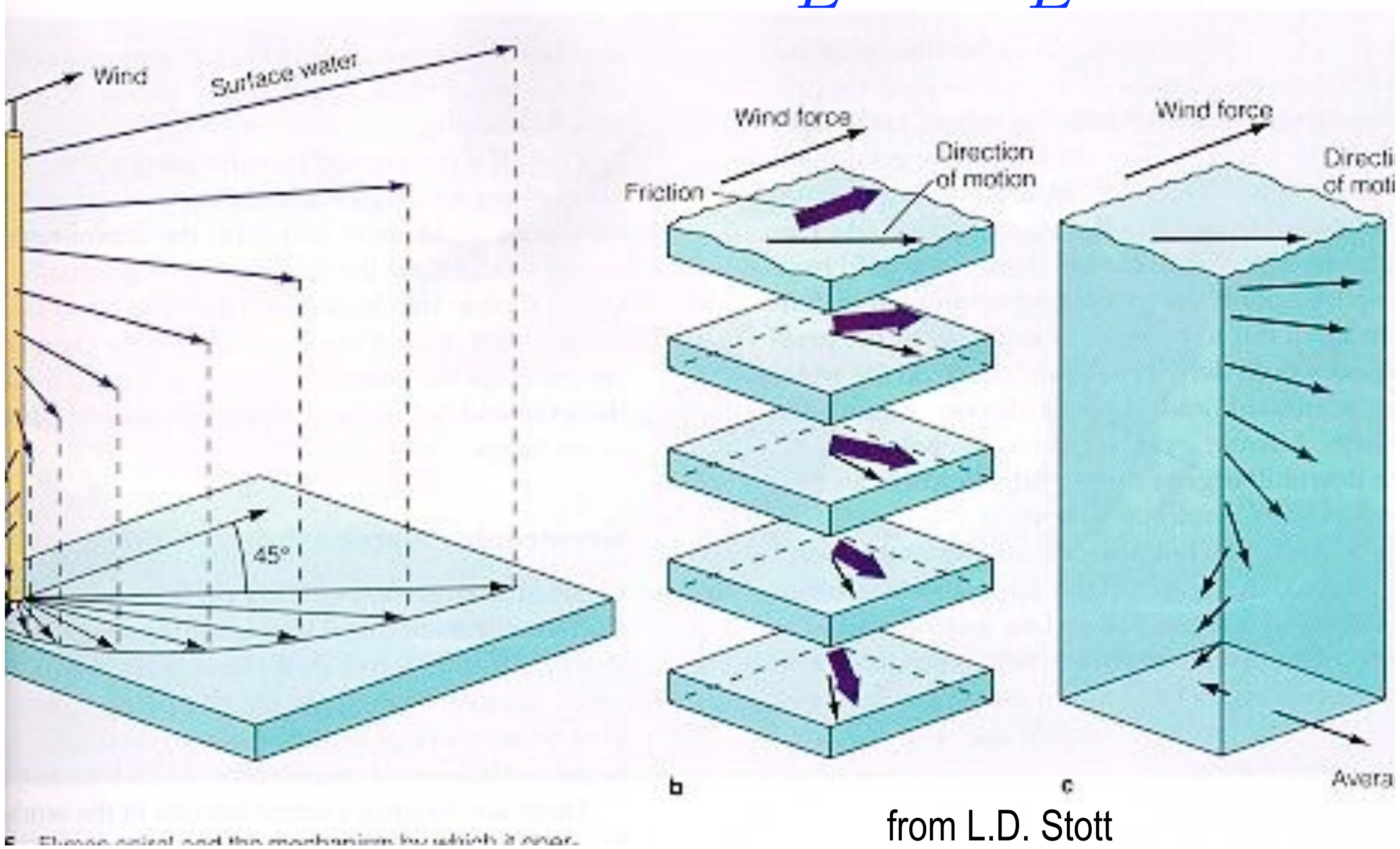
$$fU \qquad \frac{K_v U}{H^2}$$

Incompressibility

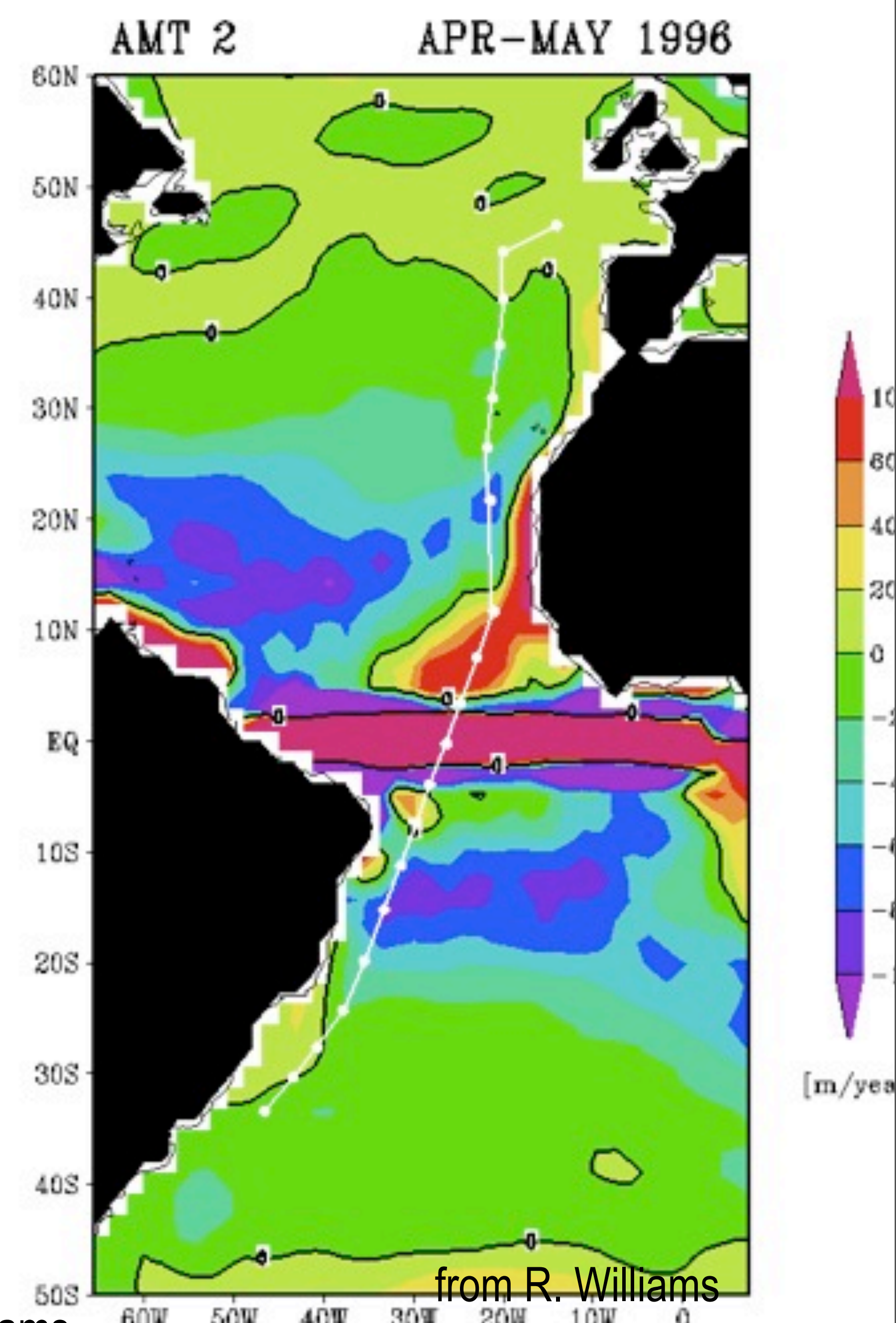
$$E_k = \frac{K_v}{f H_E^2} = O(1) \qquad H_E = (f/K_v)$$

$$\boldsymbol{M}_E = \int_{\mathrm{Ek}} \boldsymbol{u}_E \, \mathrm{d}z \qquad \boldsymbol{M}_E = -\frac{1}{f}\mathbf{k} \times \tilde{\boldsymbol{\tau}}_t$$

$$\nabla \cdot M_E = w_E = \nabla \times \tau$$



from L.D. Stott



from R. Williams

w_e from Williams

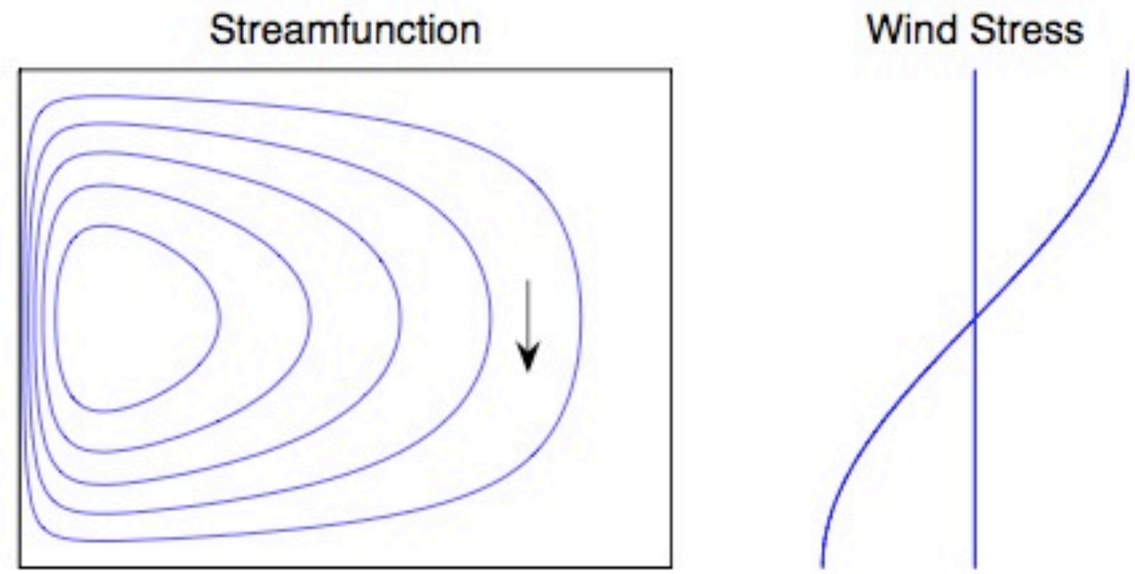
The wind-driven circulation

$$\beta \frac{\partial \psi}{\partial x} = \mathrm{curl}_z \boldsymbol{\tau} + \nu \nabla^2 \zeta = \mathrm{curl}_z \boldsymbol{\tau} + \nu \nabla^4 \psi$$

Surface boundary condition

$$\nu \frac{\partial \mathbf{u}}{\partial z} = \tau / \rho$$

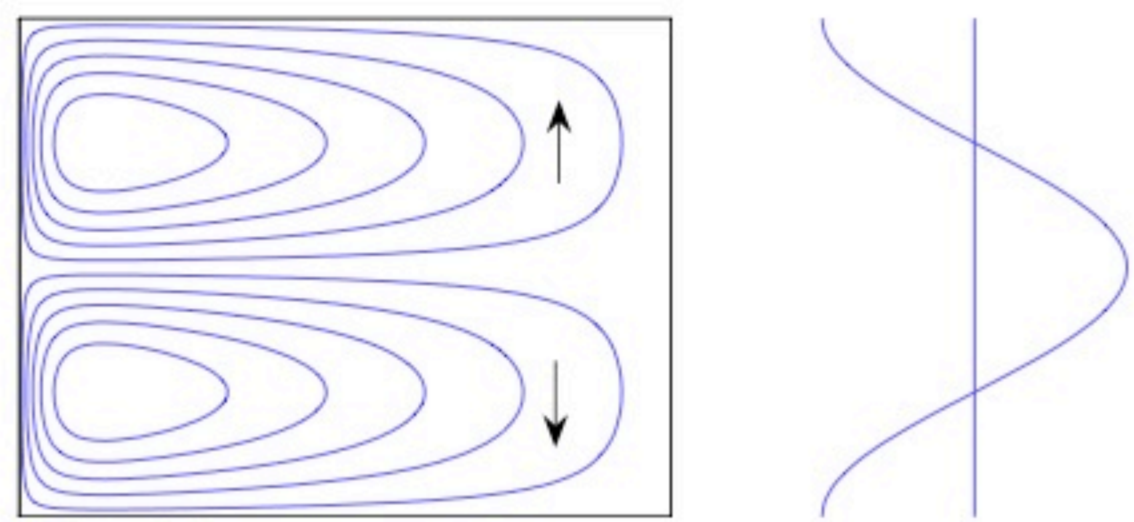
Stommel model



$$2\Omega \times \mathbf{u} = \frac{1}{\rho} \frac{\partial \tau}{\partial z}$$

Take the curl and integrate over depth of BL

$$\int \boldsymbol{f} \nabla_z \cdot \boldsymbol{u} \, \mathrm{d}z + \frac{\partial f}{\partial y} \int v \, \mathrm{d}z = \mathrm{curl}_z (\widetilde{\boldsymbol{\tau}}_t - \widetilde{\boldsymbol{\tau}}_b)$$

