“The imagination is made keener and more correct by continually studying nature and wrestling with it.”
- Vincent Van Gogh

Biocapillarity

Krogerup Summer School 2011

John W. M. Bush

Department of Mathematics
MIT

Jean Simeon Chardin (1699-1779)

Lecture IA. Fundamentals

A. Surface tension: origins, scaling, modeling

B. Statics: films, menisci, drops and bubbles

C. Dynamics: fluid jets

D. Marangoni flows and surfactants

E. Wetting and water-repellency

SEE NOTES

http://web.mit.edu/1.63/www/lecnote.html

or GOOGLE "Ifluids, MIT, 163"

Lecture IB. Applications in biology

A. Propulsion

B. Drinking strategies

Lecture II. Applications in quantum mechanics

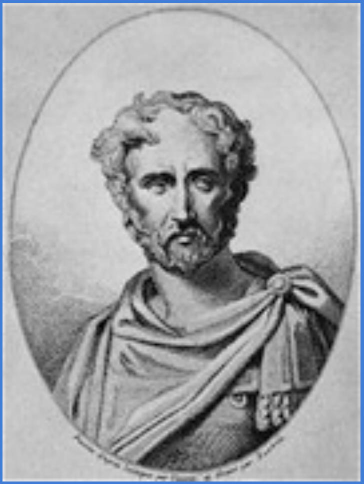
Turning σ into $\hbar$

Surface tension in antiquity

Hero of Alexandria (~ 0 BC)

- greek mathematician and engineer, `the greatest experimentalist of antiquity'
- exploited capillarity in a number of inventions described in his book *Pneumatics*, including the water clock

Pliny the Elder (~0 BC)



- author, natural philosopher, army and naval commander of the early Roman Empire
- described the glassy wakes of ships

"True glory comes in doing what deserves to be written; in writing what deserves to be read; and in so living as to make the world happier."

Surface tension in history

Leonardo da Vinci (1452-1519)

- reported capillary rise in his notebooks
- hypothesized that mountain streams are fed by capillary networks

Francis Hauksbee (1666-1713)

- conducted systematic investigation of capillary rise
- his work was described in Newton's Opticks, but no mention was made of him

Benjamin Franklin (1706-1790)

- polymath: scientist, inventor, politician
- examined the ability of oil to suppress waves

Surface tension in history

Pierre-Simon Laplace (1749-1827)

- french mathematician and astronomer
- elucidated the concept and theoretical description of the meniscus: hence, Laplace pressure

Thomas Young (1773-1829)

- polymath, solid mechanician, scientist, linguist
- demonstrated wave nature of light with ripple tank expts
- described wetting of a solid by a fluid

Joseph Plateau (1801-1883)

- Belgian physicist, continued his expts after losing his sight
- extensive study of capillary phenomena, soap films, minimal surfaces, drops and bubbles

Motivation: who cares about surface tension?

As we shall soon see, surface tension dominates gravity on a scale less than the capillary length, ~2 mm.

Biology

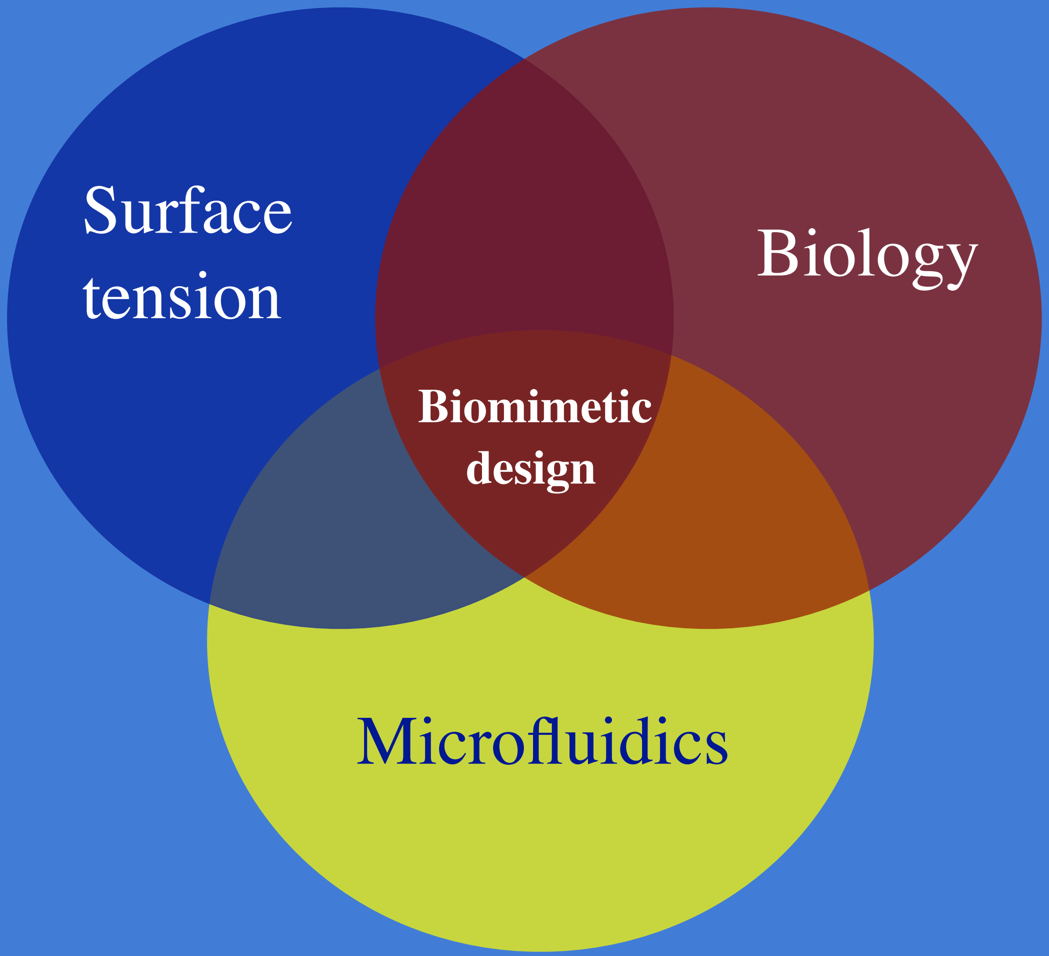
- all small creatures live in a world dominated by surface tension
- surface tension important for insects for many basic functions
- weight support and propulsion at the water surface
- water intake: drinking
- adhesion and deadhesion via surface tension
- underwater breathing and diving via surface tension
- natural strategies for water-repellency in plants and animals
- hunting with drops and bubbles

Hunting with bubbles

The Pistol Shrimp

MOTIVATION

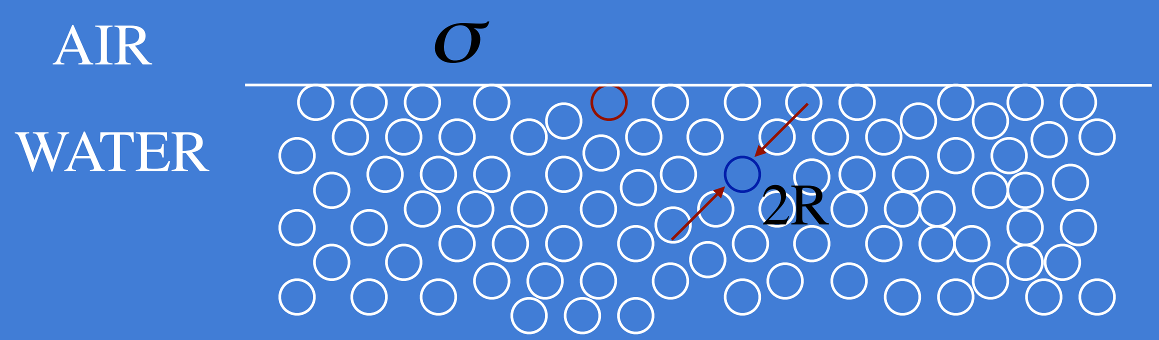
- to rationalize Nature's designs



Bonus: to inspire and inform biomimetic design

Surface Tension: molecular origins

- each molecule in a fluid feels a cohesive force with surrounding molecules
- molecules at interface feel half this force; are in an energetically unfavourable state
- the creation of new surface is thus energetically costly



- cohesive energy per molecule of radius R in bulk is U, at surface is U/2
- surface tension is this loss of cohesive energy per unit area:

$$\boxed{\sigma \sim \frac{U}{R^2}}$$

Units: $[\sigma] = \frac{\text{ENERGY}}{\text{AREA}} = \frac{\text{FORCE}}{\text{LENGTH}}$

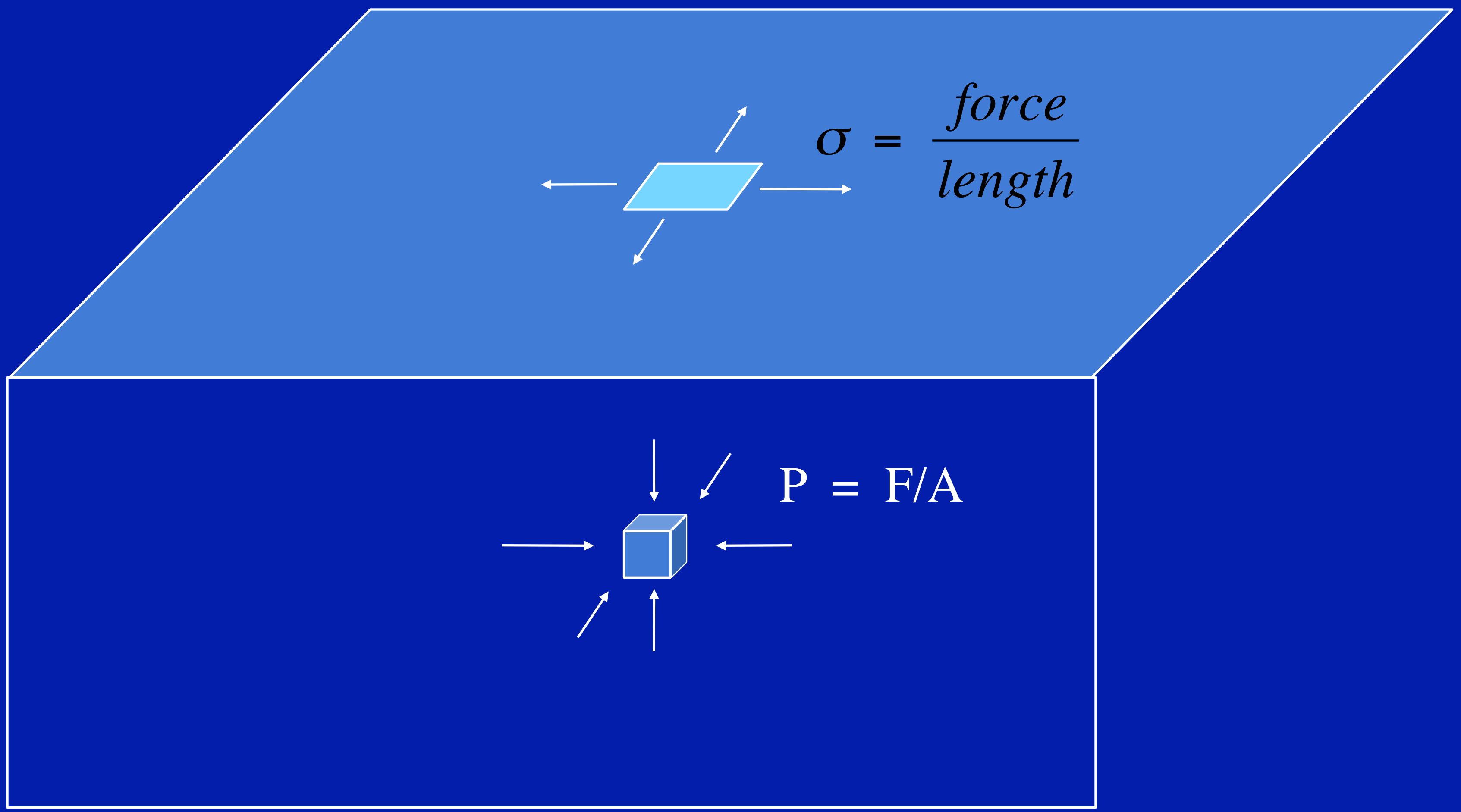
- air-water $\sigma \sim 70$ dyne/cm; oils $\sigma \sim 20$ dyne/cm; liquid metals $\sigma \sim 500$ dyne/cm

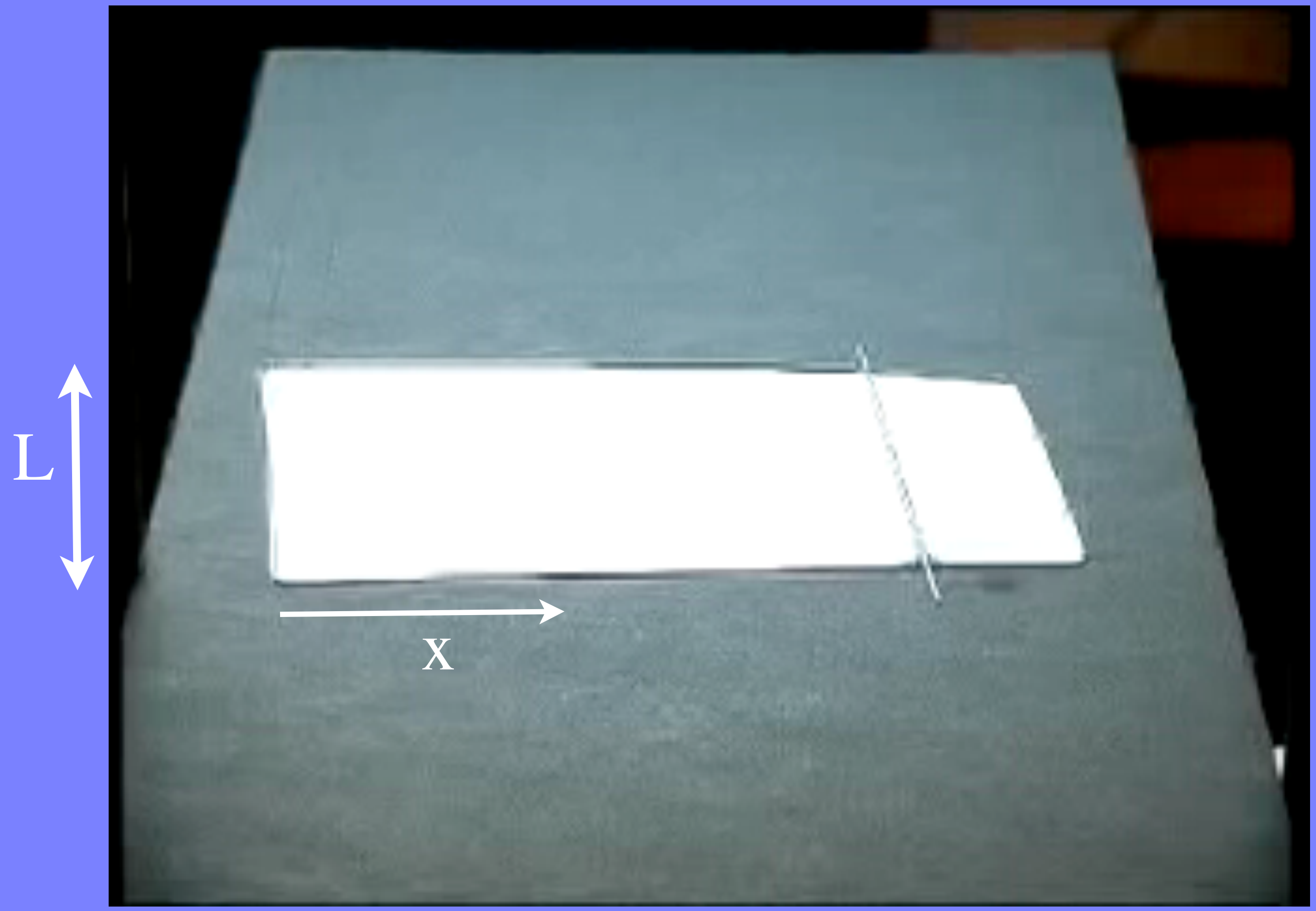
Surface tension in flocks, schools and swarms?




Might the cost of being on the edge give rise to analogous behavior?