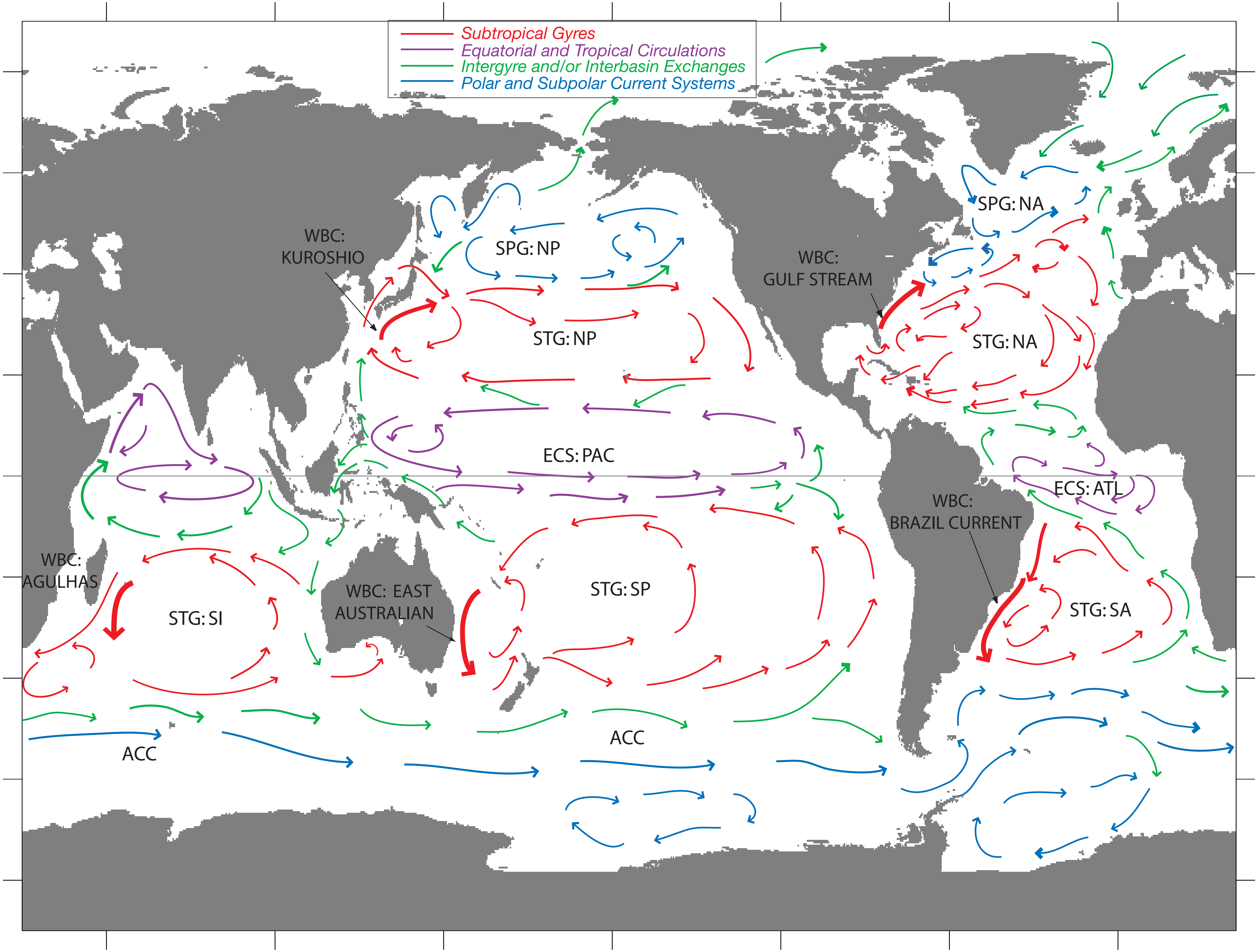


Sverdrup balance: Vertically integrated

$$\beta \overline{v} \approx \mathrm{curl}_z \widetilde{\boldsymbol{\tau}}$$

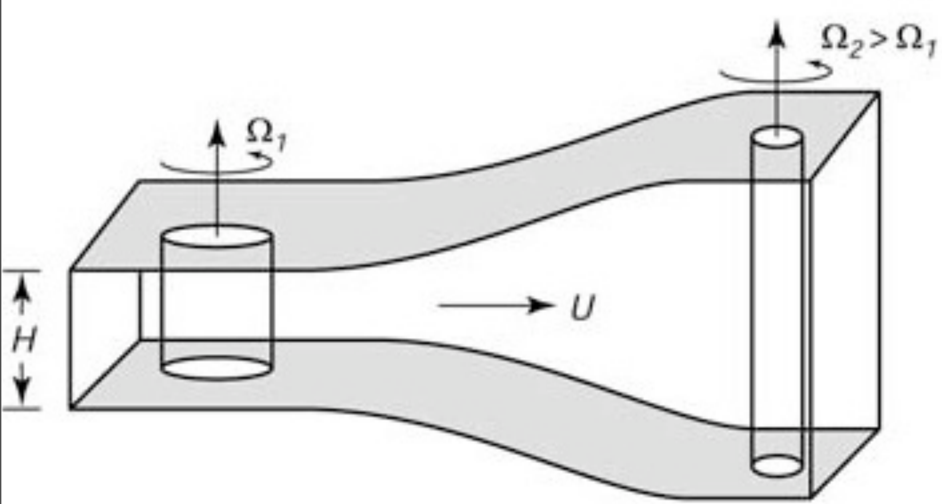
The wind-driven gyres

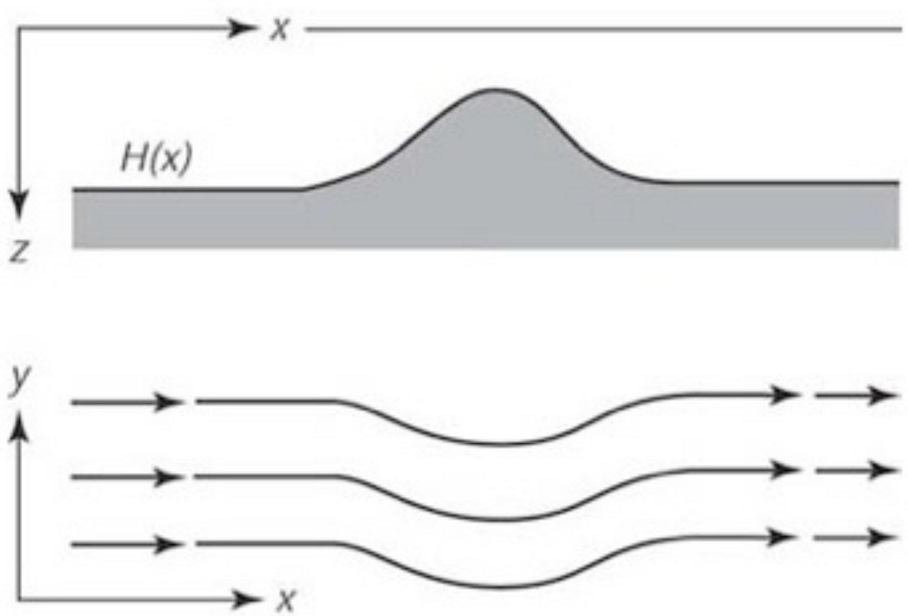
From Goeff Vallis

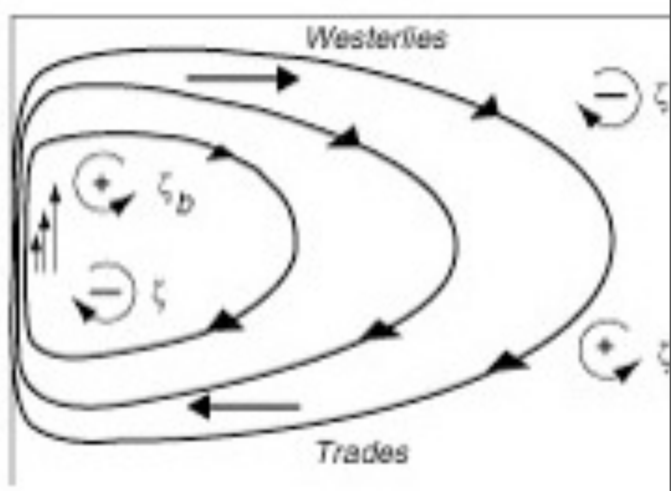


Conservation of Potential Vorticity

$$\frac{d}{dt}\left(\frac{f\ +\ \zeta}{h}\right)\ =\ 0, \qquad \zeta = v_x - u_y$$