Surface tension: analogous to a negative surface pressure





Surface tension: $[\ \sigma\ ] = \dfrac{FORCE}{LENGTH} = \dfrac{ENERGY}{AREA}$

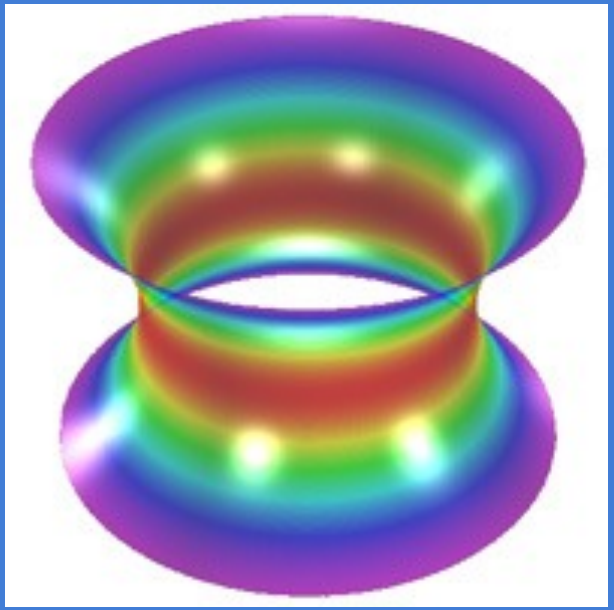
Surface energy: $E_\sigma = \int_S \sigma \, dA = 2\,\sigma\,L\,x$

Force acting on rod: $F = \dfrac{dE_\sigma}{dx} = 2\,\sigma\,L$

Minimal surfaces: surface energy/area minimized by soap films

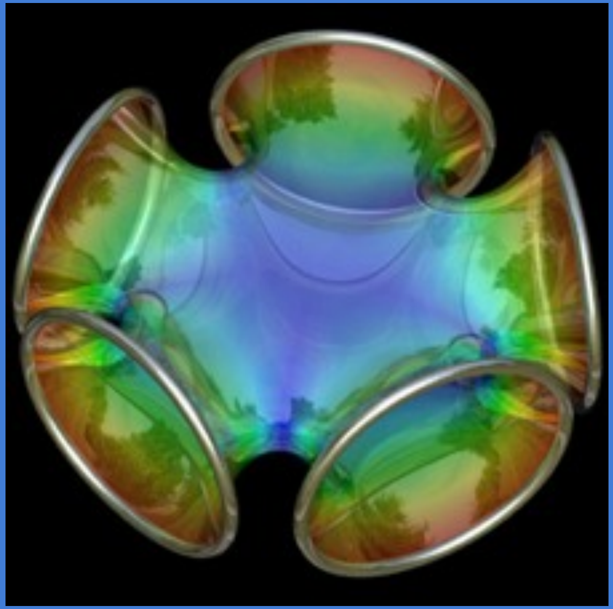
The creation of surface is energetically costly

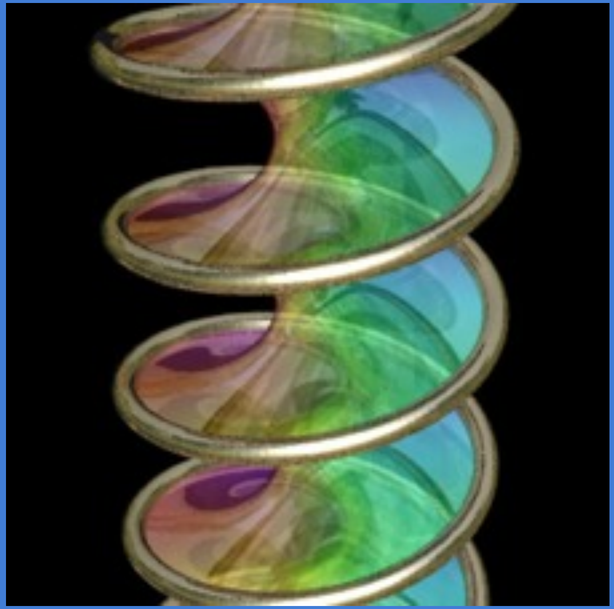
- quasi-static soap films (for which gravity, inertia are negligible) take the form of minimal surfaces

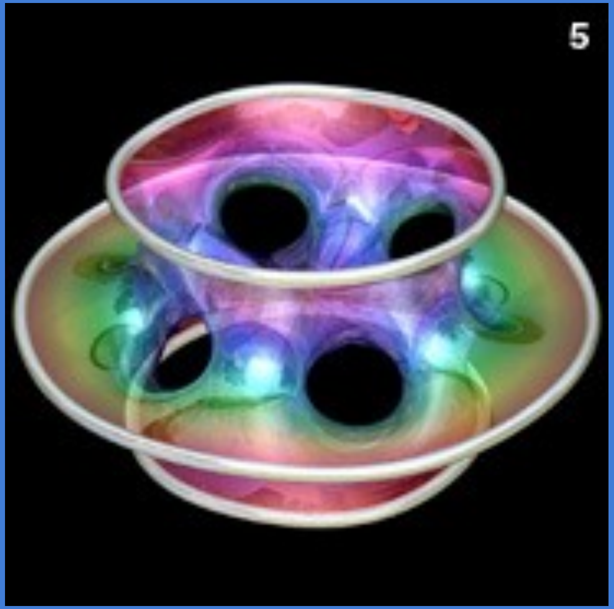


- hence their interest to mathematicians:

"Find the minimal surface bound by the multiply connected curve C, where C ...."







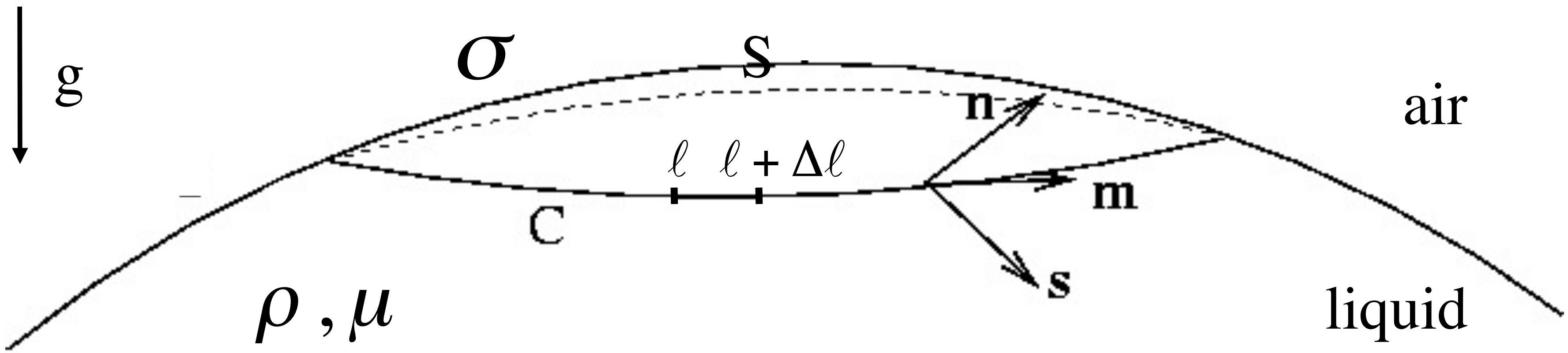
The creation of surface is energetically costly

Thus:

- small drops are nearly spherical
- fluid jets pinch off into droplets
- fluid atomization results in spherical drops
- wet hair sticks together: the “wet look”
- bubbles and films are fragile

Surface tension: Geometry

Along a contour C bounding a surface S there is a tensile force per unit length σ acting in the $\mathbf{s}$ direction



Net force on S:

$$\boxed{\oint_C \sigma\, \mathbf{s}\; d\ell = \iint_S \sigma\, (\nabla \cdot \mathbf{n})\, \mathbf{n}\; dS \;+\; \iint_S \nabla\sigma\; dS}$$

curvature pressure

Marangoni stress

1) normal curvature pressure $\sigma \nabla \cdot \mathbf{n}$ resists surface deformation

2) tangential Marangoni stresses may arise from $\nabla\sigma$

Curvature pressures, $\sigma \nabla \cdot \mathbf{n}$, make the surface behave as a trampoline.

Marangoni effects

- gradients in surface tension drive flow tangent to surface
- $\nabla\sigma$ may arise due to dependence of $\sigma(T, c, \Gamma)$

The cocktail boat (with Lisa Burton)

Interfacial Fluid Dynamics: Governing Equations

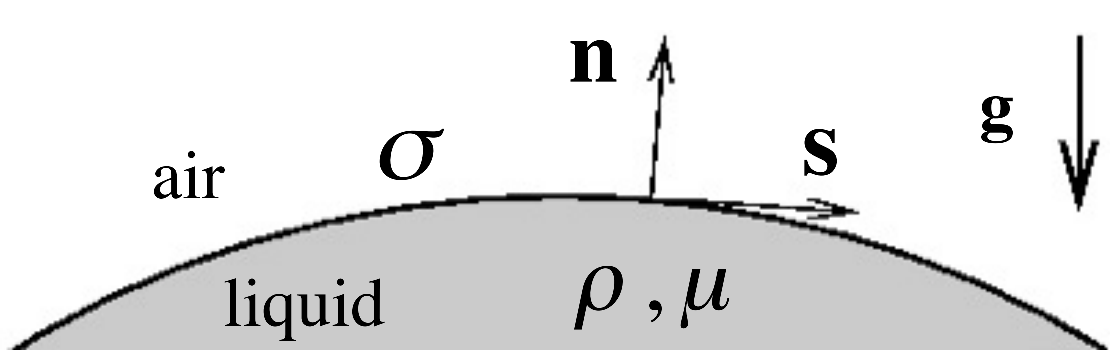
Navier-Stokes:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \rho\mathbf{g} + \mu\nabla^2\mathbf{u} \quad , \qquad \nabla\cdot\mathbf{u} = 0$$

Boundary Conditions

Normal stress: $\Delta\, \mathbf{n}\cdot\mathbf{T}\cdot\mathbf{n} = \sigma\,\nabla\cdot\mathbf{n}$

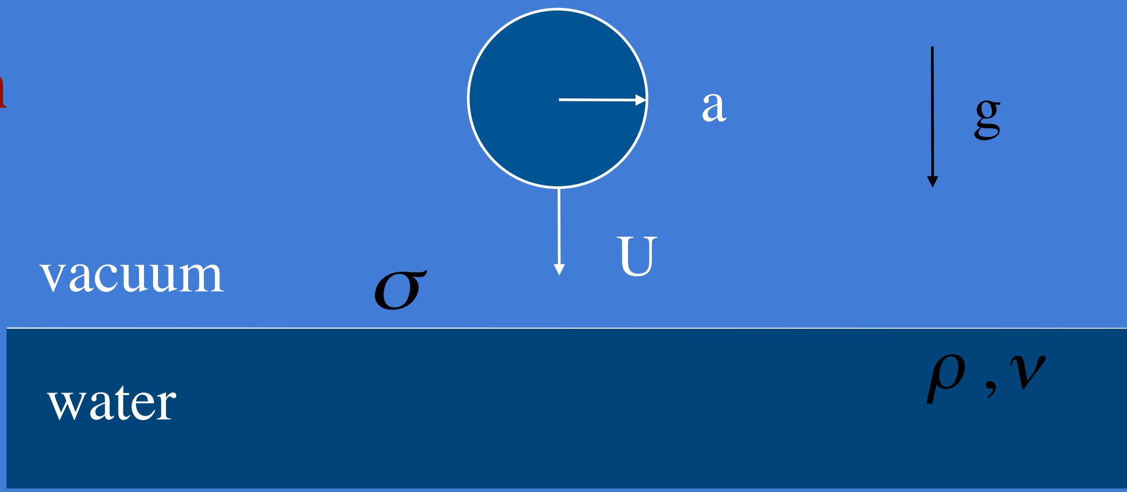
Tangential stress: $\Delta\, \mathbf{n}\cdot\mathbf{T}\cdot\mathbf{s} = \nabla_s\sigma$



Stress tensor

$$\mathbf{T} = -p\mathbf{I} + \mu\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right)$$

The scaling of surface tension



$$W_e = \frac{\rho U^2 a}{\sigma} = \frac{\text{INERTIA}}{\text{CURVATURE}} = \text{Weber number}$$

$$C_a = \frac{\rho \nu U}{\sigma} = \frac{\text{VISCOSITY}}{\text{CURVATURE}} = \text{Capillary number}$$

$$B_o = \frac{\rho g a^2}{\sigma} = \frac{\text{GRAVITY}}{\text{CURVATURE}} = \text{Bond number}$$

Note: σ is dominant relative to gravity when $B_o < 1$

i.e. $a < \left(\frac{\sigma}{\rho g}\right)^{1/2} = \ell_c =$ capillary length ~ 2mm for air-water

When is surface tension important relative to gravity?

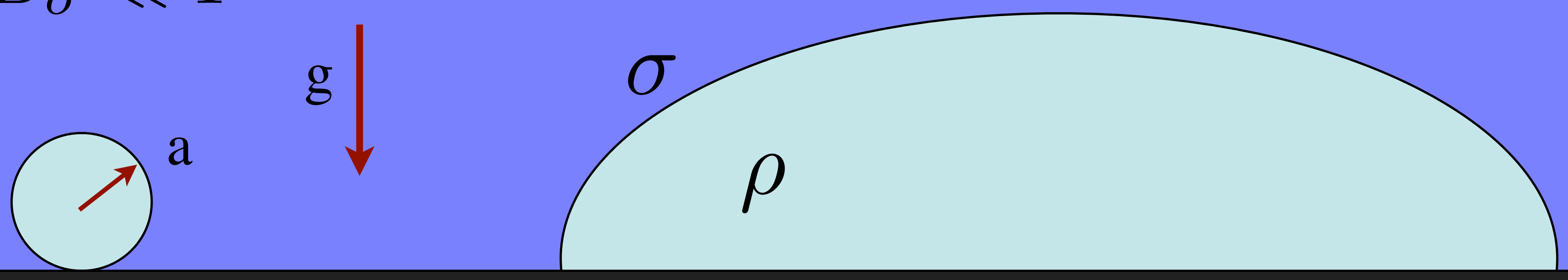
- when curvature pressures are large relative to hydrostatic:

Bond number: $$B_o = \frac{\rho g a}{\sigma / a} = \frac{\rho g a^2}{\sigma} < 1$$

i.e. for drops small relative to the capillary length:

$$\boxed{a < l_c = \left(\frac{\sigma}{\rho g}\right)^{1/2}} \sim$$ 2 mm for air-water ($\sigma = 70$ dynes/cm)

$B_o \ll 1$ $\quad$ $B_o > 1$



Surface tension dominates the world of insects - and of microfluidics.

A key question in a bug’s life...

What sets the size of raindrops?

Falling drops

Force balance:

$$\rho_a U^2 a^2 \sim \; M g = \tfrac{4}{3}\pi a^3 \rho g$$

Fall speed: $U \sim \sqrt{ga\ \rho/\rho_a}$

Drop integrity requires:

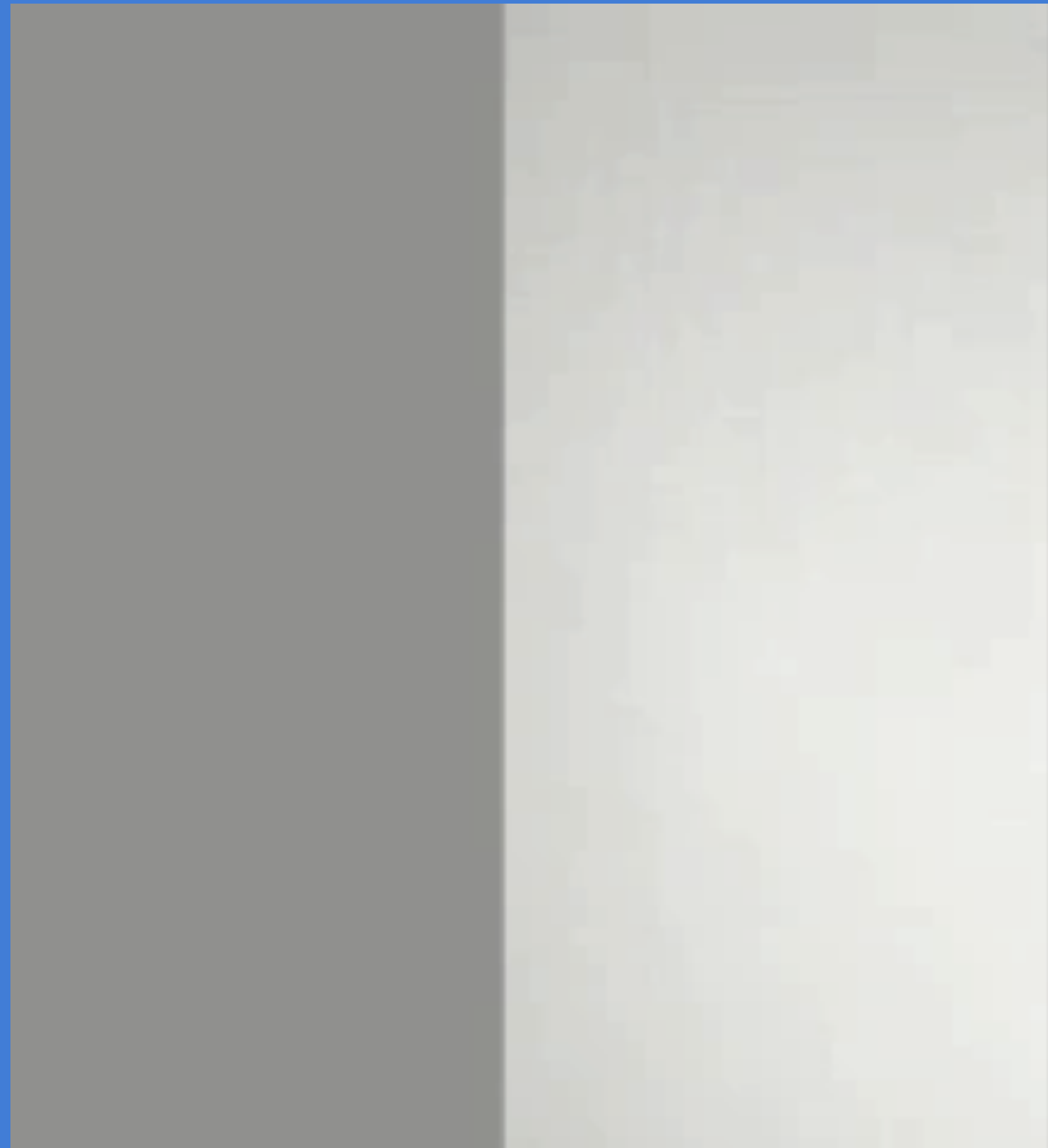
$$\rho_a U^2 = \; \rho g a \; < \; \sigma/a$$

Small drops

If a drop is small relative to the capillary length

$$a < \ell_c = \sqrt{\sigma/\rho g} \approx 2\text{mm} \; ,$$

σ maintains it against the destabilizing influence of aerodynamic stresses.