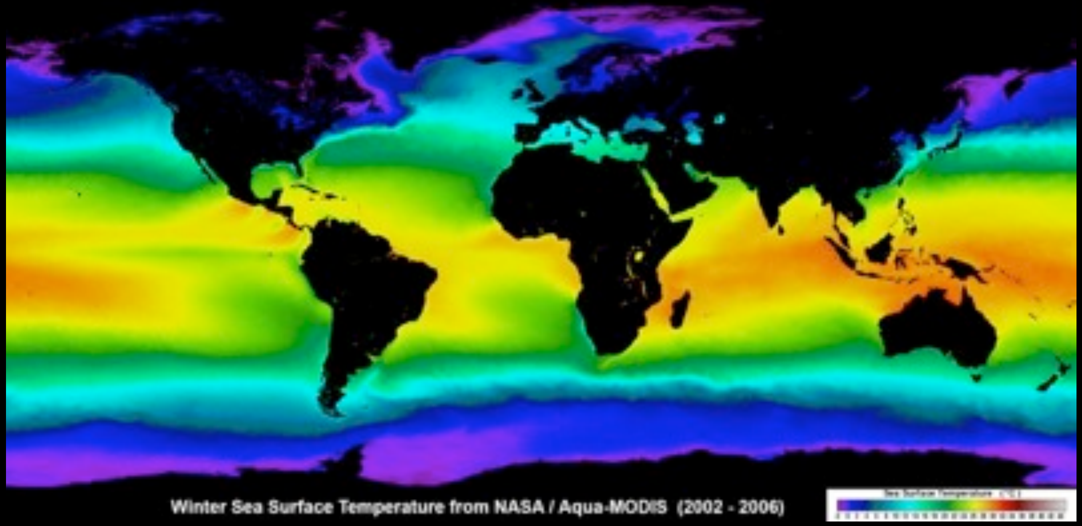


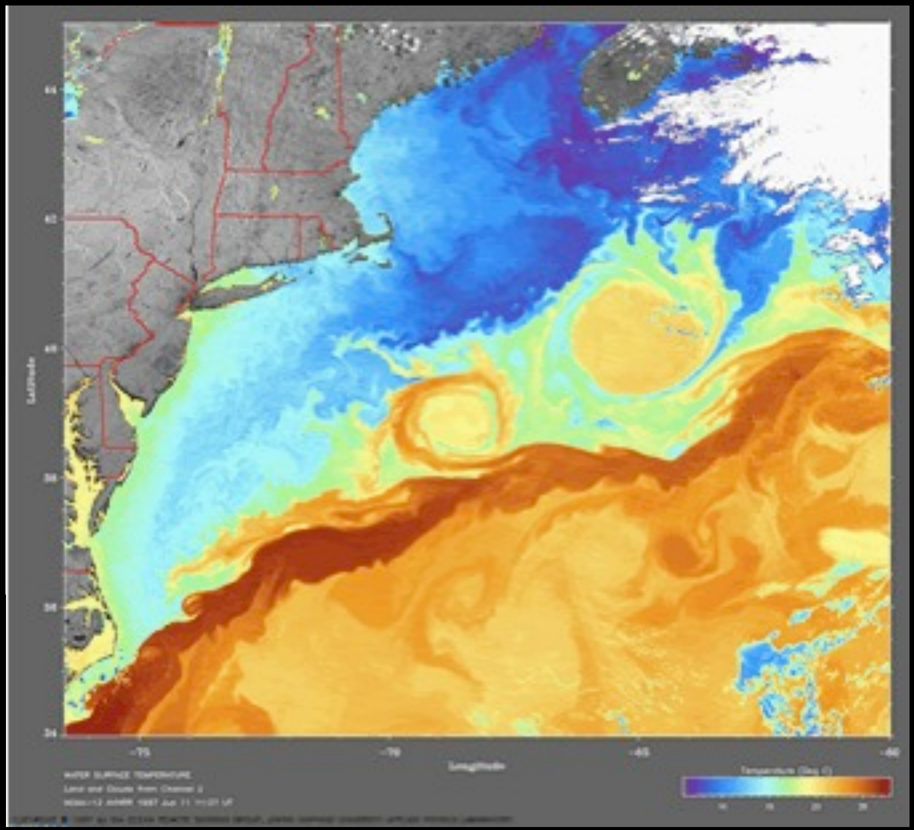




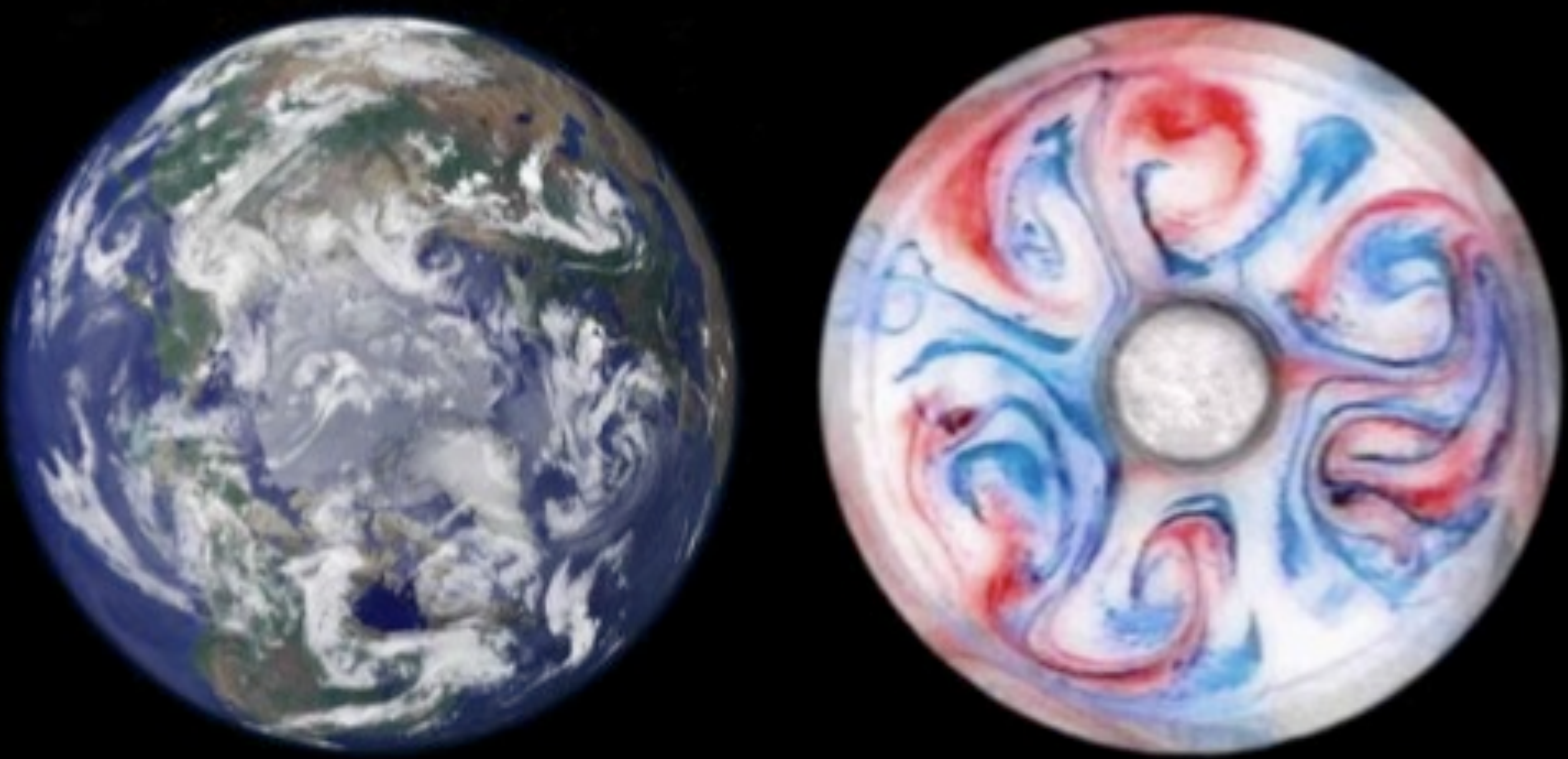
Lateral variations in density

Sea Surface Temperature from Satallite



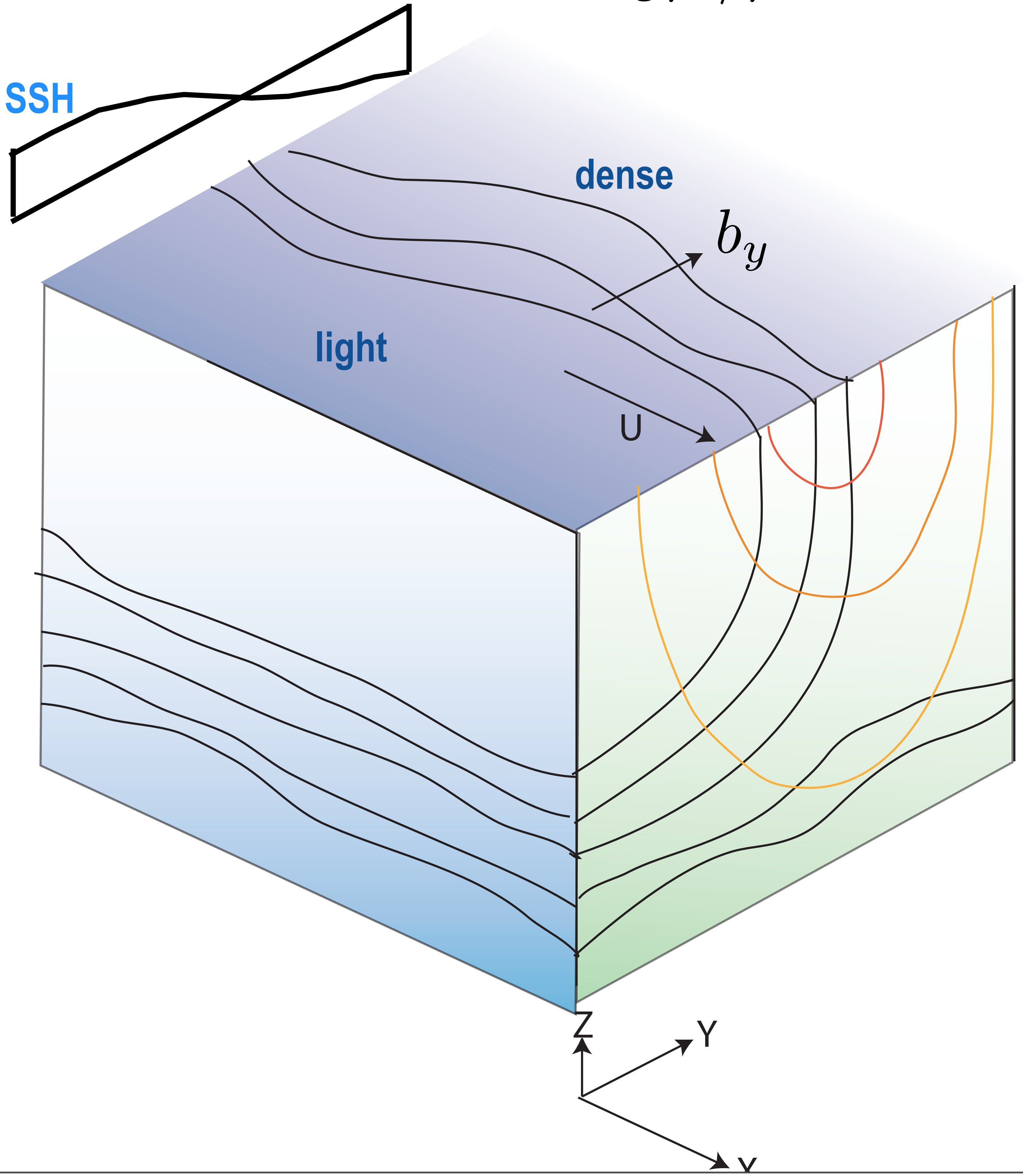


Gulf stream



from Marshall and Plumb 2006

$$b = -g\rho'/\rho_0$$



Hydrostatic balance

$$p_z = -\rho g$$

Geostrophic balance

$$fu = -p_y$$

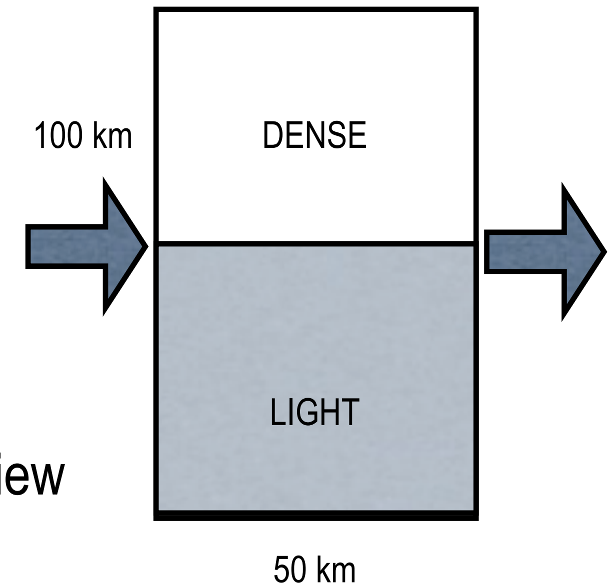
Thermal wind balance

$$fu_z = g\rho_y$$

Numerical Experiments

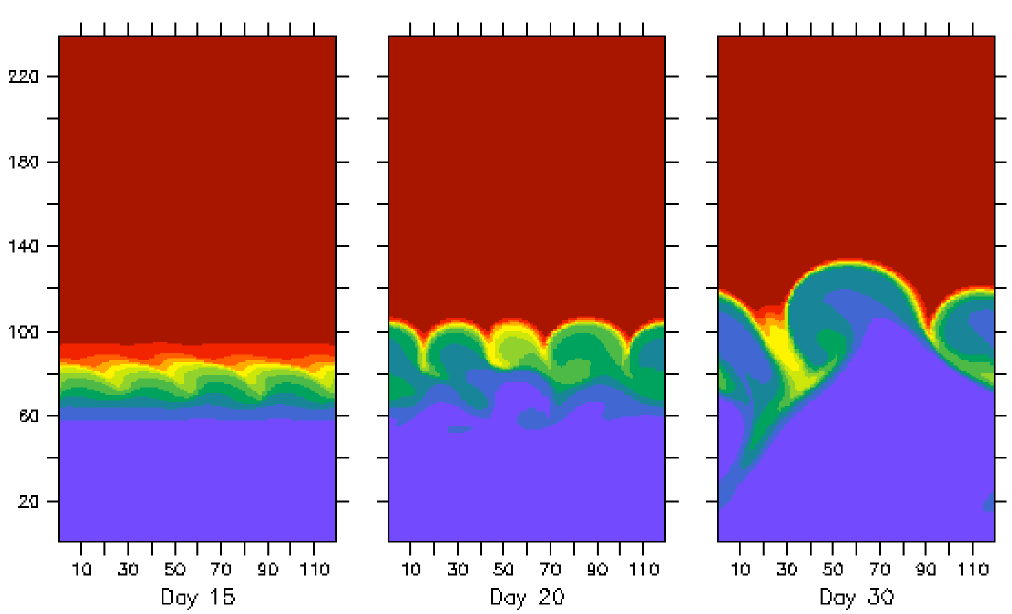
Periodic Channel

Initial Conditions



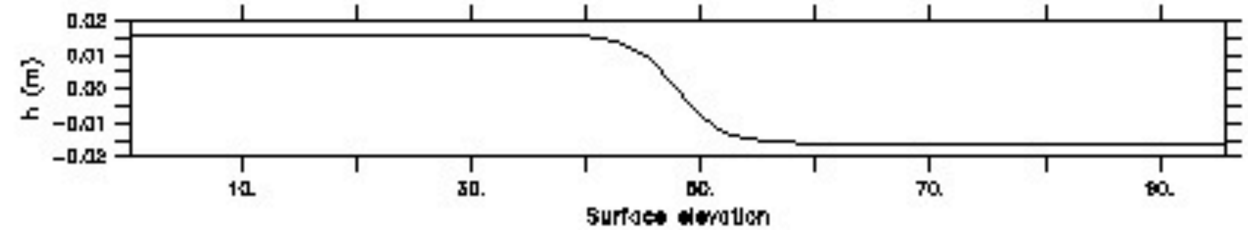
Surface View

Varying resolution 1000-250m
Wind / no wind
Hydrostatic/Nonhydrostatic

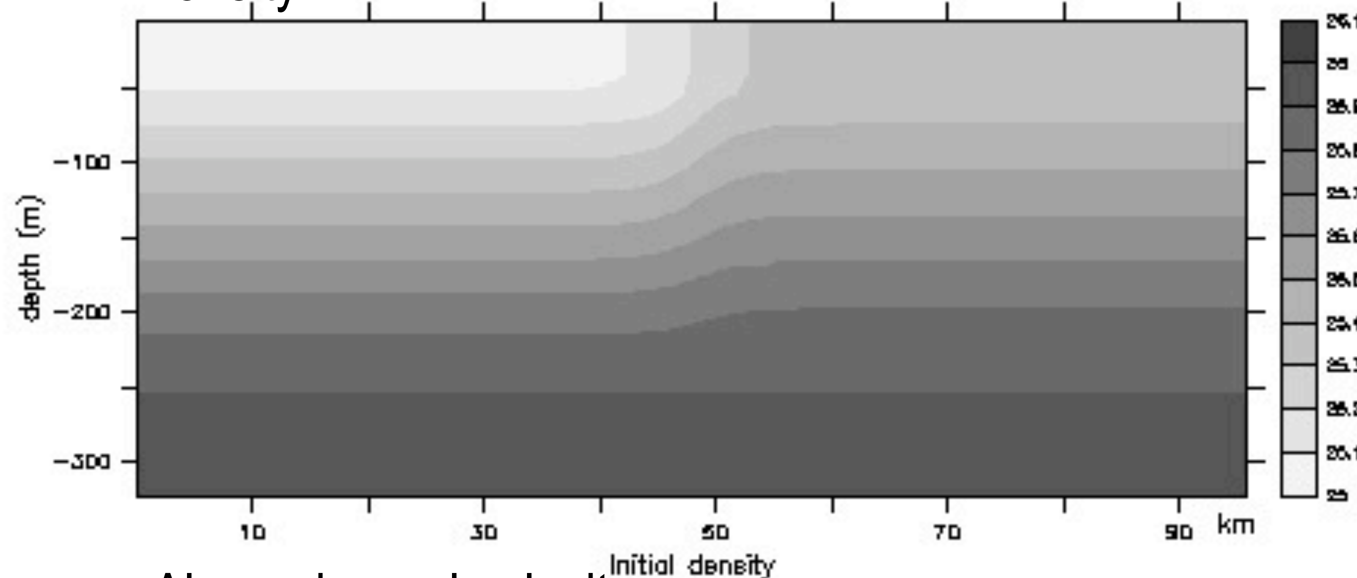


Sectional View

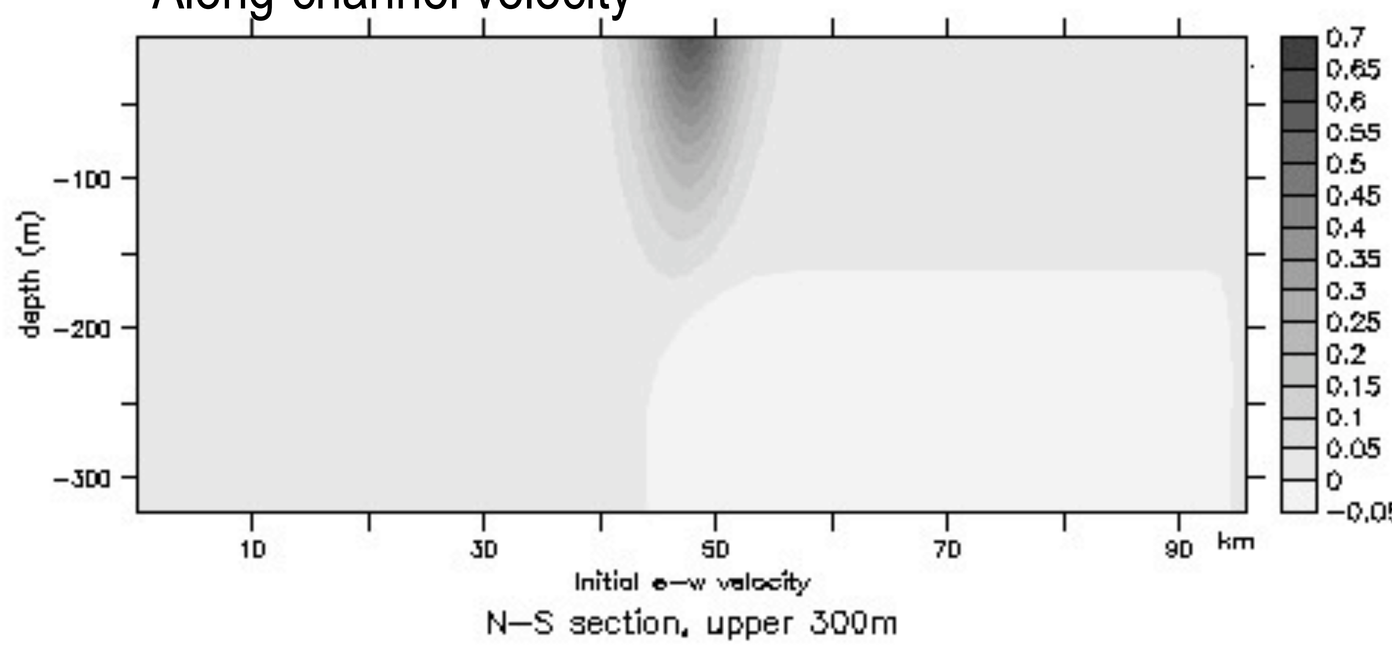
Free Surface



Density



Along-channel velocity



Instability of a Front

Dispersion/ cross-front exchange

Cold -blue
Warm - red

Vorticity / f

200 km

z (m)

y

y (km)

SECTIONAL VIEW

x (0-100 km)

x (0-100 km)