David Quere, MFM

Big drops

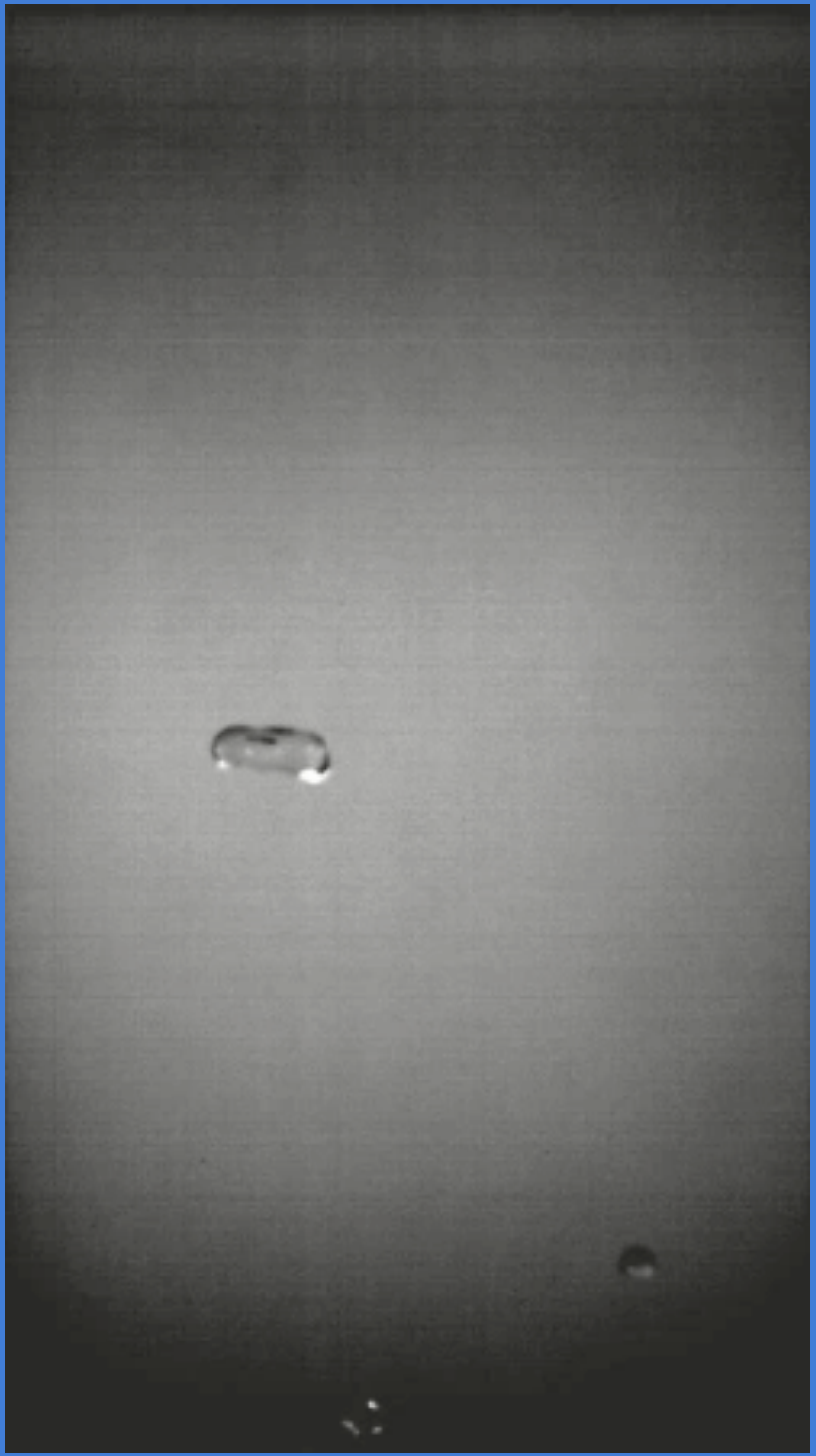
Drops larger than the capillary length ,

$$a > \ell_c \approx 2\text{mm}$$

break up under the influence of aerodynamic stresses.

The break-up yields drops with size of order:

$$\ell_c \approx 2\text{mm}$$



David Quere, MFM

Fluid Statics

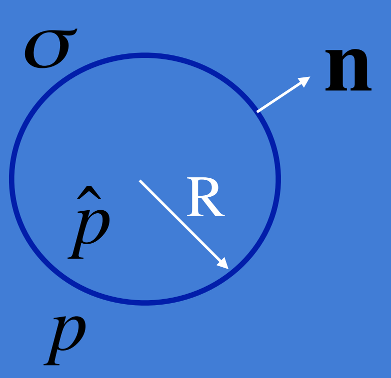
$$\mathbf{T} = -p\mathbf{I} \quad , \quad \hat{\mathbf{T}} = -\hat{p}\,\mathbf{I}$$

Normal stress balance: $\Delta\, \mathbf{n}\cdot\mathbf{T}\cdot\mathbf{n} = \sigma\,(\nabla\cdot\mathbf{n}) \longrightarrow \hat{p} - p = \sigma\,\nabla\cdot\mathbf{n}$

Tangential stress balance: $\Delta\, \mathbf{n}\cdot\mathbf{T}\cdot\mathbf{t} = \nabla\sigma \longrightarrow 0 = \nabla\sigma$

Stationary bubble: what is the pressure drop across a bubble surface?

$$\hat{p} - p = \sigma\,\nabla\cdot\mathbf{n} = \frac{2\sigma}{\mathrm{R}}$$



- smaller bubbles burst more loudly than large ones
- champagne is louder than beer

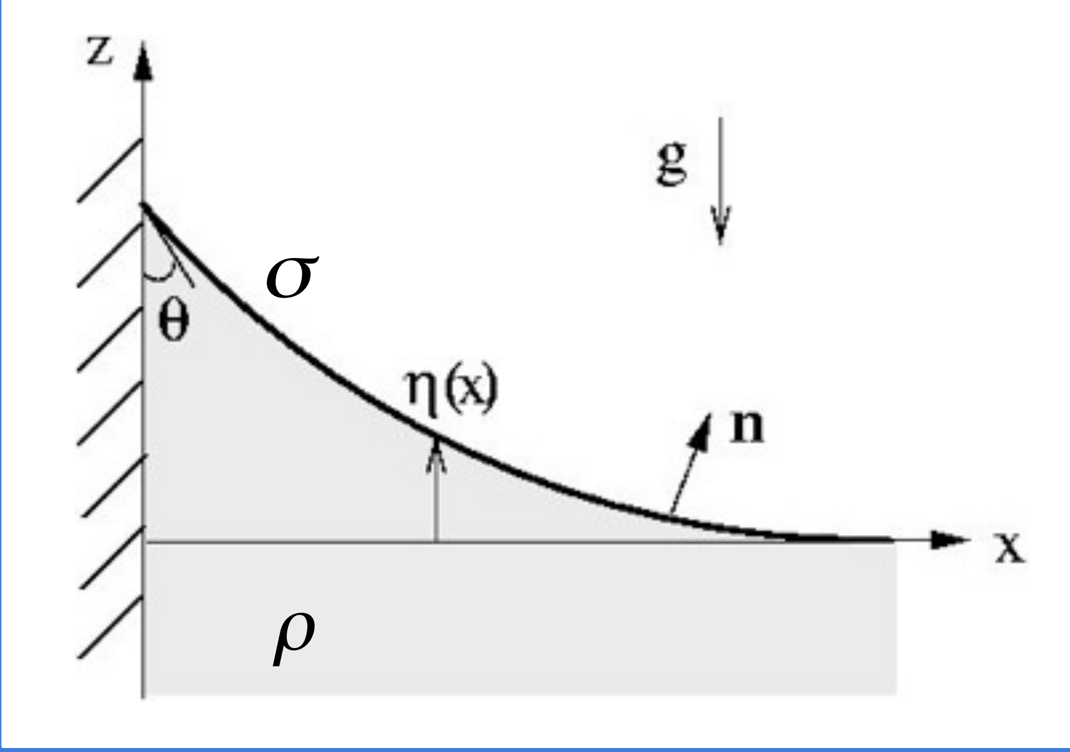
Capillary pressure

Which way does the air go?

(MFM)

Which drop to drink from?

Static 2D meniscus



Young-Laplace equation

$$\boxed{\rho g \eta = \sigma \nabla \cdot \mathbf{n}}$$

Define: $\mathrm{f}(\mathrm{x},\mathrm{z}) = \mathrm{z} - \eta(\mathrm{x})$

so that $\mathbf{n} = \dfrac{\nabla \mathrm{f}}{|\nabla \mathrm{f}|} = \dfrac{\mathbf{z} - \eta_{\mathrm{x}}(\mathrm{x})\mathbf{x}}{[1+\eta_{\mathrm{x}}^2(\mathrm{x})]^{1/2}}$

and $\nabla \cdot \mathbf{n} = \dfrac{\eta_{\mathbf{xx}}}{(1+\eta_{\mathbf{x}}^{\mathbf{2}})^{3/2}} \approx \eta_{\mathrm{xx}}$ provided $|\eta_{\mathrm{x}}| << 1$

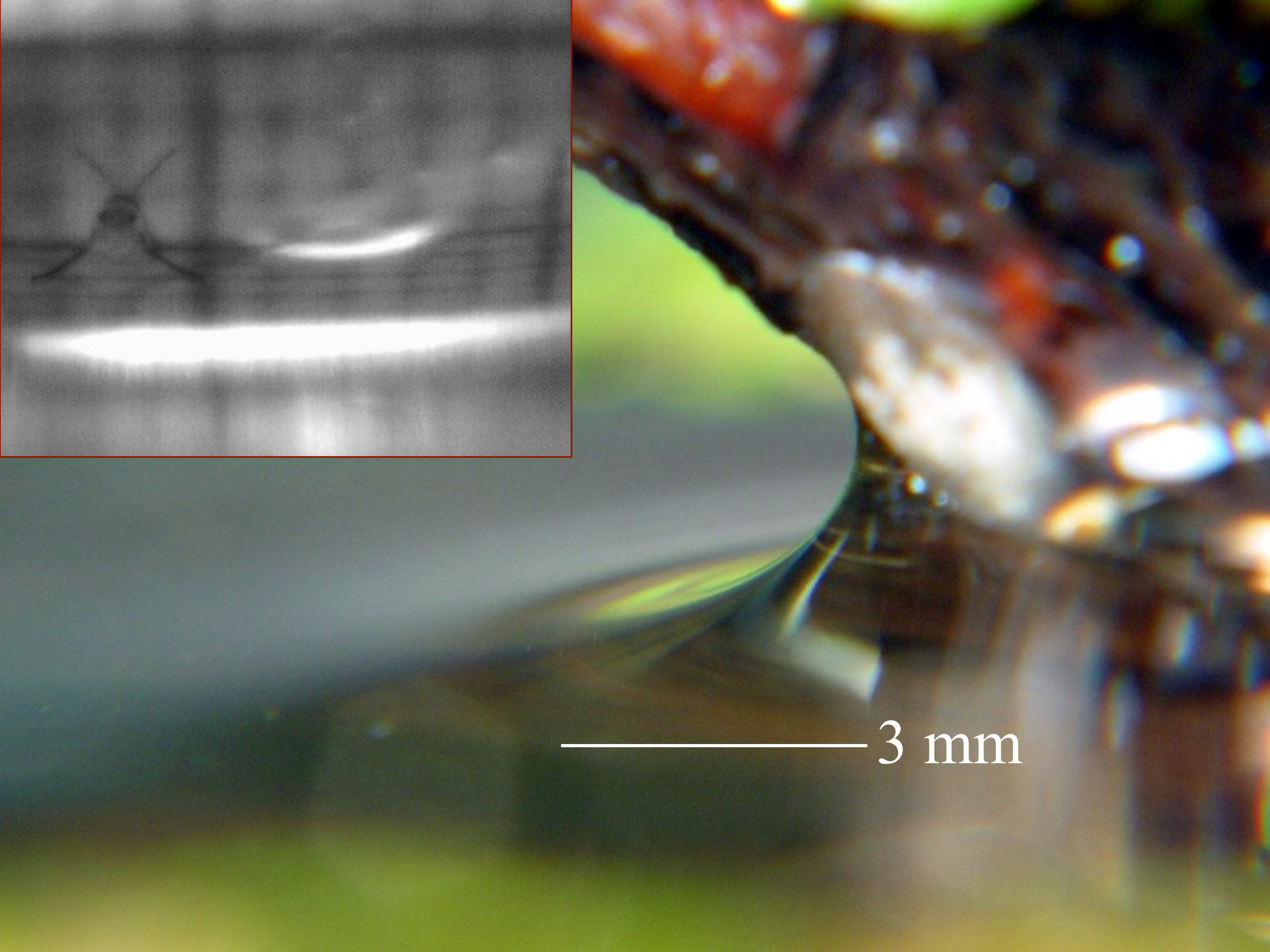
Linearized: $\rho g \eta(\mathrm{x}) = \sigma \eta_{\mathbf{xx}}$ BCs: $\eta_{\mathrm{x}}(0) = -\cot\theta \;,\; \eta(\infty) = 0$

$$\longrightarrow \boxed{\eta(x) = \ell_c \cot\theta \exp(-x/\ell_c)}$$

Meniscus decays exponentially over the capillary length $\ell_c = \left(\dfrac{\sigma}{\rho g}\right)^{1/2}$

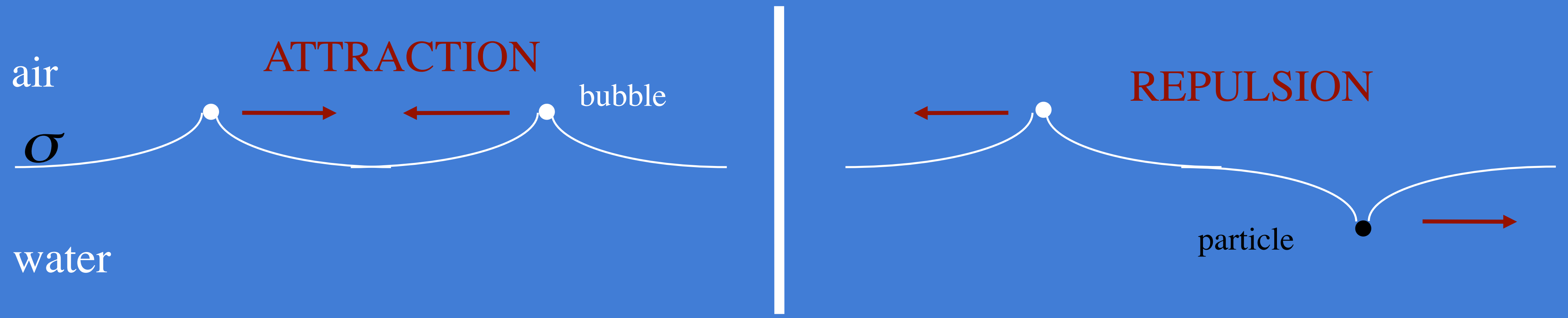
Who cares? **Meniscus-climbing insects**

What if $U < \sqrt{g \ell_c} \sim 35\ cm/s$, the capillary escape velocity?


3 mm

Capillary forces: The Cheerios effect

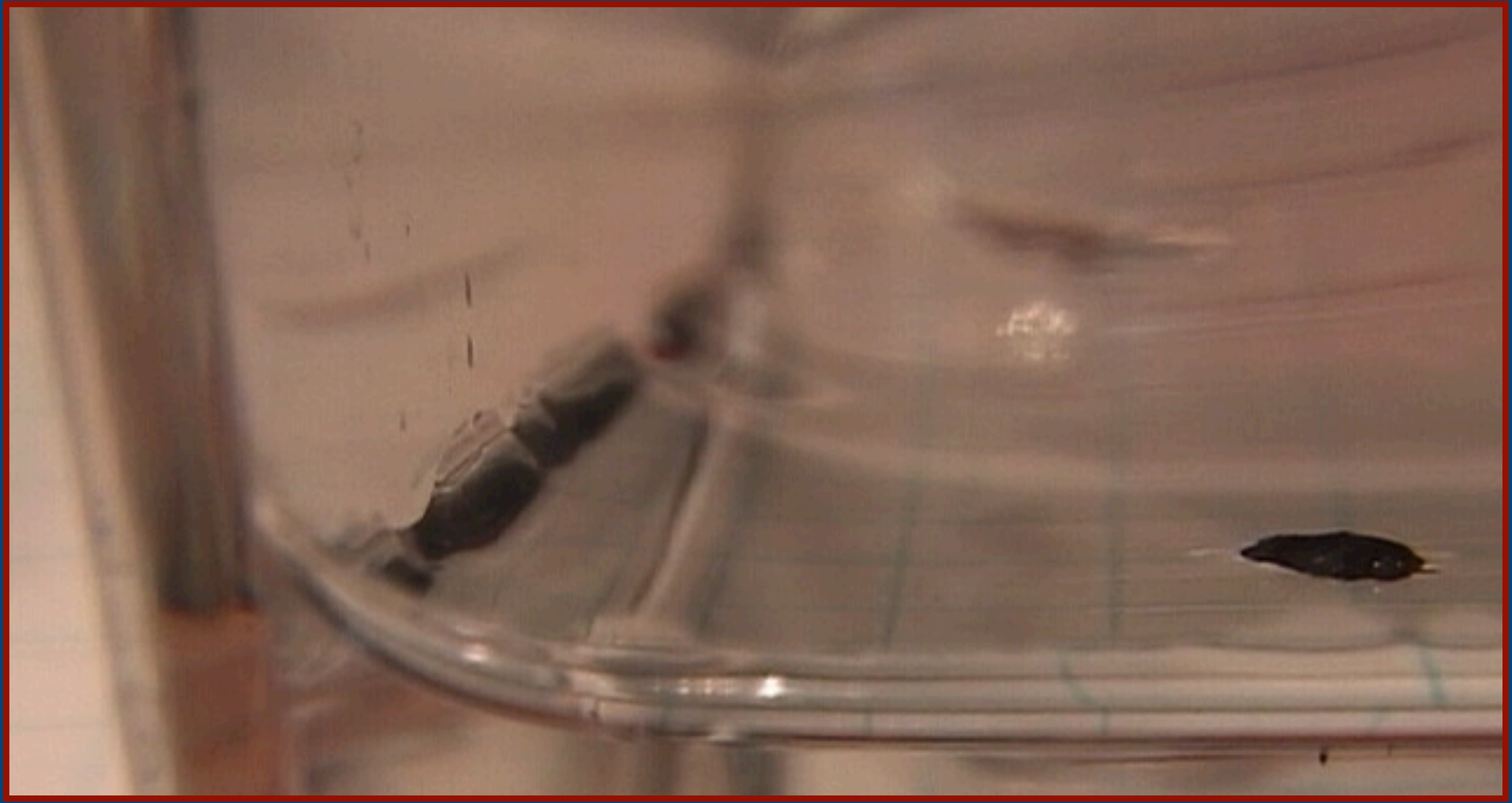
- exist between objects floating at a free surface
- attractive/repulsive for meniscii of the same/opposite sense



- explains the formation of bubble rafts in champagne
- explains the attraction of Cheerios in a bowl of milk
- used by small insects to move themselves along the free surface