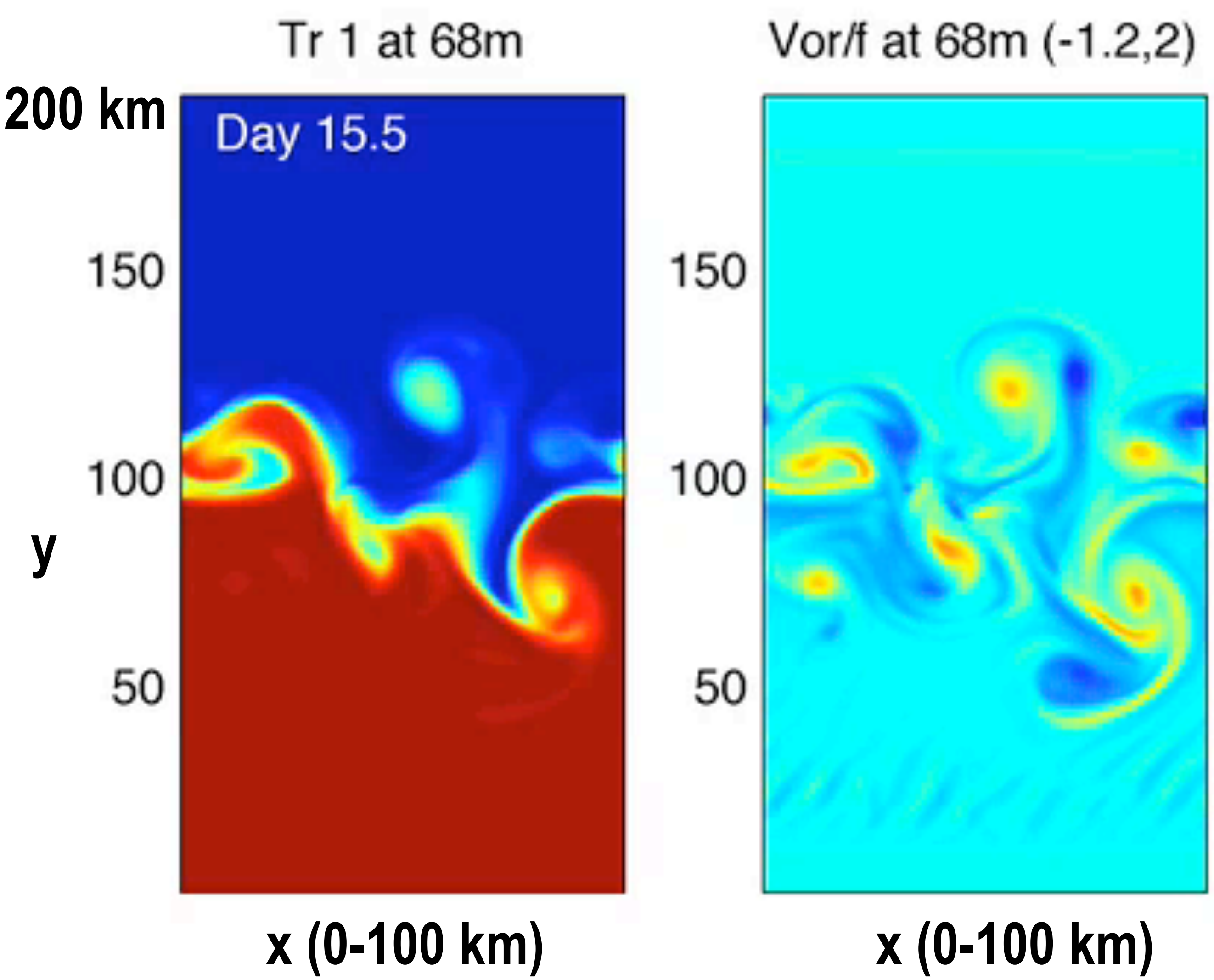
PLAN VIEWS

Instability of a Front

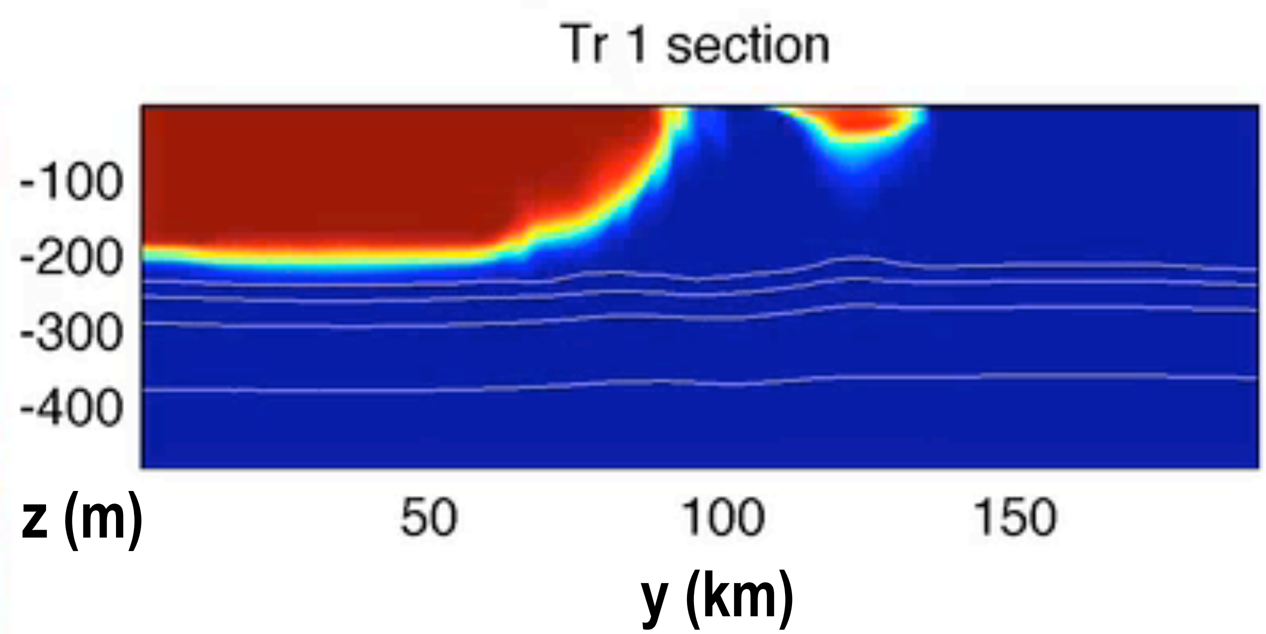
Dispersion/ cross-front exchange

Cold -blue
Warm - red

Vorticity / f

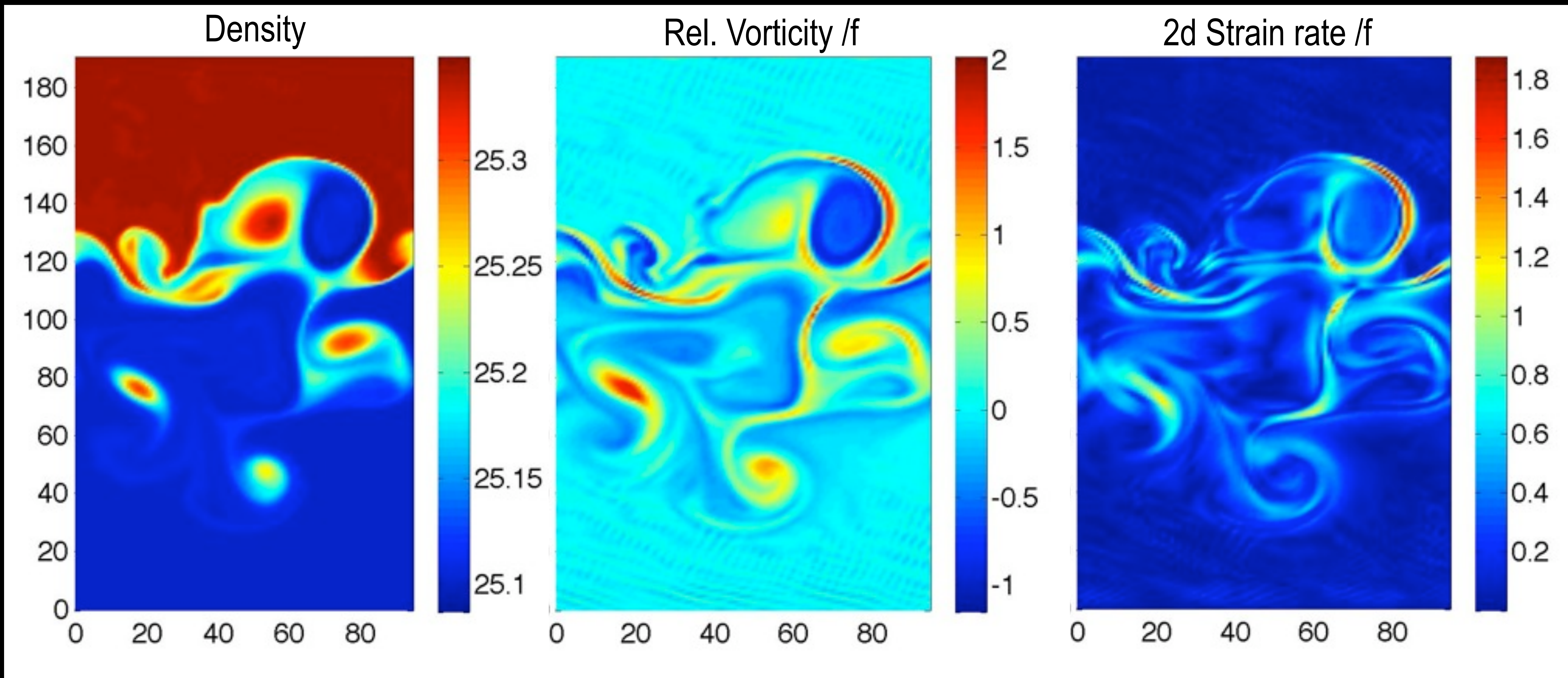


PLAN VIEWS

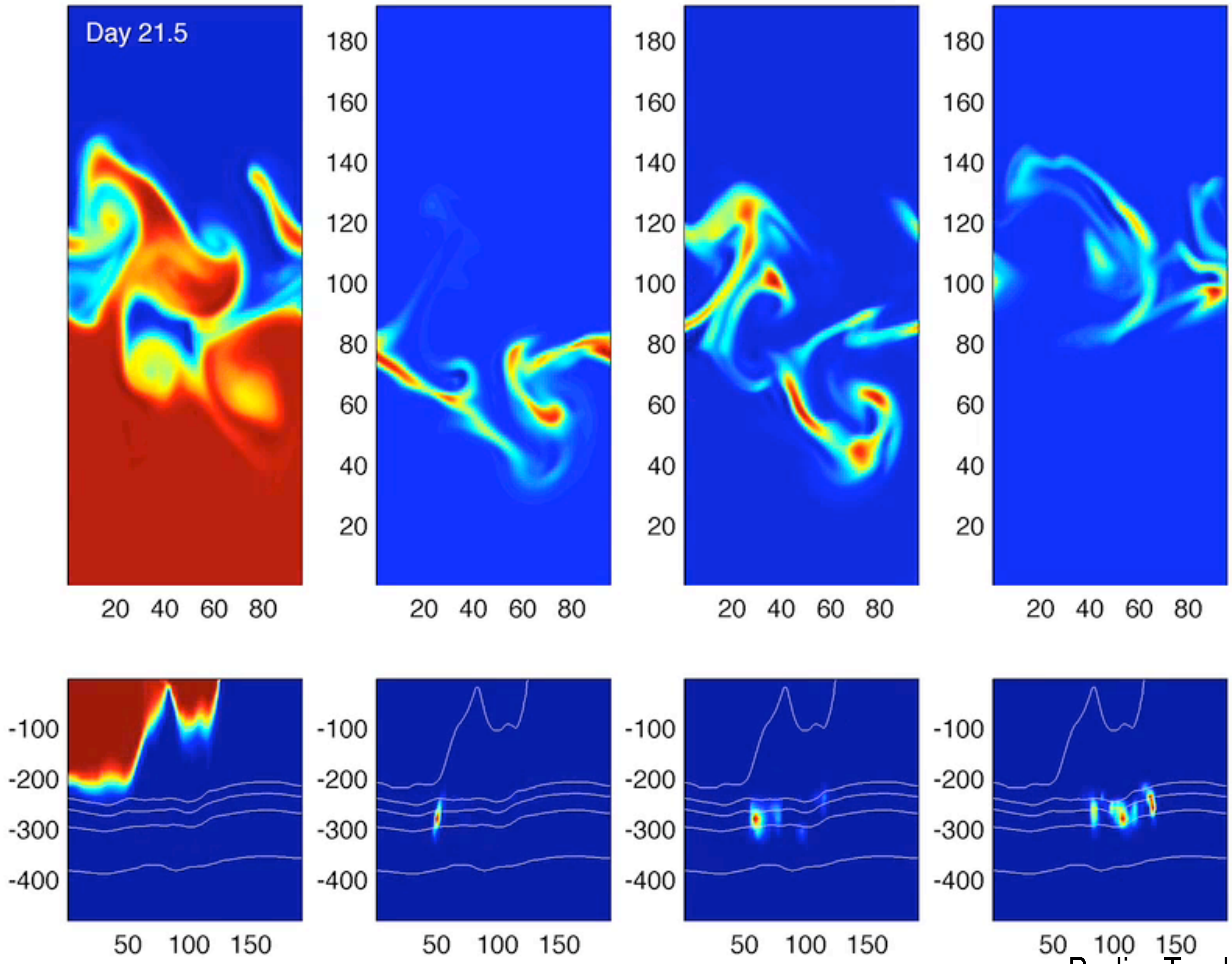


SECTIONAL VIEW

Front - forms eddies and filaments



Tracer dispersion

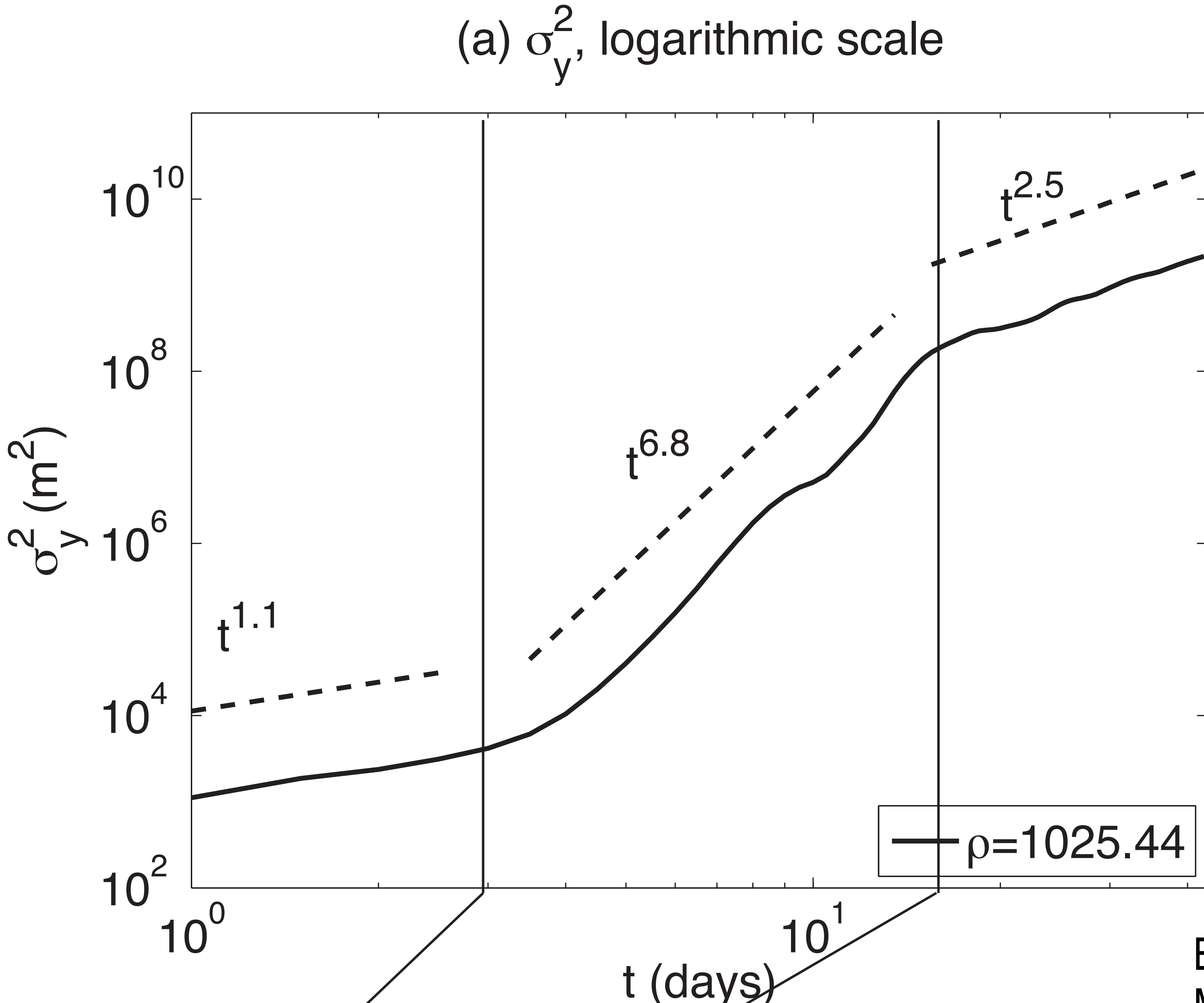


Badin, Tandon, Mahadevan, 2011

Tracer variance

First Moment $d_y = \frac{\langle yC \rangle}{\langle C \rangle}$ Position of center of mass

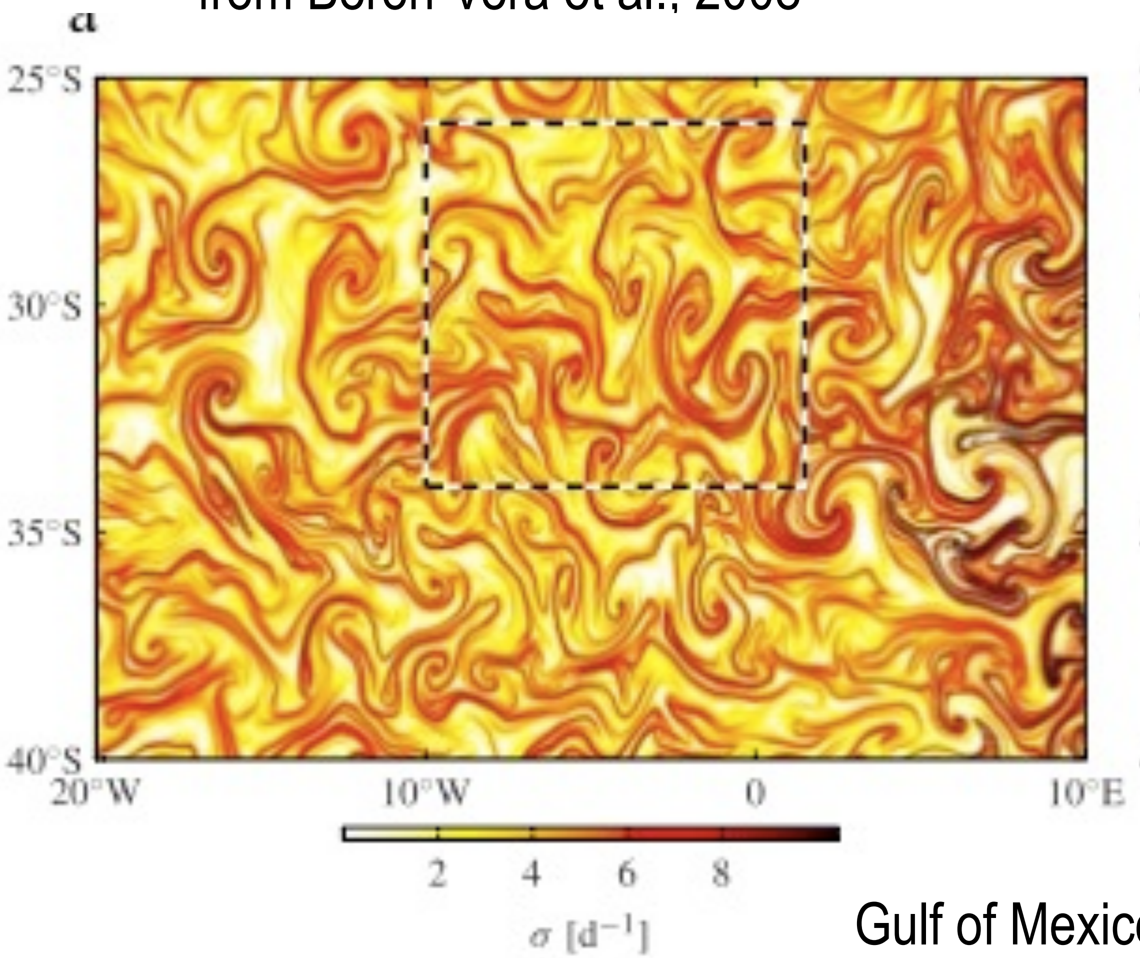
Dispersion ~Variance $\sigma_y^2 = \frac{\langle y^2 C \rangle}{\langle C \rangle} - d_y^2$