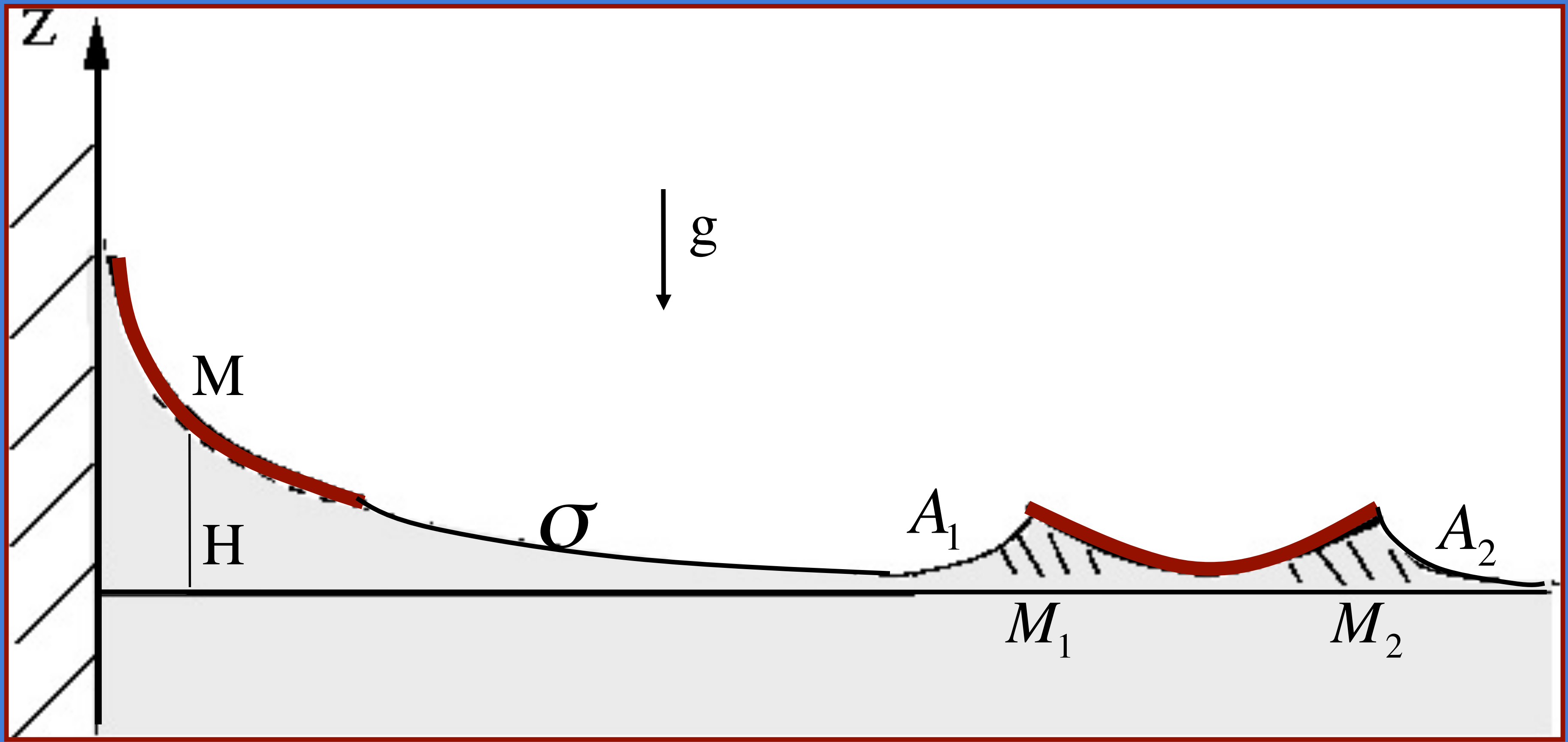
Meniscus climbing

Hu & Bush (2005)



- Anurida arches its back to match curvature of meniscus
- anomalous surface energy exceeds GPE associated with climb

Meniscus-climbing: Energetics



Body climbs provided total energy minimized:

$$\sigma(A_1 + A_2) + M_1 g h_1 + M_2 g h_2 > MgH$$

Microvelia

- pulls up with its front legs to generate lateral force
- pulls up with rear legs to balance torques
- pushes down with middle legs to support its weight

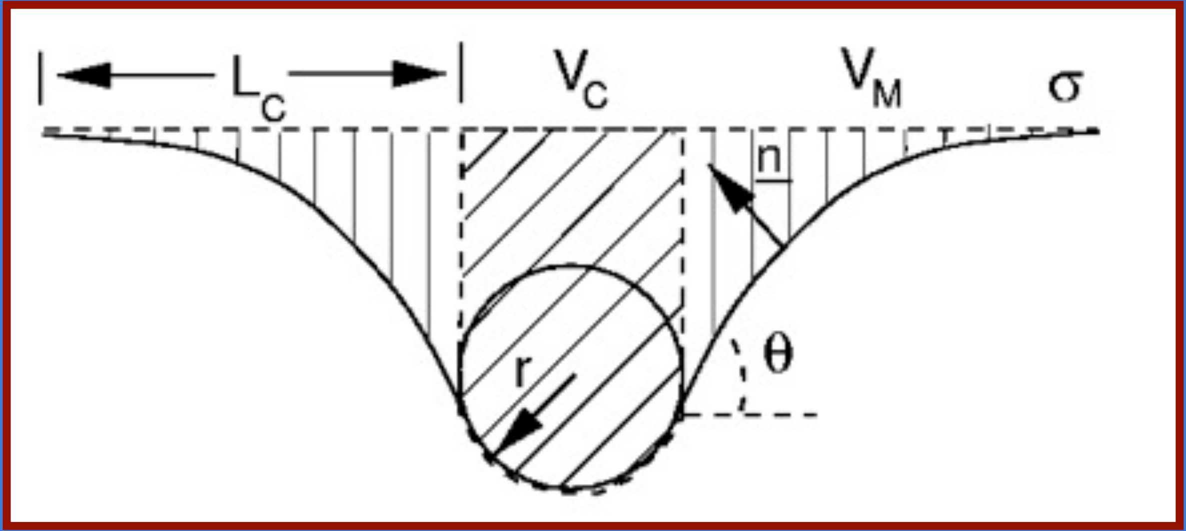
Other uses for capillary attraction

Heavy things sink, light things float.

Not exactly…..

Statics of 2D floating bodies

Keller (1998)



$\Rightarrow \; F_b = \rho\, g\, V_c =$ wt. of fluid displaced above body

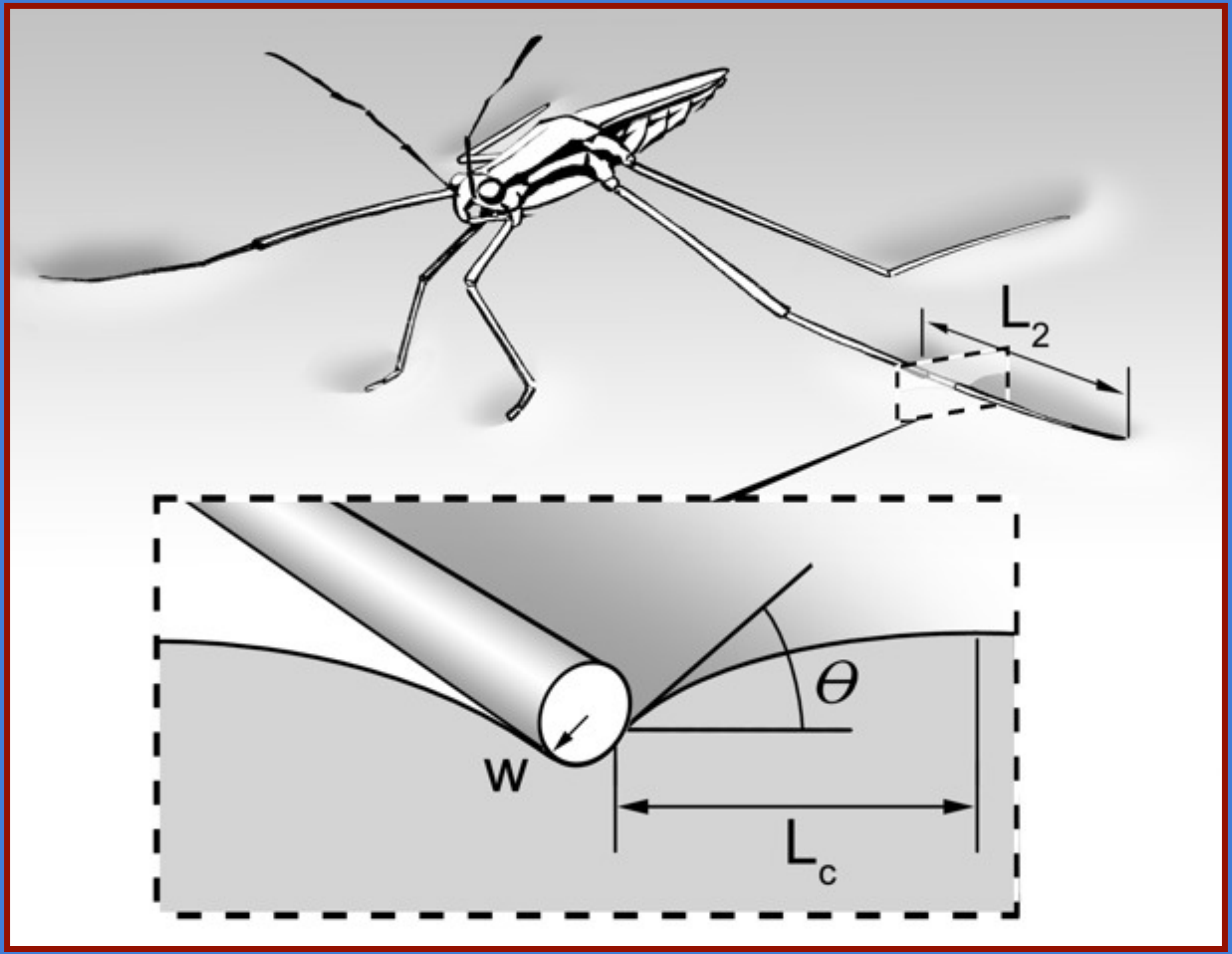
$\Rightarrow F_c = 2\sigma \sin\theta = \rho g V_M =$ wt. of fluid above meniscus

$$\Longrightarrow \boxed{\frac{F_b}{F_c} = \frac{V_c}{V_M} \approx \frac{r}{L_c}}$$

where $L_c = \left(\dfrac{\sigma}{\rho g}\right)^{1/2} \approx 0.3\,\mathrm{cm}$

$\Longrightarrow$ small objects (eg. insects) supported primarily by σ

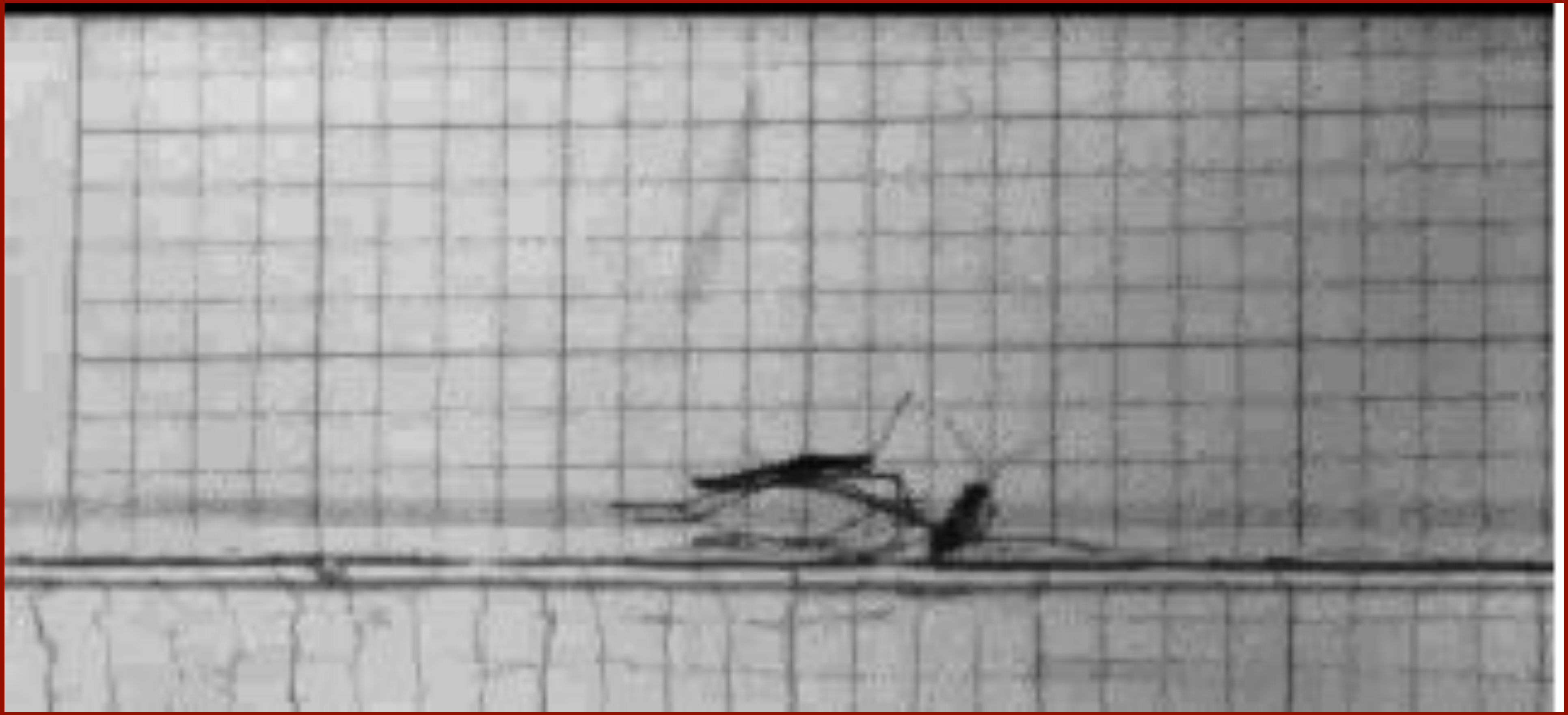
$\Longrightarrow$ objects more dense than water can float



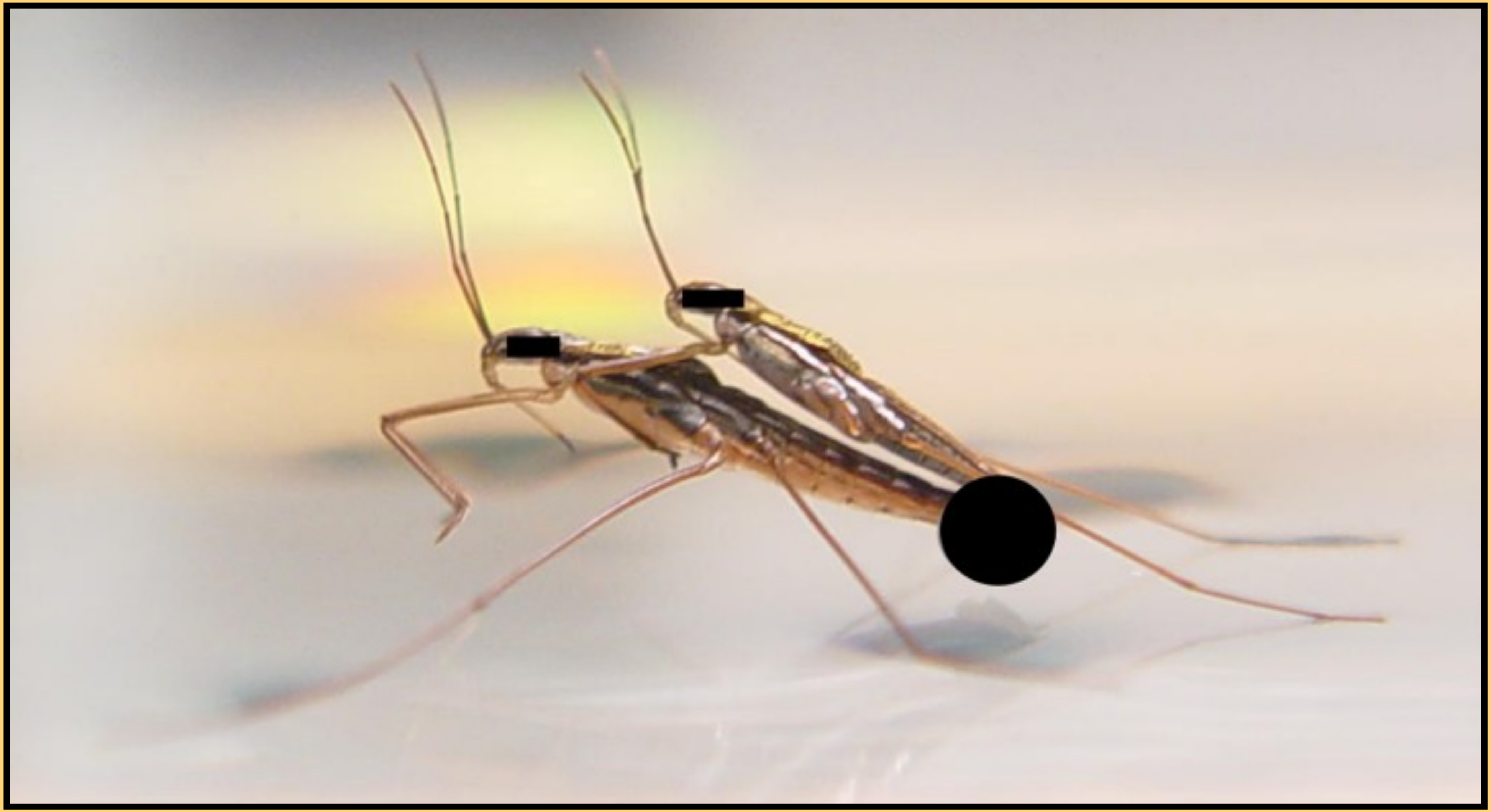
Static weight support requires: $$M\,g\;<\;2\,\sigma\,P\cos\theta$$

where P is total contact length

Water strider combat/courtship



Interfacial Love

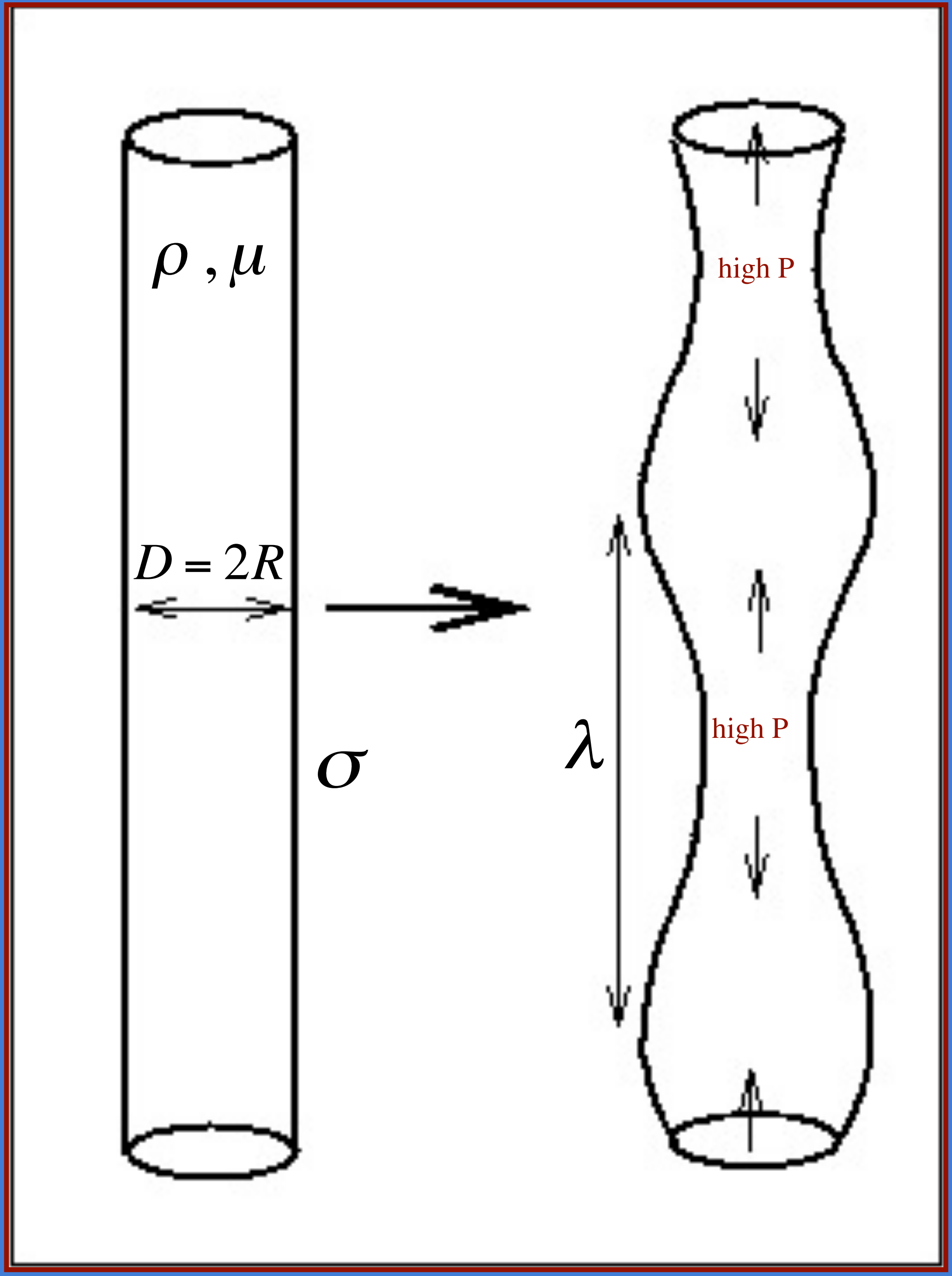


Fluid jets

Capillary self-defence

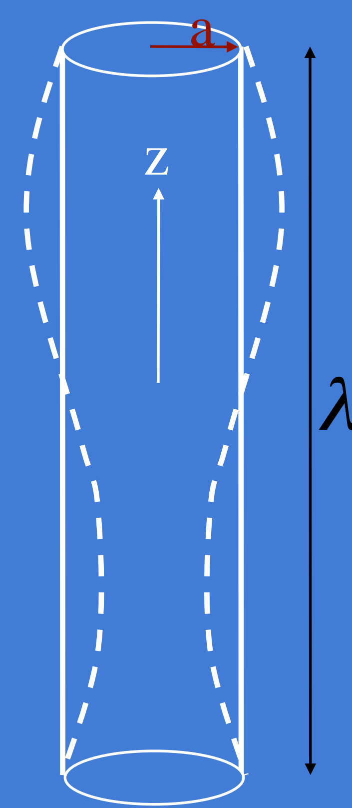
Rayleigh-Plateau Instability (Rayleigh 1900)

Capillary pinch-off of a fluid thread



Rayleigh-Plateau instability

$$k = 2\pi / \lambda$$



Seek normal modes:

$$r = a + \varepsilon\, e^{\omega t + ikz} \quad , \quad u_r = R(r)\, e^{\omega t + ikz}$$

$$u_z = Z(r)\, e^{\omega t + ikz} \quad , \quad p = P(r)\, e^{\omega t + ikz}$$

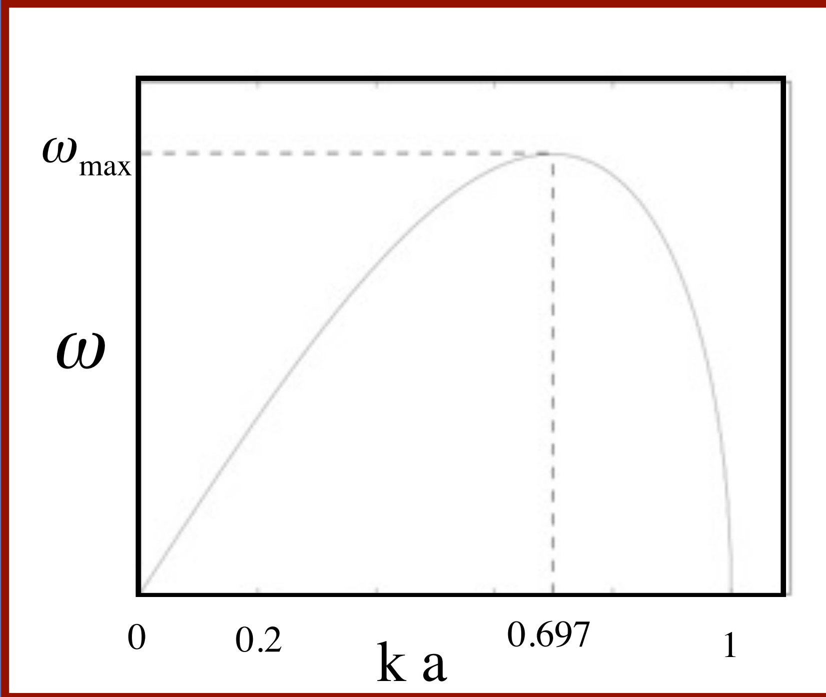
Sub into Navier-Stokes and linearize to solve for disturbance fields

Dispersion relation:

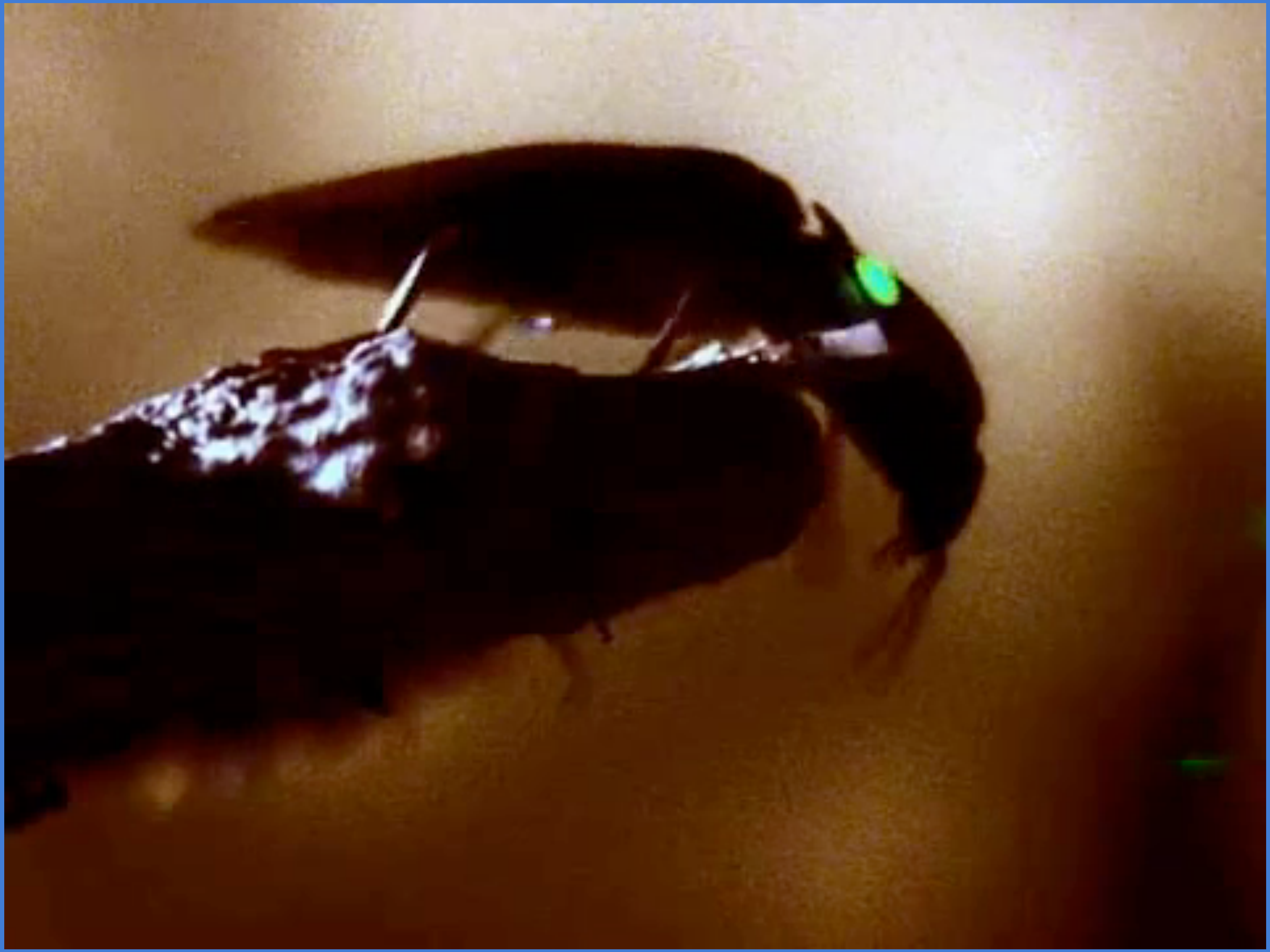
$$\boxed{\omega^2 = \frac{\sigma\, k}{\rho\, a^2} \frac{I_1(ka)}{I_0(ka)} (1 - k^2 a^2)}$$

- instability for modes with $\lambda > 2\pi\, a$
- fastest growing mode: $\lambda = 9.02\, a$

Break-up time: $\tau_{break-up} = 2.91 \left(\dfrac{\rho\, a^3}{\sigma} \right)^{1/2}$



Hunting with drops



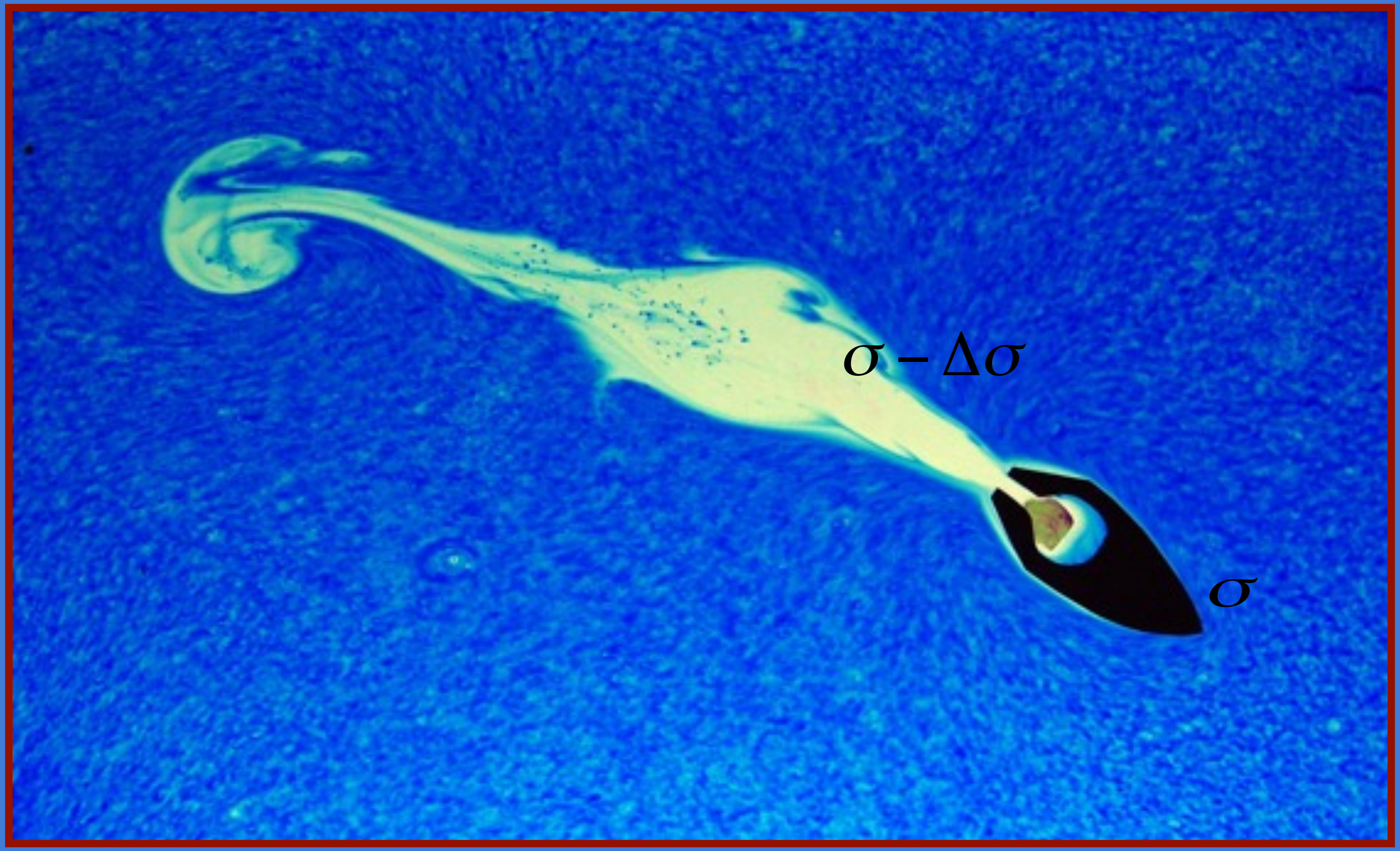
The Archer Fish

Marangoni Flows

- flows dominated by the influence of surface tension gradients

Recall tangential stress BC: $\Delta\, \mathbf{n}\cdot\mathbf{T}\cdot\mathbf{s} = \nabla\sigma$

- $\nabla\sigma$ may arise due to dependence of $\sigma(T, c, \Gamma)$

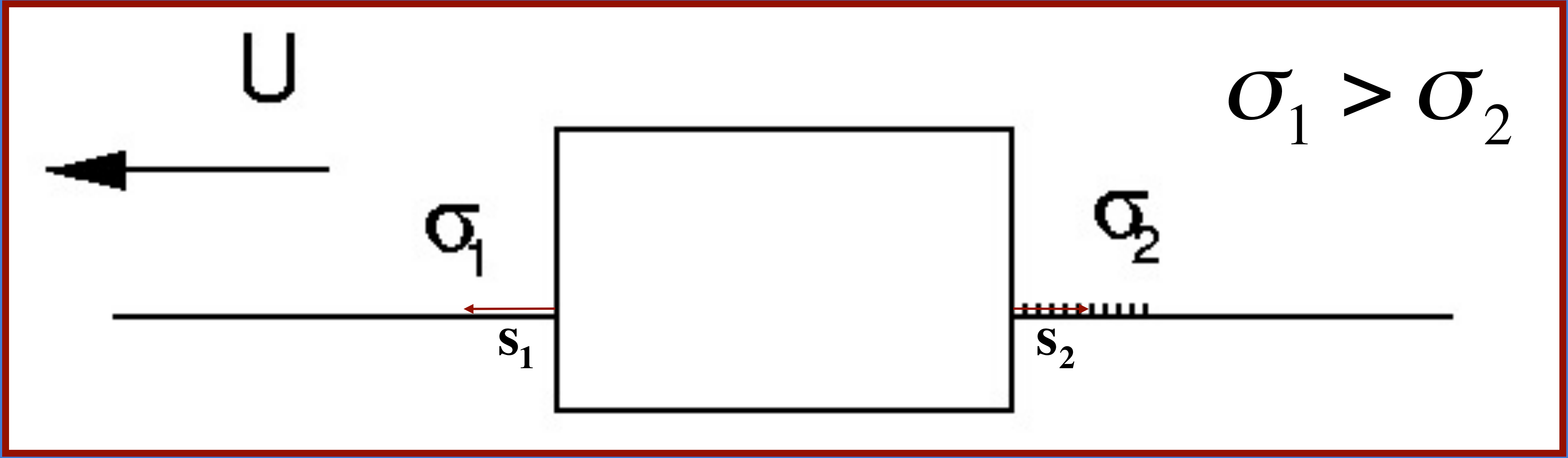


Marangoni propulsion

- lateral force may be generated by surface tension gradient

$$\boxed{F = \int_C \sigma \, \mathbf{s} \, d\ell}$$

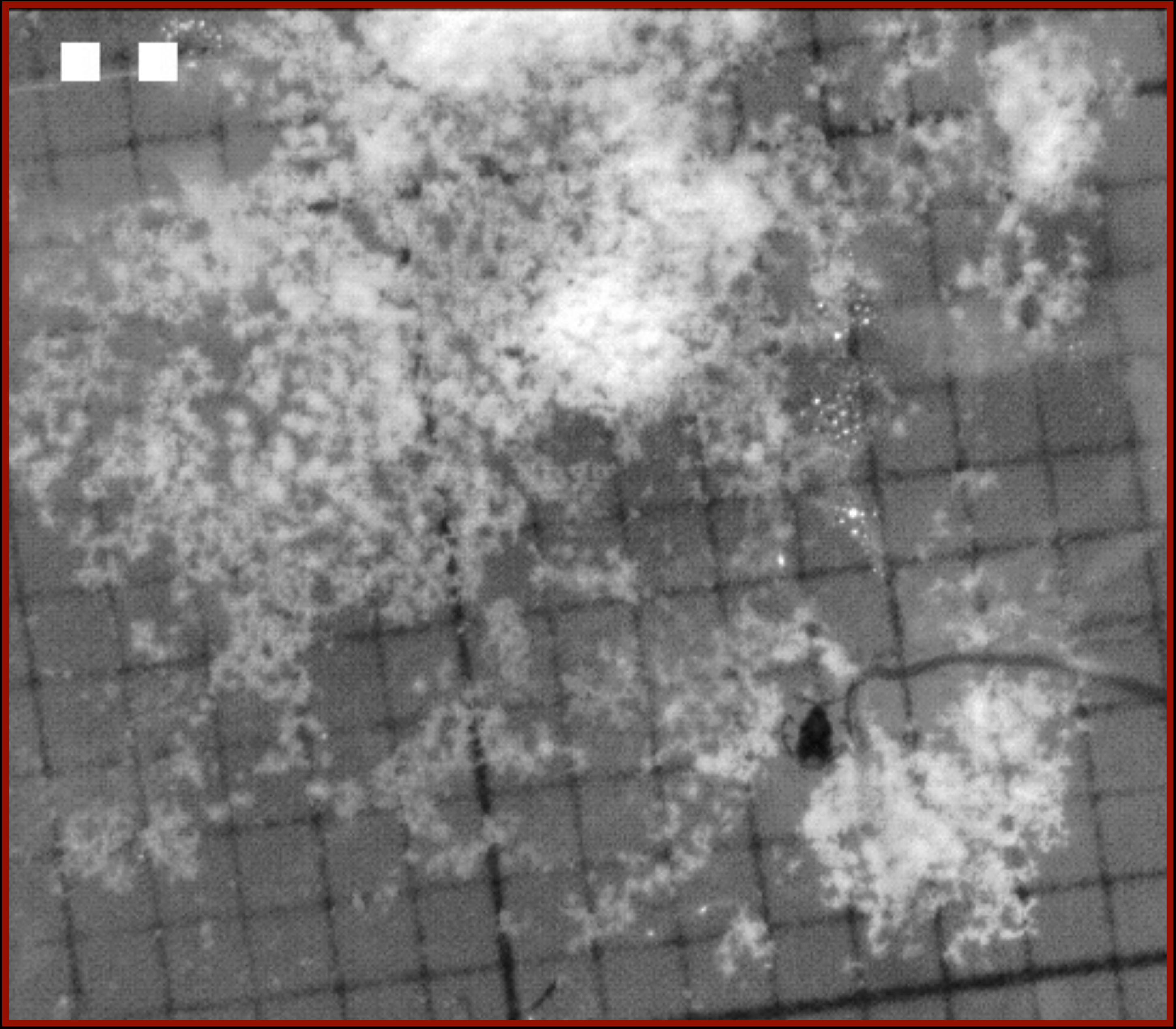
integrate around contact line



e.g. water-walking insects, dispersal of pine needles

- motion driven by soap cannot be sustained in a closed container
- motion may be sustained if driven by a volatile component (e.g. camphor)

Tangential stress, $\boxed{\nabla \sigma}$, may drive lateral motion.