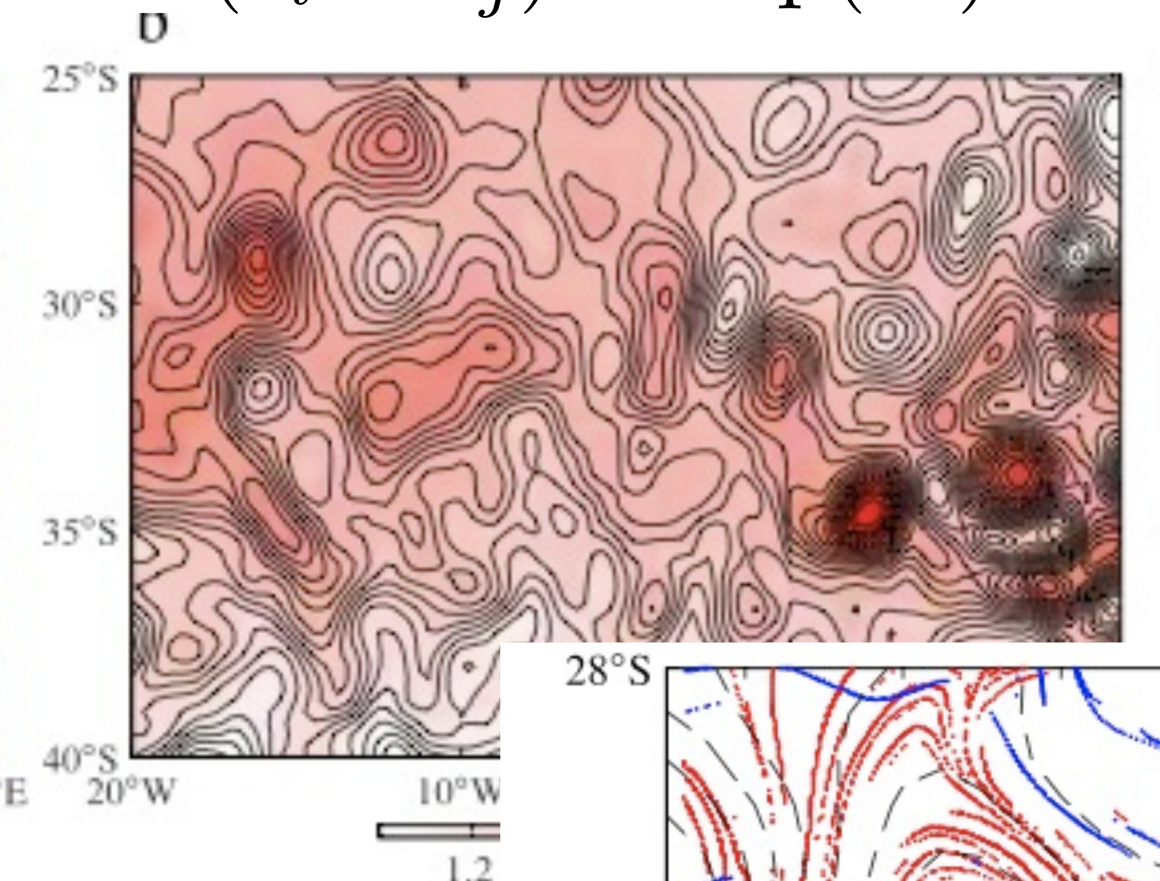


Badin, Tandon, Mahadevan, 2011

from Beron-Vera et al., 2008

$$(r_i - r_j) = \exp(\lambda t)$$





Gulf of Mexico, June 2010

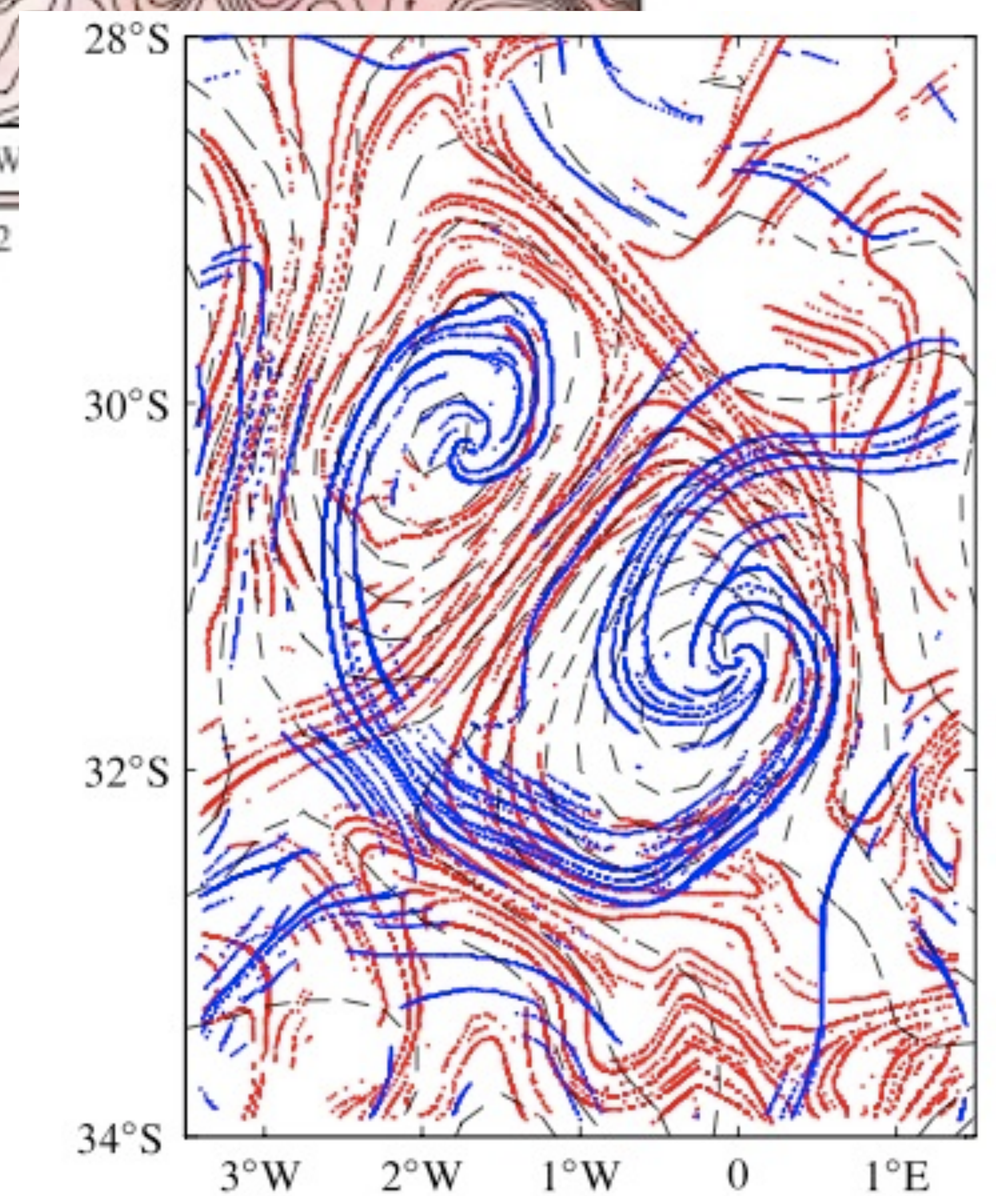
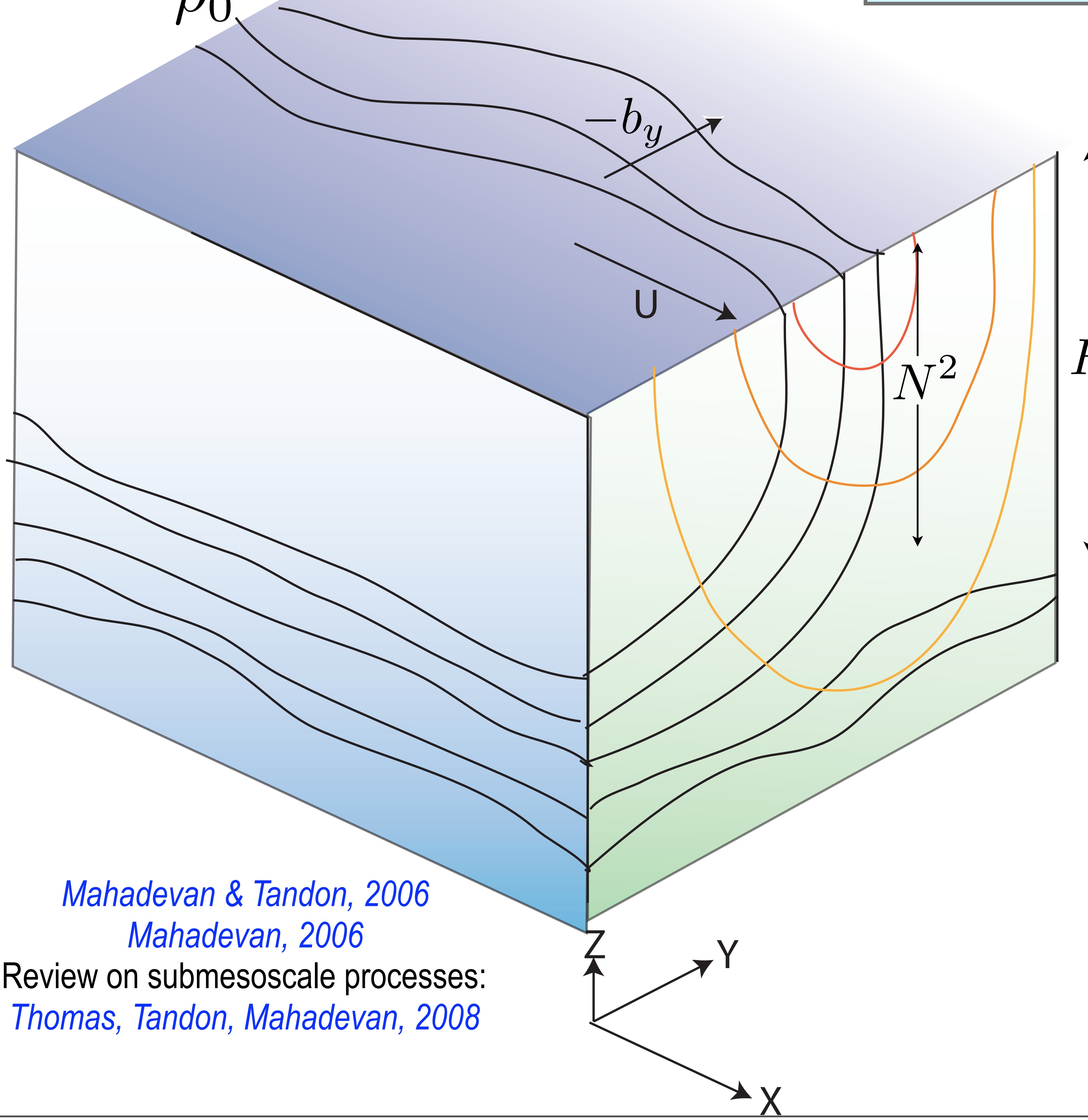


Figure 4. Attracting (red) and repelling (blue) LCSs

Fronts - Lateral gradients in density

$$b = -\frac{g}{\rho_0}\rho'$$



Vertical transport

$$\zeta/f = O(1), \quad Ro = U/fL = O(1)$$

Vertical velocity $$W \sim Ro\,\delta\,U = \delta\,U$$

where $\delta = H/L = f/N$

$$\frac{\partial}{\partial y} \quad \frac{Db}{Dt} = 0$$

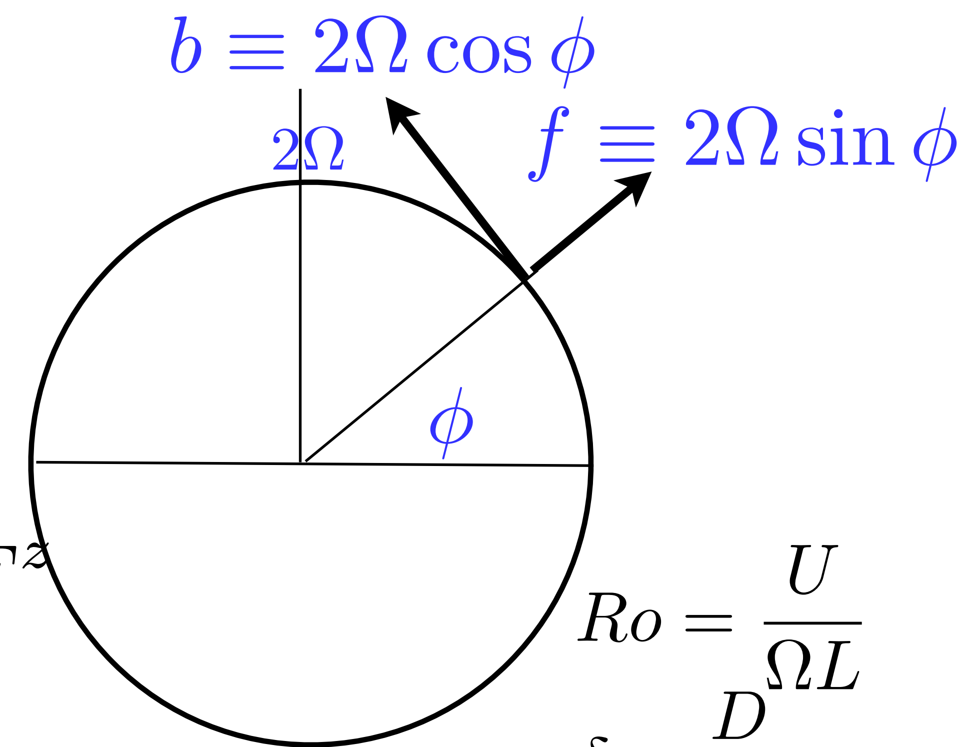
Frontogenesis

$$\frac{Db_y}{Dt} = -u_y b_y - v_y b_y$$

Mahadevan & Tandon, 2006
Mahadevan, 2006
Review on submesoscale processes:
Thomas, Tandon, Mahadevan, 2008

Modeling

p = Hydrostatic pressure
q = Nonhydrostatic pressure
$P = p + \delta q$



$$D_t u + Ro^{-1}(p_x + \delta q_x - f v + Ro\, \delta\, bw) = F^x$$

$$D_t v + Ro^{-1}(p_y + \delta q_y + f u) = F^y$$

$$D_t w + Ro^{-2}\delta^{-2}(\rho^{-1} p_z + g + \delta q_z - \delta\, bu) = F^z$$

$$u_x + v_y + Ro\, w_z = 0$$

$$Ro = \frac{U}{\Omega L} \qquad \delta = \frac{D}{L} \qquad W \sim Ro\, \delta\, U$$

Hydrostatic $\delta \rightarrow 0$

$$p_z + \rho g = 0$$

$$w_z = -Ro^{-1}(u_x + v_y)$$

Free-surface height
...and density => p

$$h_t + \partial_x \int_{z_b}^{h} u dz + \partial_y \int_{z_b}^{h} v dz = 0$$

Nonhydrostatic (δ does not $\rightarrow$ 0)

$$D_t w + Ro^{-2}\delta^{-1}(q_z - bu) = F^z$$

Well-posed with open boundaries

Mahadevan et al., 1996a,b, Mahadevan & Archer 1998

Nonhydrostatic Model

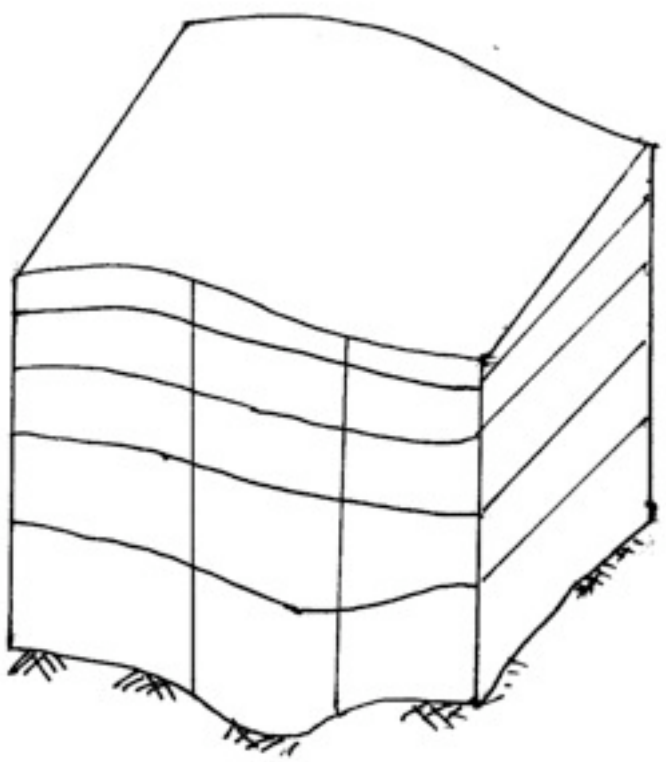
3-D pressure field to be determined

Using incompressibility

$$q_{xx} + q_{yy} + \delta^{-2} q_{zz} = F$$

Discretized ...

$$(q_{i+1} - 2q_i + q_{i-1}) + (q_{j+1} - 2q_j + q_{j-1}) + \delta^{-2}(q_{k+1} - 2q_k + q_{k-1}) = F$$



$$\begin{bmatrix} 2/\delta^2 & \delta^{-2} & .. & 1 & 1 & .. \\ 1/\delta^2 & 2/\delta^2 & 1/\delta^2 & .. & 1 & 1 \\ .. & 1/\delta^2 & 2/\delta^2 & 1/\delta^2 & .. & 1 \\ .. & .. & .. & .. & .. & .. \\ 1 & 1 & .. & 1/\delta^2 & 2/\delta^2 & 1/\delta^2 \\ .. & 1 & 1 & .. & 1/\delta^2 & 2/\delta^2 \end{bmatrix} \begin{bmatrix} .. \\ ... \\ q_{ijk} \\ .. \\ .. \\ .. \end{bmatrix} = \begin{bmatrix} .. \\ ... \\ F_{ijk} \\ .. \\ .. \\ .. \end{bmatrix}$$

Solved efficiently using the multigrid method and line by line (block) relaxation.