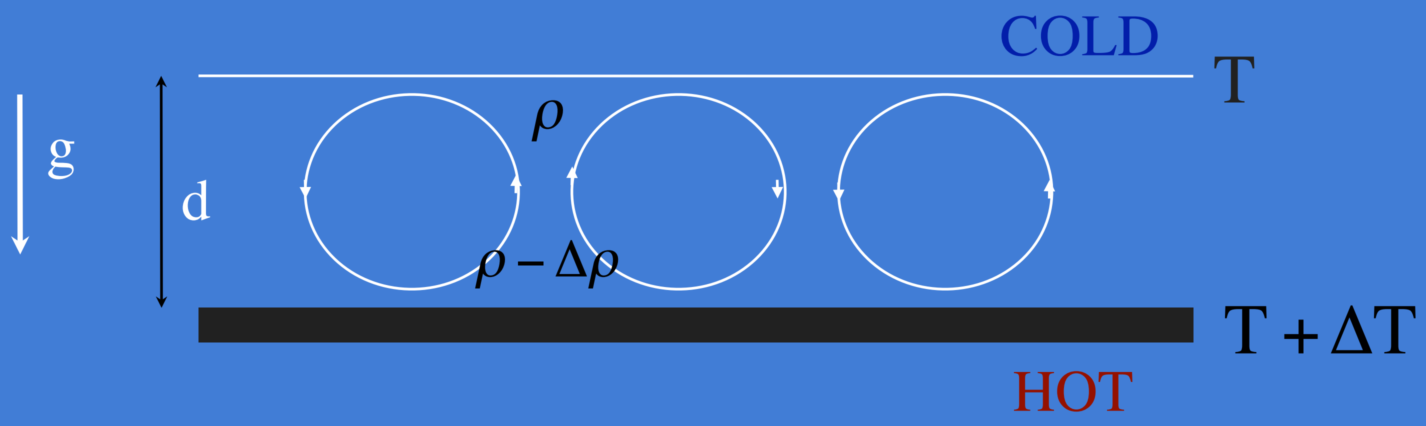


Marangoni propulsion: insect uses lipid as fuel.

THERMAL CONVECTION

Rayleigh-Benard $\rho(T) = \rho_0\left[1 + \alpha(T - T_0)\right]$

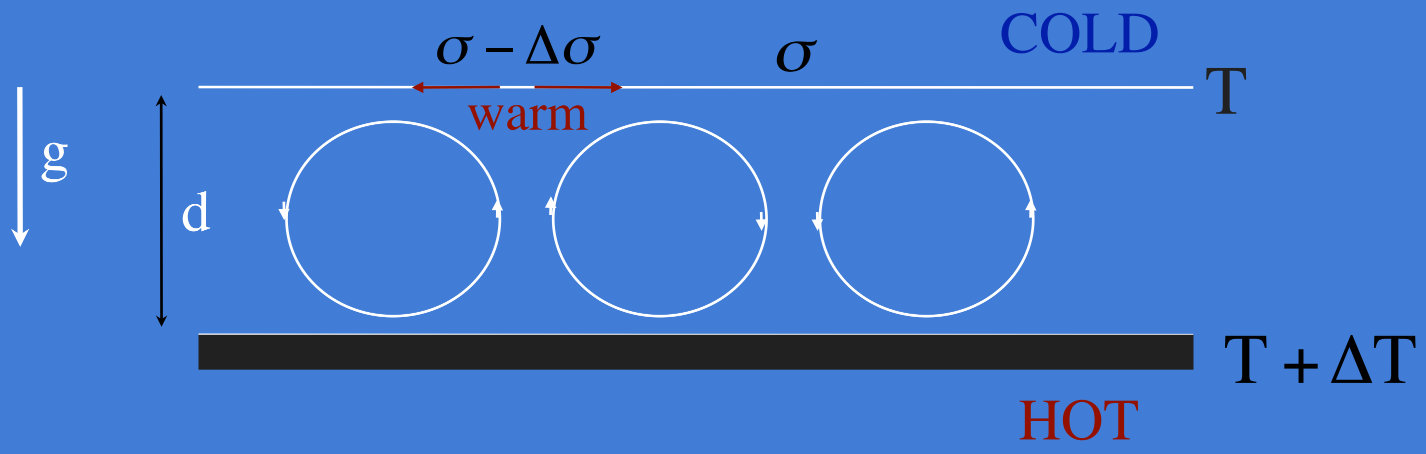
Stability prescribed by:



$$\boxed{Ra = \frac{g\alpha\Delta T d^3}{\kappa\nu}}$$

Rayleigh number

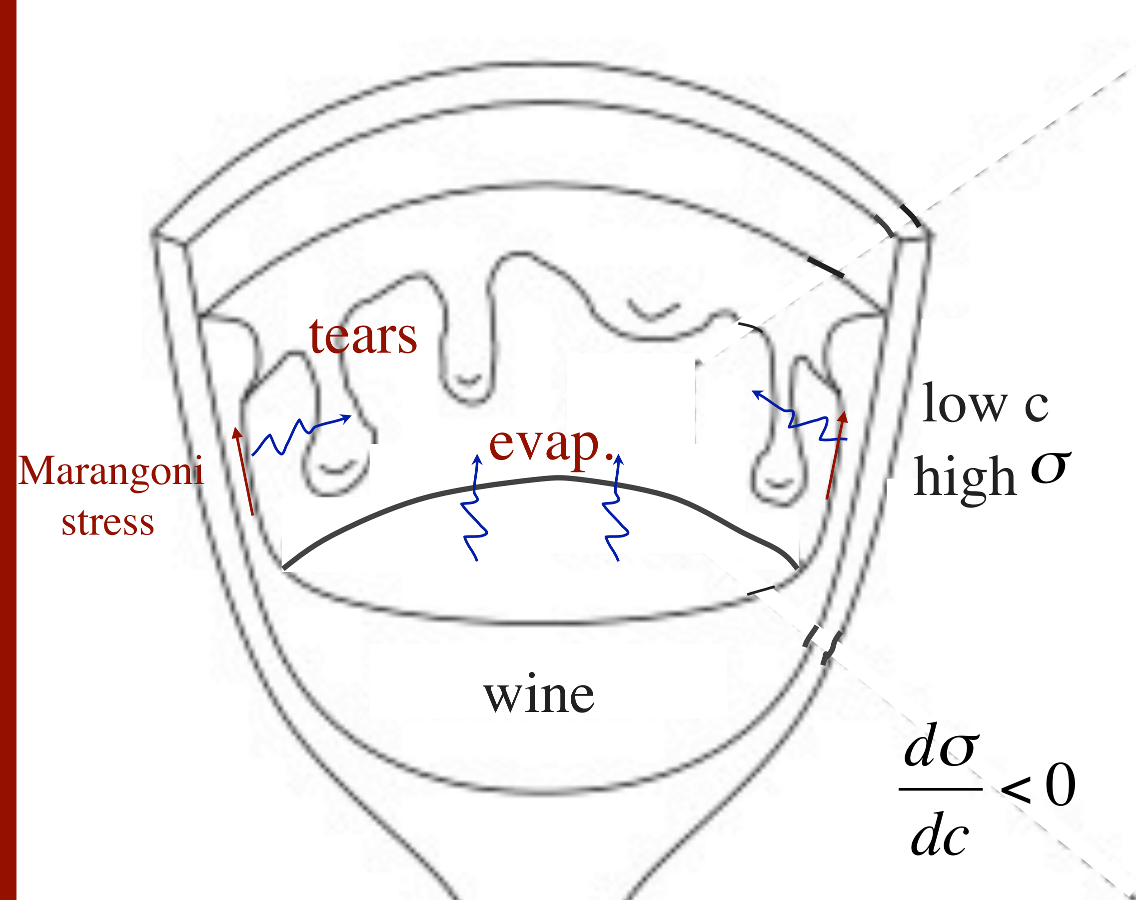
Marangoni-Benard $\sigma(T) = \sigma_0 - \Gamma(T - T_0)$



$$\boxed{Ma = \frac{\Gamma\Delta T d^2}{\kappa\mu}}$$

Marangoni number

Note: Marangoni convection dominates for thin films



The Tears of Wine (Thomson 1855)

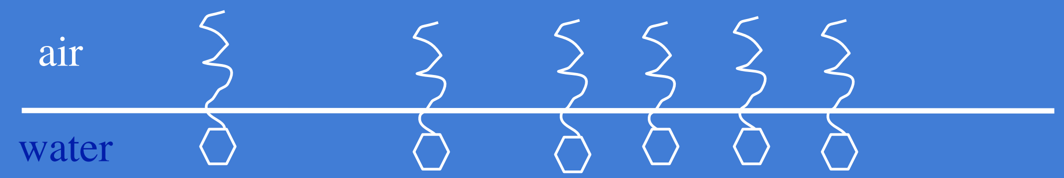
"Who hath sorrow? Who hath woe? They that tarry long at the wine.
Look not though upon the strong red wine that moveth itself aright.
At the last it biteth like a serpent and stingeth like an adder."

- Proverbs 23: 29-32

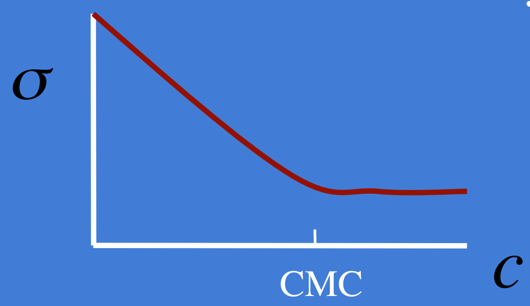
Surfactants: surface-active reagents

- molecules that find it energetically favourable to reside at an interface

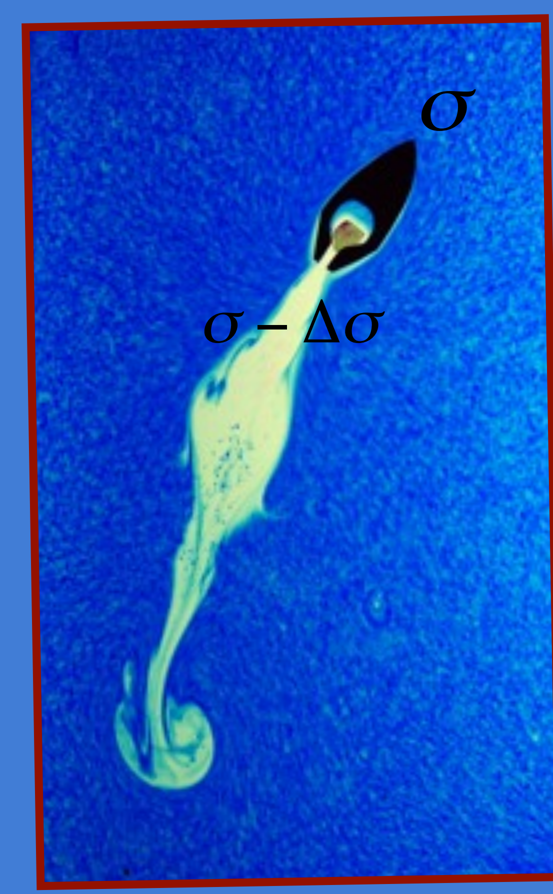
  e.g. commercial detergents



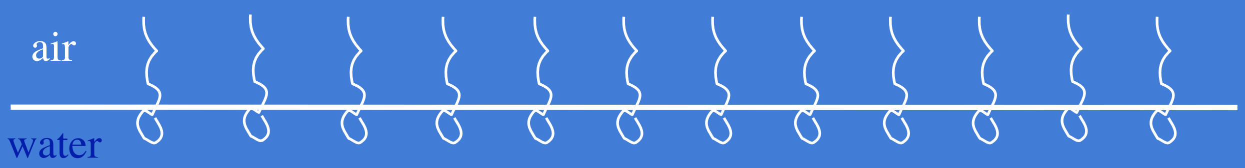
- generally act to reduce σ locally, $\dfrac{d\sigma}{d\Gamma} < 0$: may induce Marangoni flows



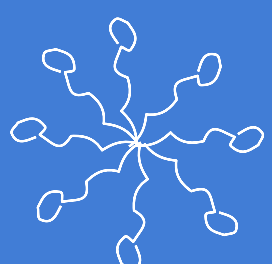
e.g. Soap boat



- beyond the CMC (critical micell concentration), there is no further dependence of σ on Γ

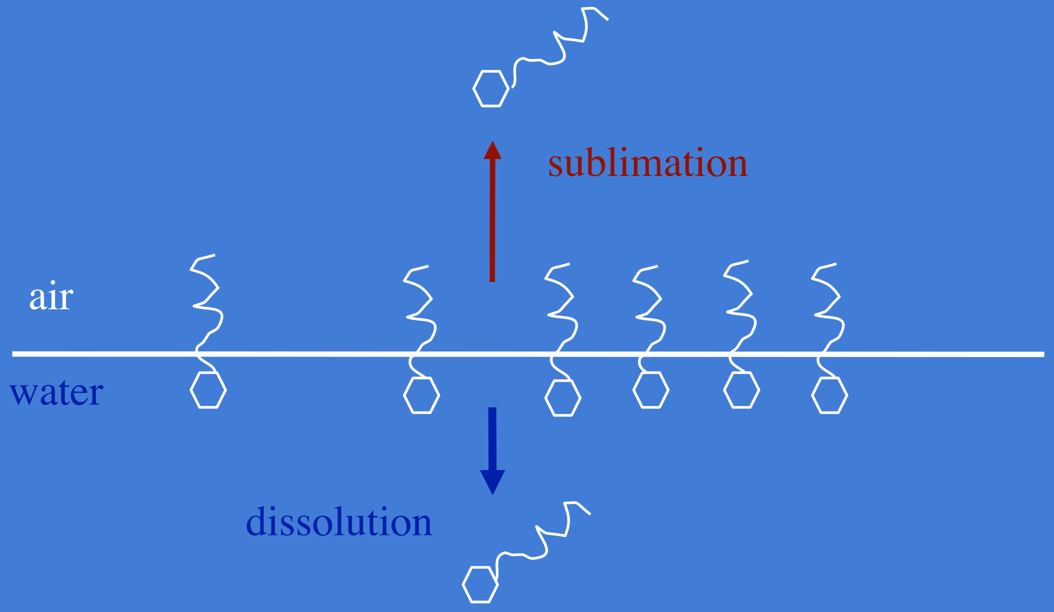


MICELL



micells shed by saturated interface, desorbed into bulk

Surfactant properties



Diffusivity

- prescribes the rate of diffusion, D_s , of a surfactant along an interface

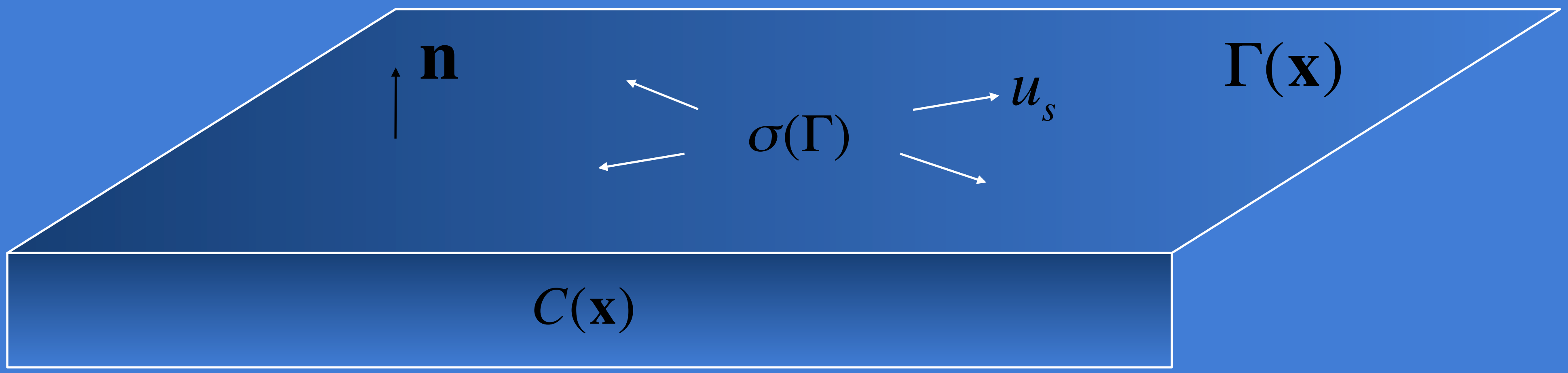
Solubility

- prescribes the ease with which surfactant passes from the surface to the bulk
- an insoluble surfactant cannot dissolve into the bulk, must remain on surface

Volatility

- prescribes the ease with which surfactant sublimates

The evolution of a surfactant-laden interface



Since $\sigma(\Gamma)$, N-S equations and BCs must be augmented by

Surfactant evolution equation:

$$\boxed{\frac{\partial \Gamma}{\partial t} + \nabla_s \cdot \left(\Gamma u_s\right) + \Gamma\,(\nabla_s \cdot n)(u \cdot n) = J(\Gamma, C) + D_s \nabla_s^2 \Gamma}$$

advection

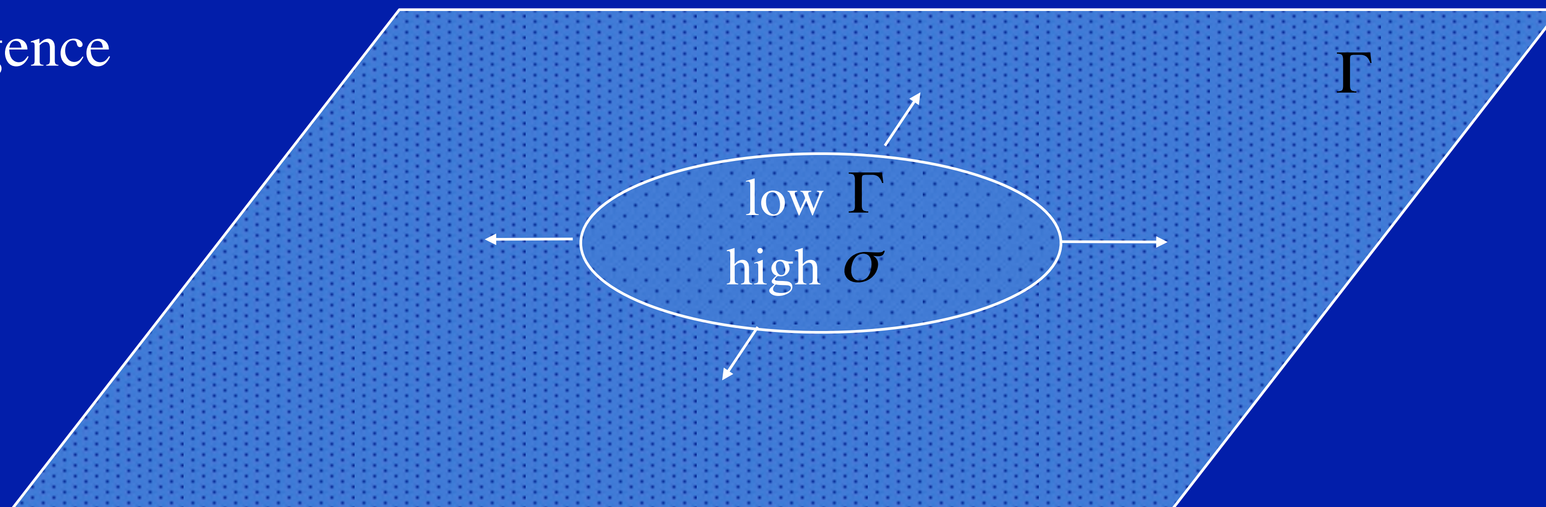
surface expansion

exchange with bulk

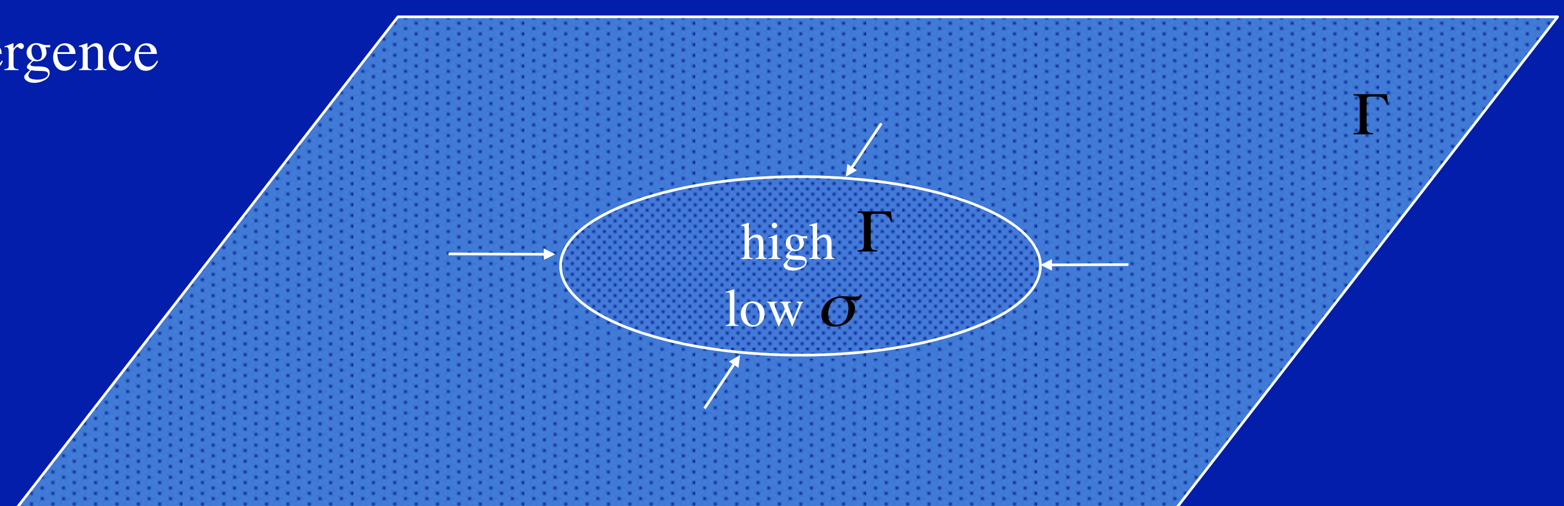
surface diffusion

Surfactants: impart effective elasticity to contaminated interfaces through resisting flows with non-zero surface divergence

Surface divergence

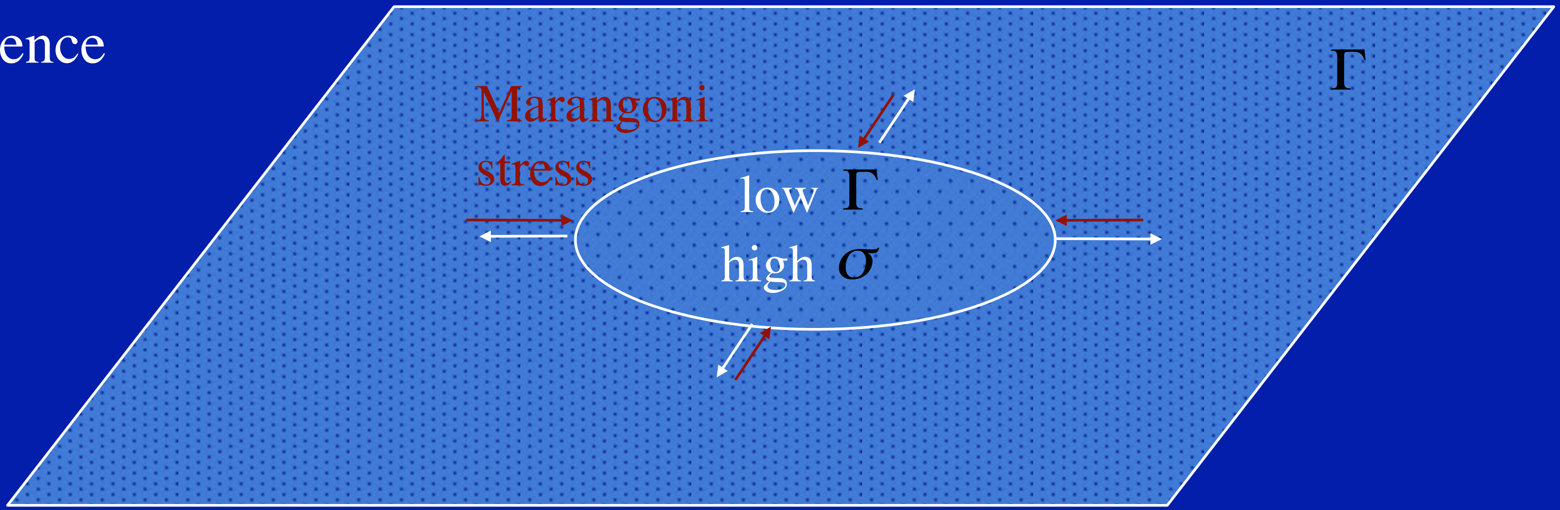


Surface convergence

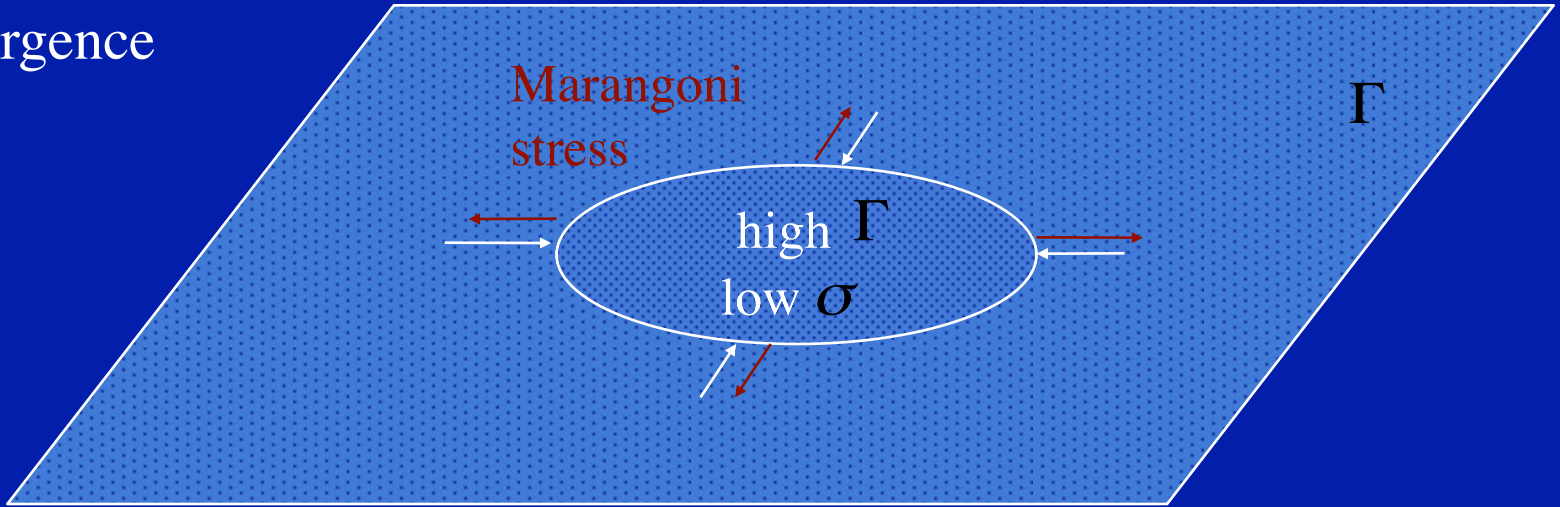


Surfactants: impart effective elasticity to contaminated interfaces through resisting flows with non-zero surface divergence

Surface divergence

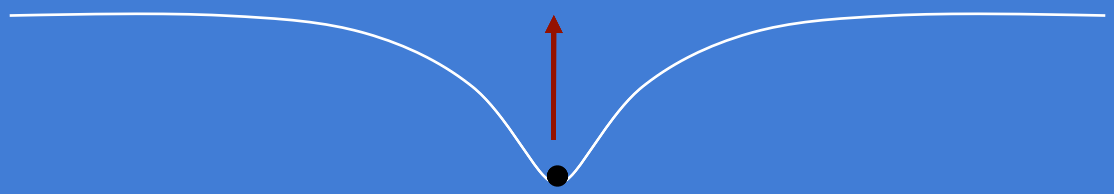


Surface convergence



Clean interface = `slippery trampoline'

- resists deformation through generation of normal curvature pressures
- cannot generate traction on the interface

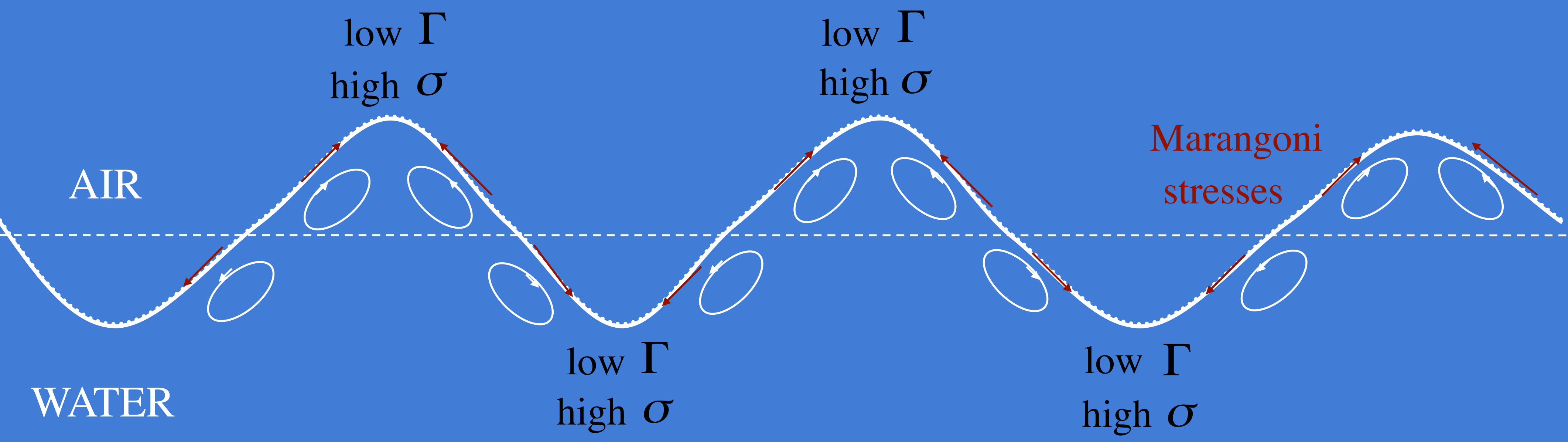


Surfactant-laden interface = trampoline

- resists surface deformation as does a clean interface
- can support tangential stresses via Marangoni elasticity

The suppression of capillary waves by surfactant

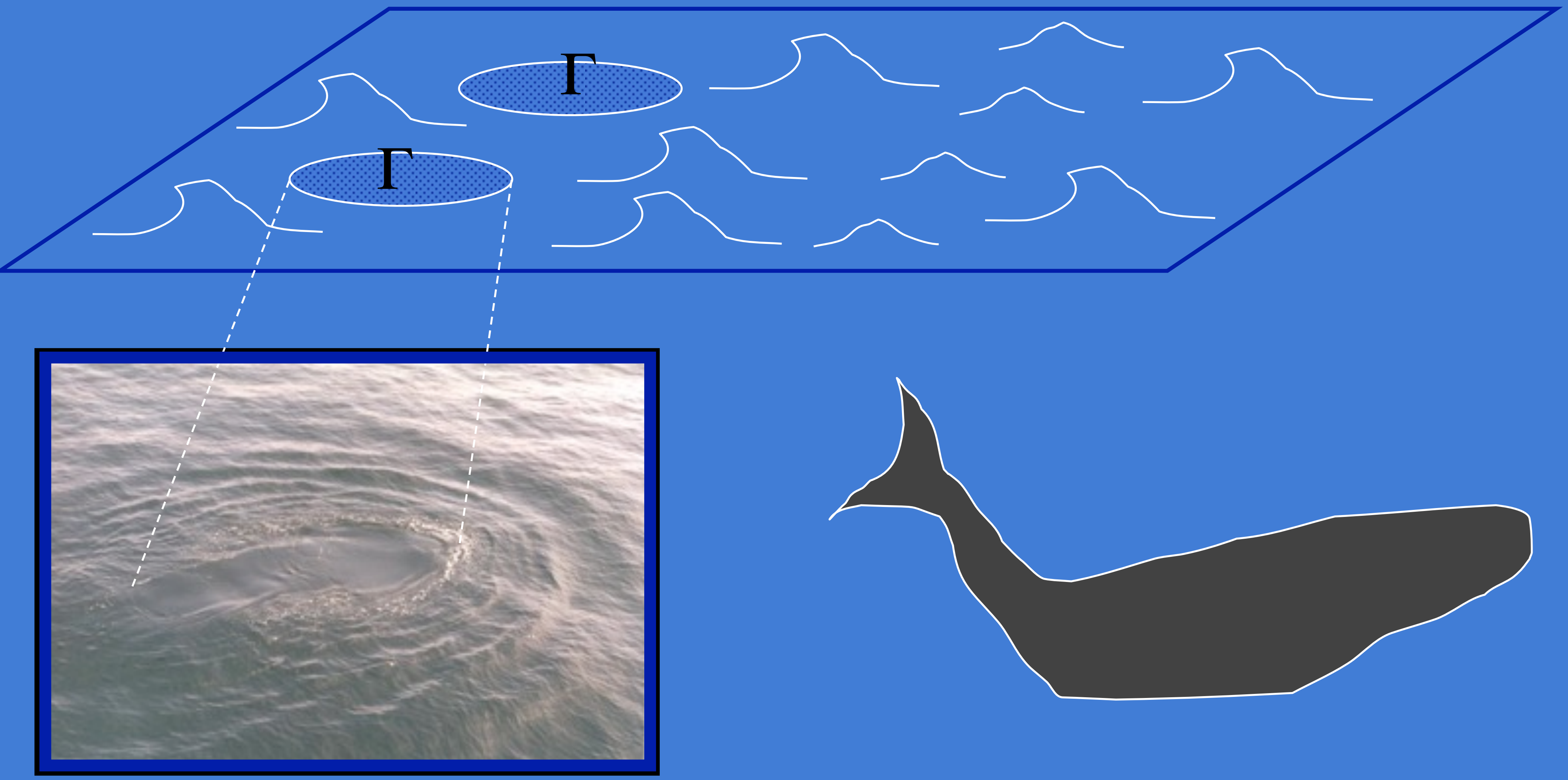
- wave motion generates regions of surface divergence
- concomitant surfactant gradients generate Marangoni stresses $\frac{d\sigma}{d\Gamma} < 0$
- resulting small scale flows extremely dissipative