Mahadevan et al., 1996a,b, Mahadevan & Archer 1998

Strong vertical velocities

At higher (1 km) model resolution, we find that:

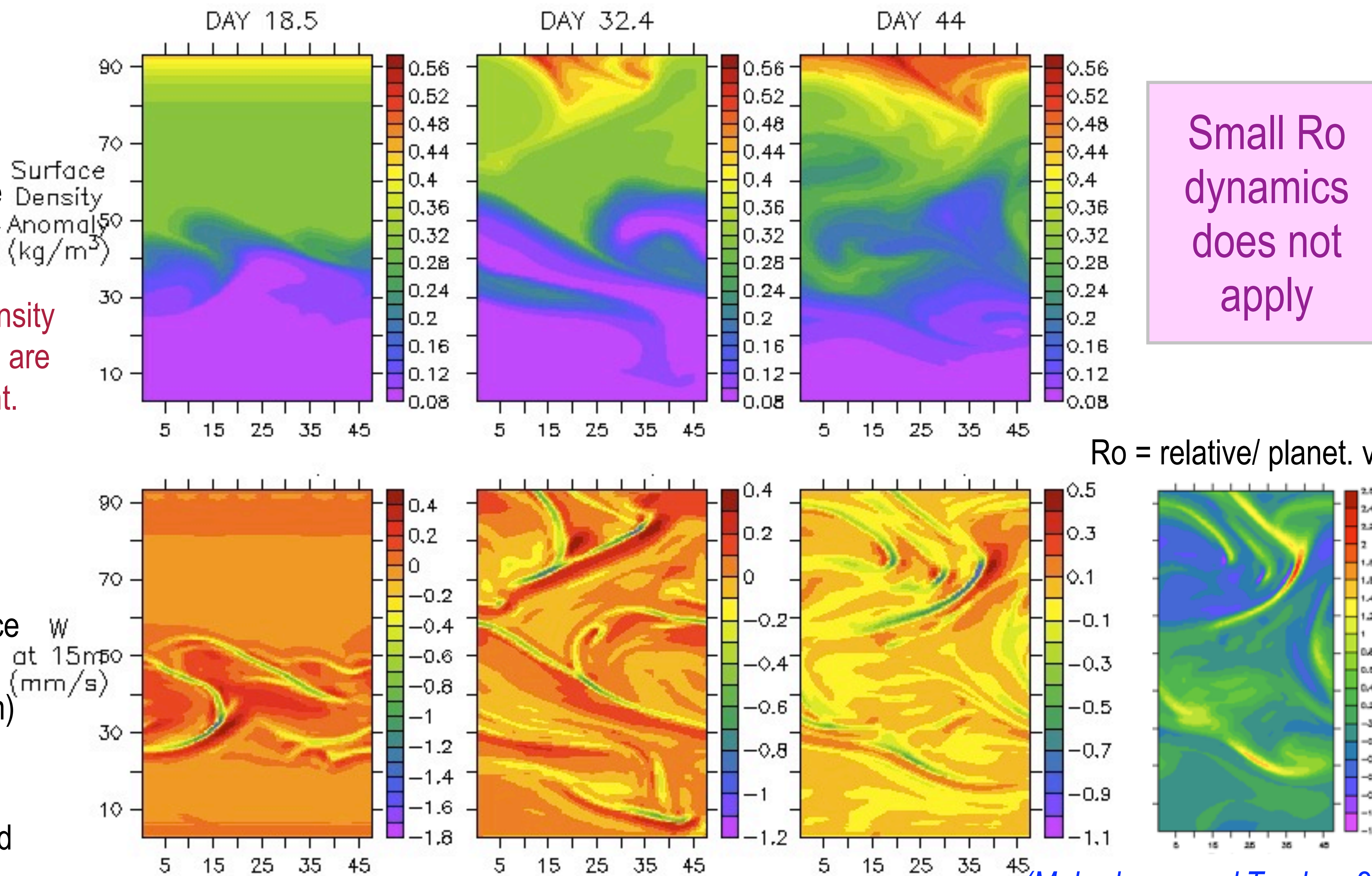
The largest vertical velocities O(100m/day) occur where the Rossby number becomes O(1).
Circulation not in geostrophic / thermal-wind balance -- has a large vertical component.

Surface Density

lateral density gradients are present.

Near-surface w (upper 50m)

~100 m/d

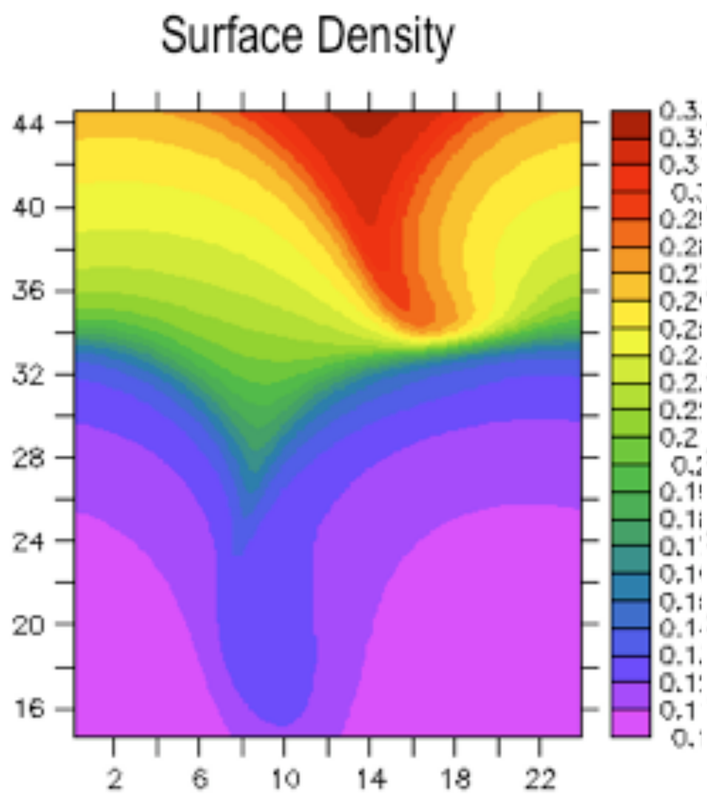


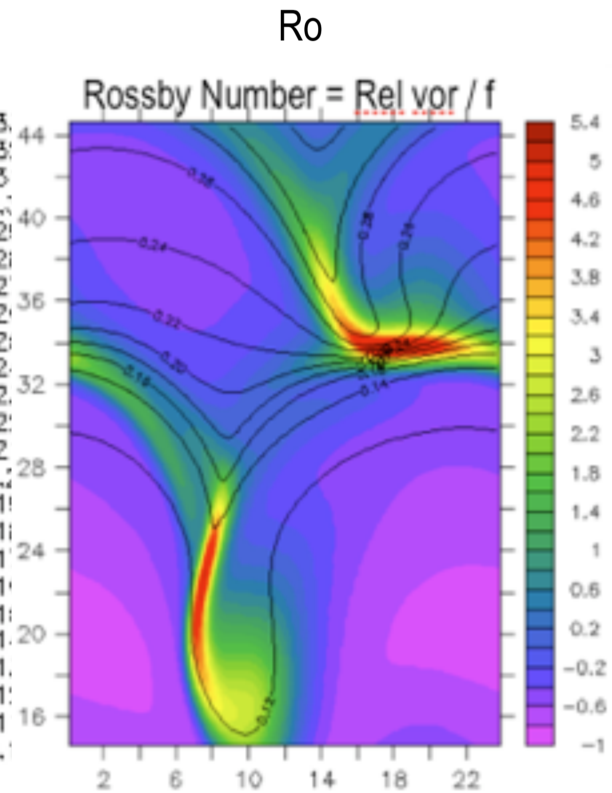
Small Ro dynamics does not apply

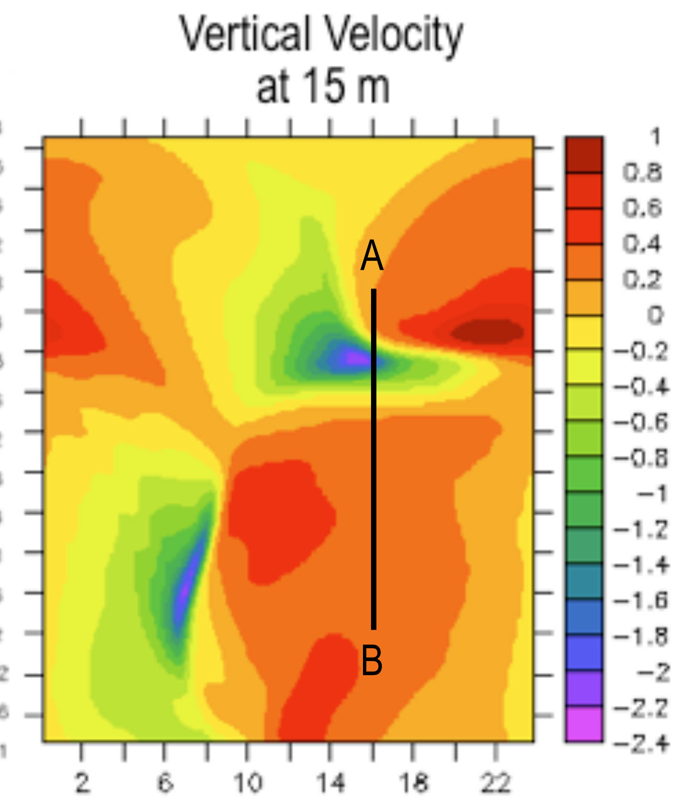
Ro = relative/ planet. vort

(Mahadevan and Tandon, 2006)

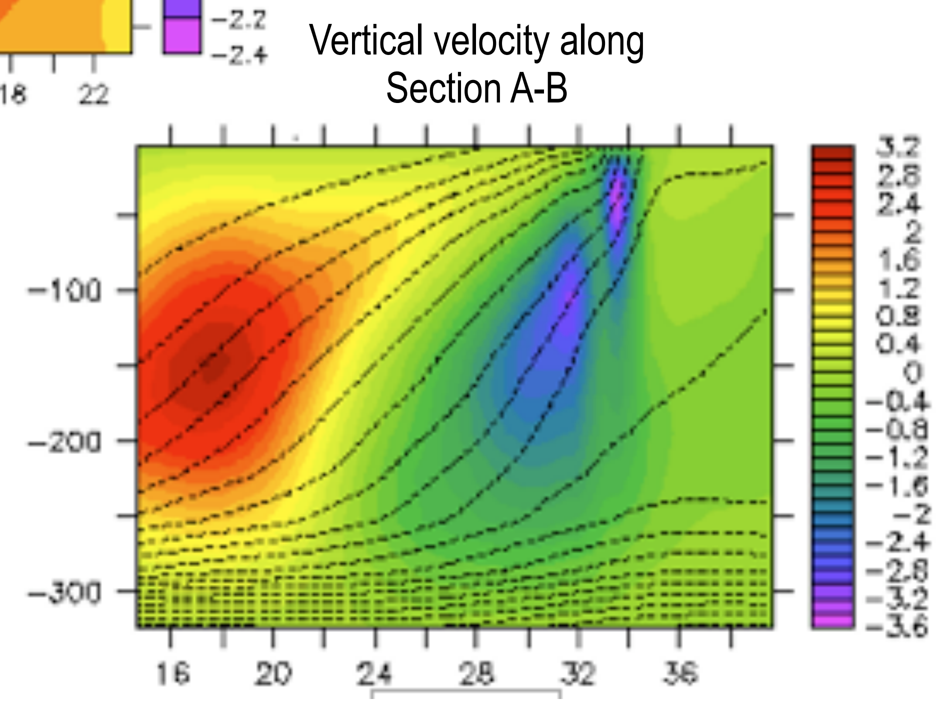
A closer look at a single feature







Frontogenesis



A simpler model for circulation in the vertical plane

Semi-geostrophic: higher order in Ro

$$b = \frac{-g\rho;}{\rho_0}$$

$$F_2^2 \frac{\partial^2 \psi}{\partial z^2} + 2S_2^2 \frac{\partial^2 \psi}{\partial z \partial y} + N^2 \frac{\partial^2 \psi}{\partial y^2} = -2Q_2^g,$$

where $N^2 = b_z,\ S_2^2 = -b_y = f u_{gz},\ F_2^2 = f(f - u_{gy})$

$$\text{Potential vorticity} \quad = \quad q_{2D} = \frac{1}{f}(F_2^2 N^2 - S_2^4)$$

< generally positive, but when it changes sign, this is not solvable

Loss of balance -- leads to vertical motion and mixing.

$$\mathbf{Q}^g = (Q_1^g, Q_2^g) = \left(-\frac{\partial \mathbf{u}_g}{\partial x} \cdot \nabla b, -\frac{\partial \mathbf{u}_g}{\partial y} \cdot \nabla b\right)$$

Summary

On large scales, the wind drives the ocean

Stratification and rotation inhibit vertical motion

Lateral dispersion is highly non-uniform

Fronts - generate large vertical velocities