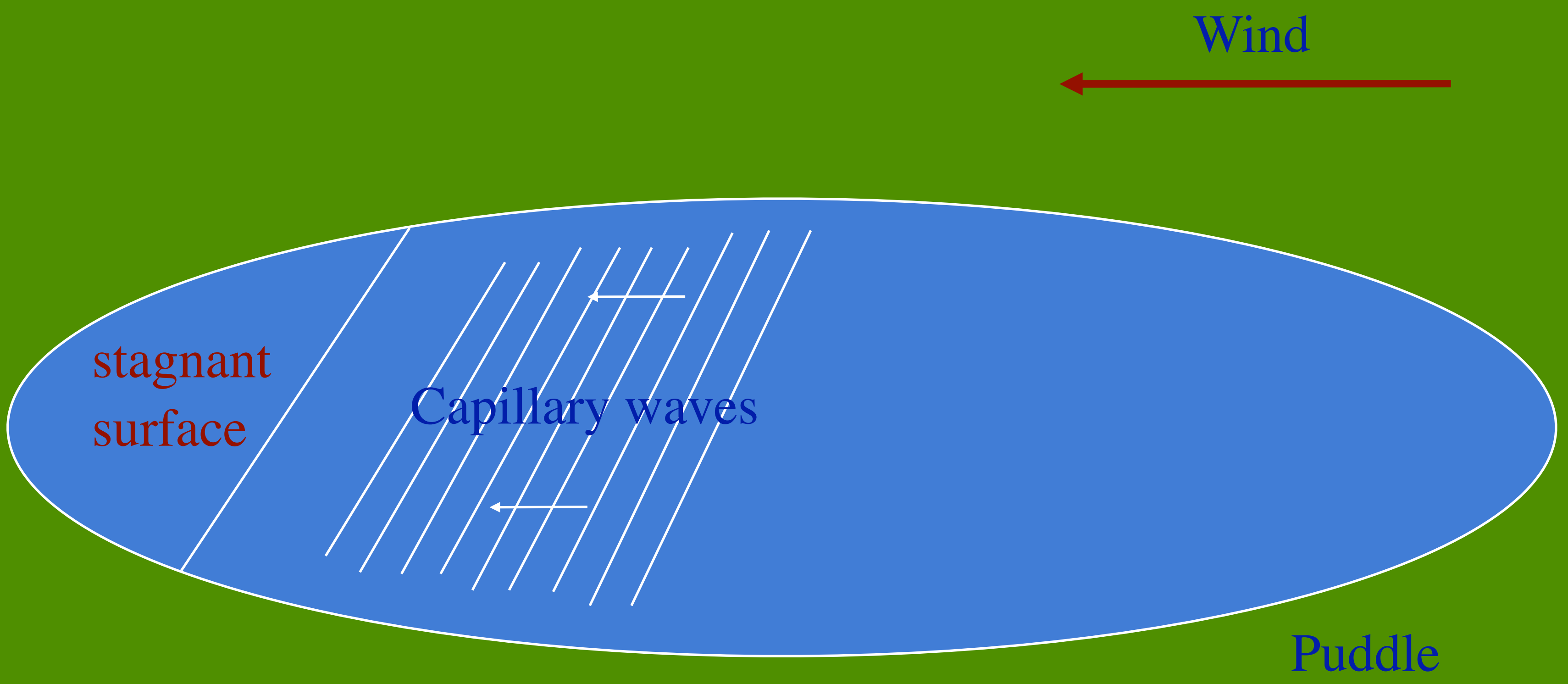
- flat ship wakes first remarked upon by Pliney the Elder
- examined by Benjamin Franklin, motivated by Bermudan spear fishermen
- now used to track submarines: flat wakes visible on satellite images

The footprints of whales

- surfactants (biomaterial in water column) swept to surface by diving whales
- suppress capillary waves and cascade to larger scale waves



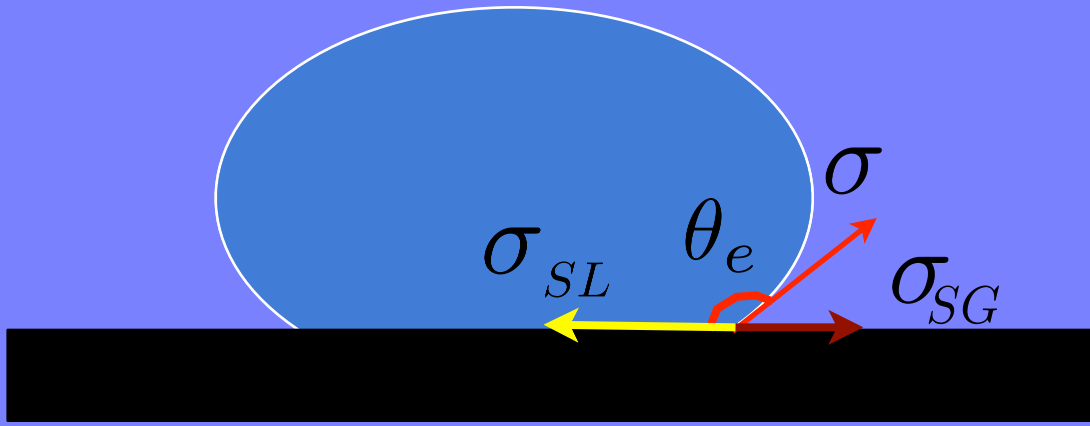
The dynamics of puddles



- surfactants swept to lee of puddle by wind stress
- Marangoni stress balances wind stress $\Longrightarrow$ stagnant surface
- capillary waves suppressed by surfactant in lee of puddle

The brazilian pygmy gecko

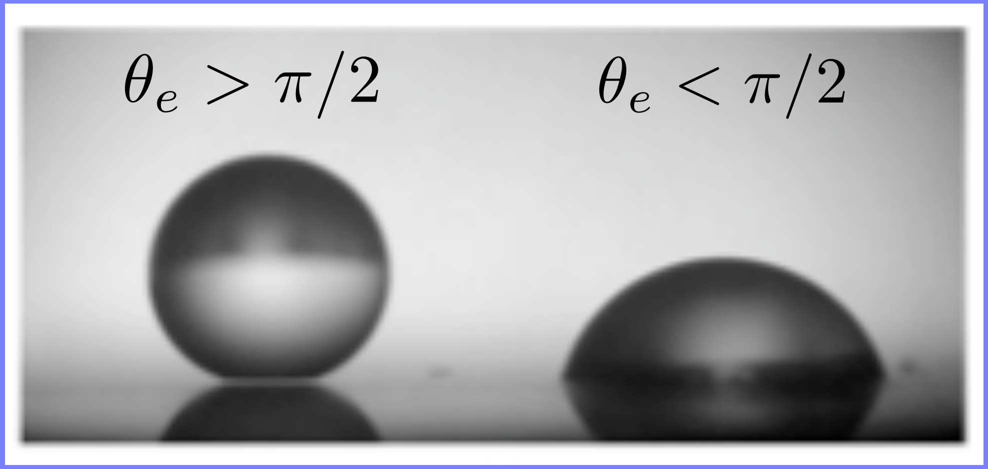
Fluid-Solid Contact: WETTING



Equilibrium contact angle θ_e

Energy differential: $dW = dx\,(\sigma_{SG} - \sigma_{SL}) - dx\,\sigma\,\cos\theta_e$

Young's relation: $\boxed{\sigma\,\cos\theta_e = \sigma_{SL} - \sigma_{SG}}$



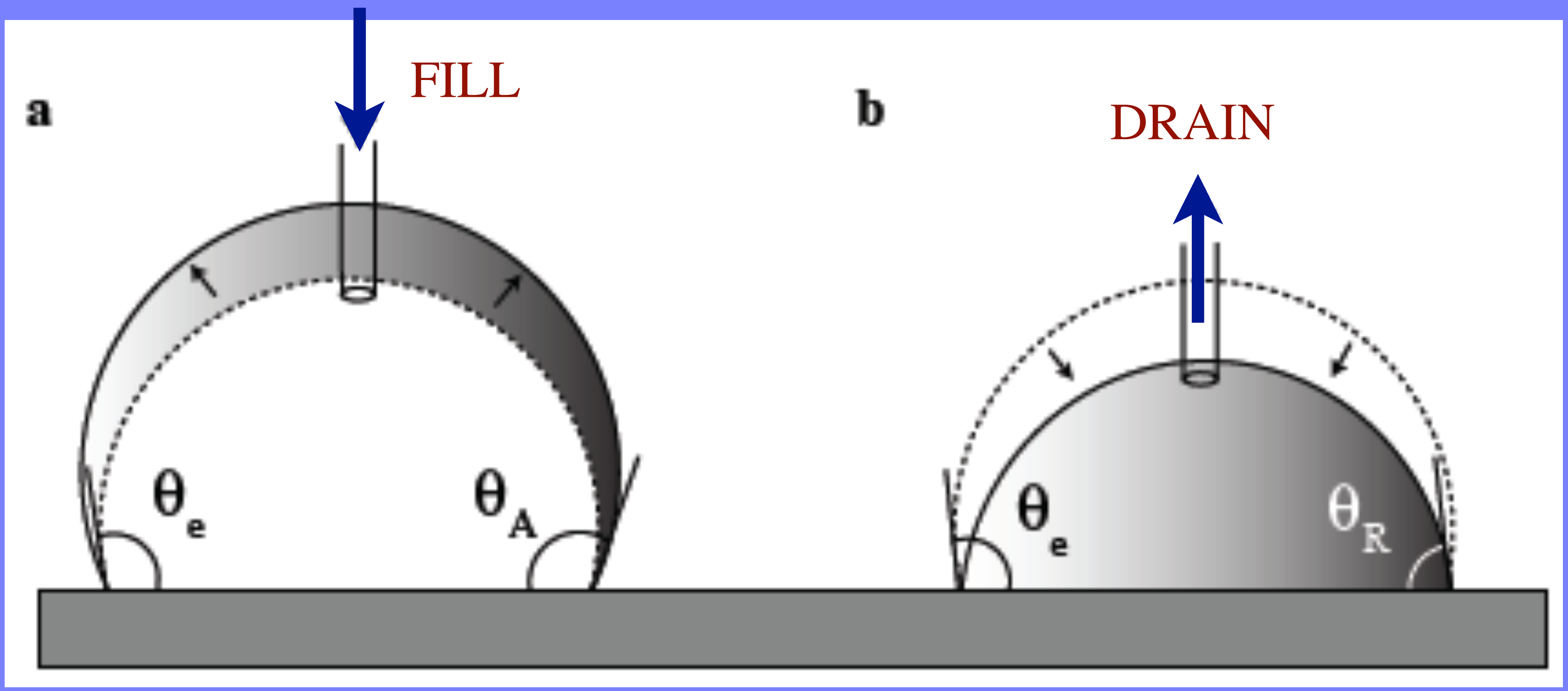
Hydrophobic surface

Hydrophilic surface

Contact angle hysteresis

Static contact angle is not uniquely θ_e

Reality: drop is stable over a range of $\theta_r < \theta < \theta_a$

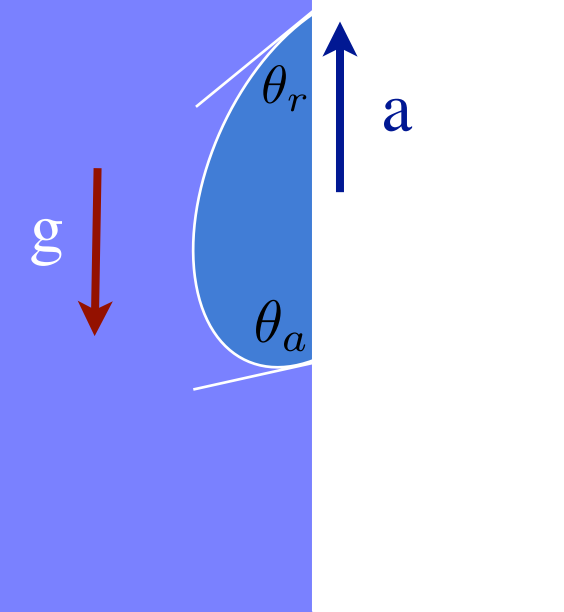


→ FORCE of ADHESION resists drop motion

increases with $\Delta\theta = \theta_a - \theta_r$

Origins: advancing contact lines pinned on surface irregularities

The force of adhesion (Dussan & Chow 1983)



Raindrop stuck on a window

- small drops supported by contact line resistance

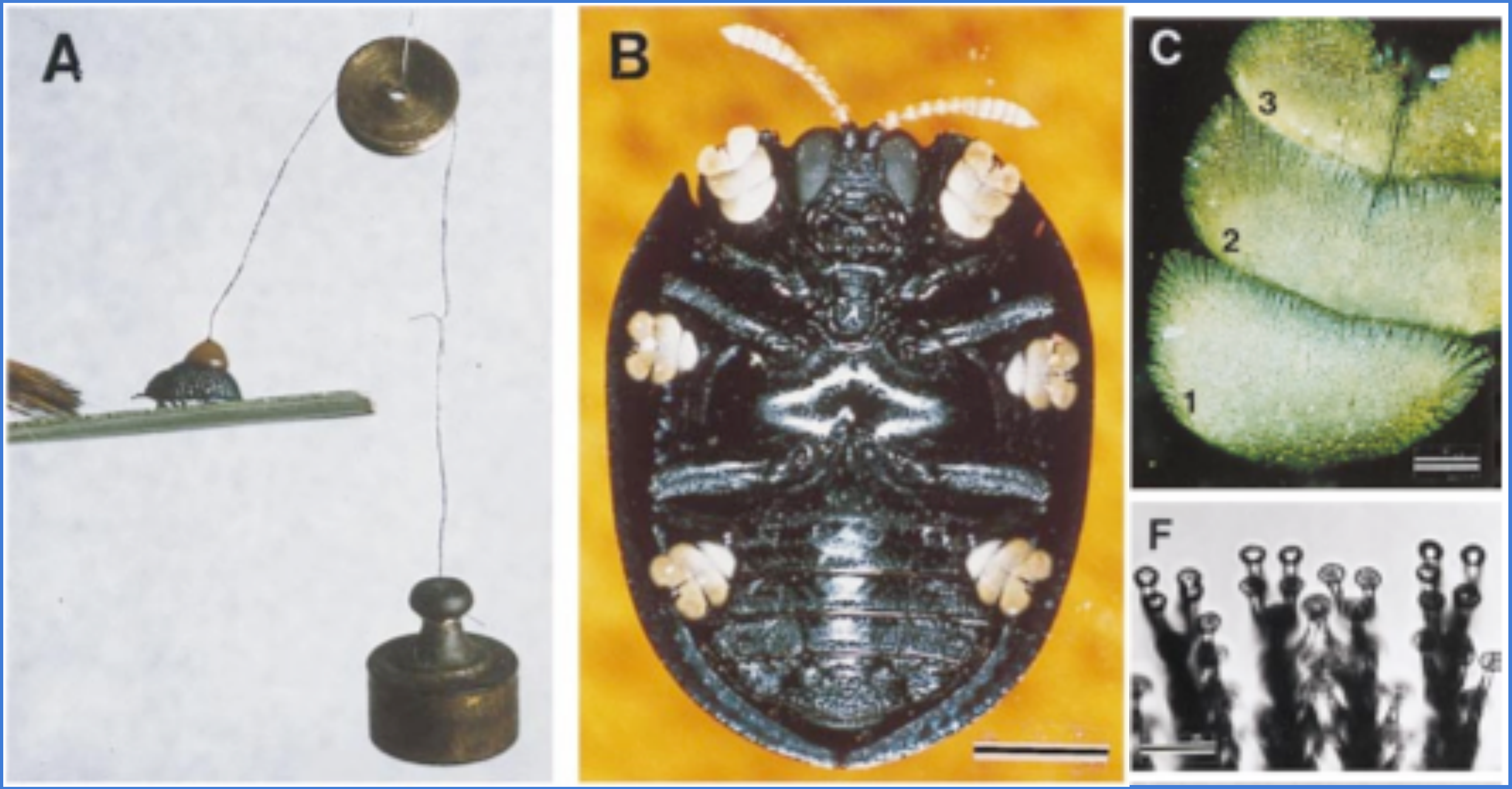
$$F_c \sim 2\pi a \,\sigma \,(\cos\theta_r - \cos\theta_a)$$

- drops grow by accretion until weight prompts rolling

But who cares?

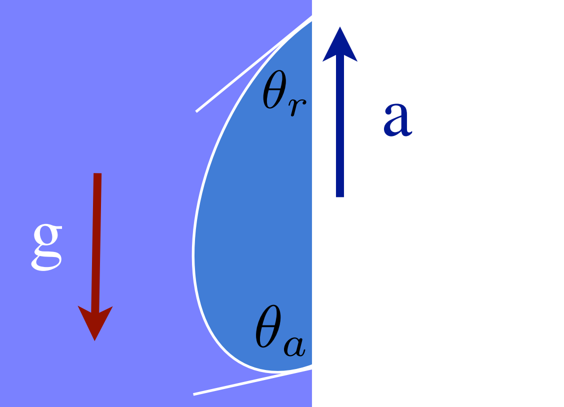
Self-defense via capillary adhesion

Eisner & Aneshansley (2000)



Hemisphaerota cyanea (Chrysomelidae; Cassidinae)

The force of adhesion (Dussan & Chow 1983)



Raindrop stuck on a window

- small drops supported by contact line resistance

$$F_c \sim 2\pi a \, \sigma \, (\cos\theta_r - \cos\theta_a)$$

- drops grow by accretion until weight prompts rolling

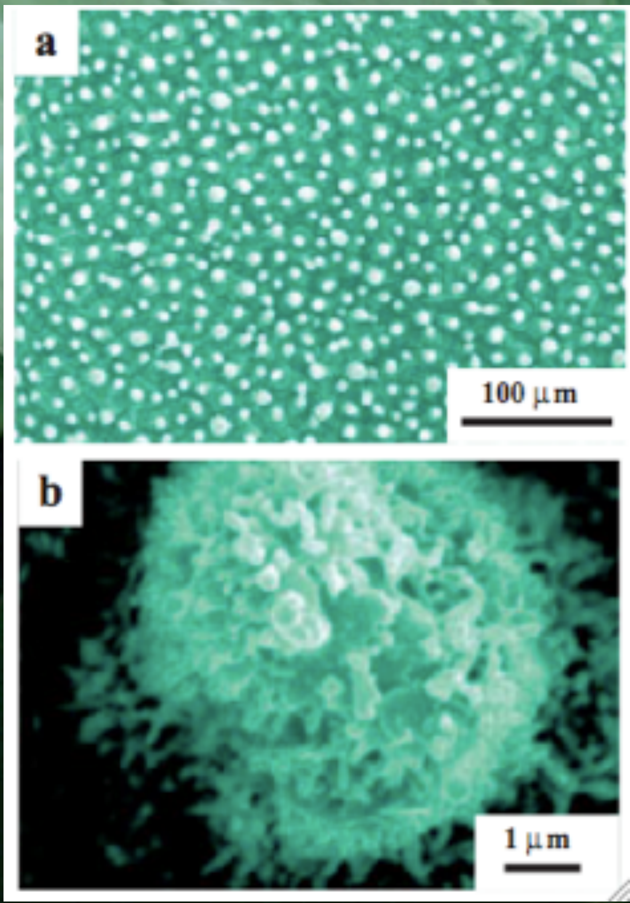
Water-repellency

- impinging drops roll off rather than adhering
- requires large θ_e , small $\Delta\theta = \theta_a - \theta_r$

How can we reduce the force of adhesion?

Water repellency in nature

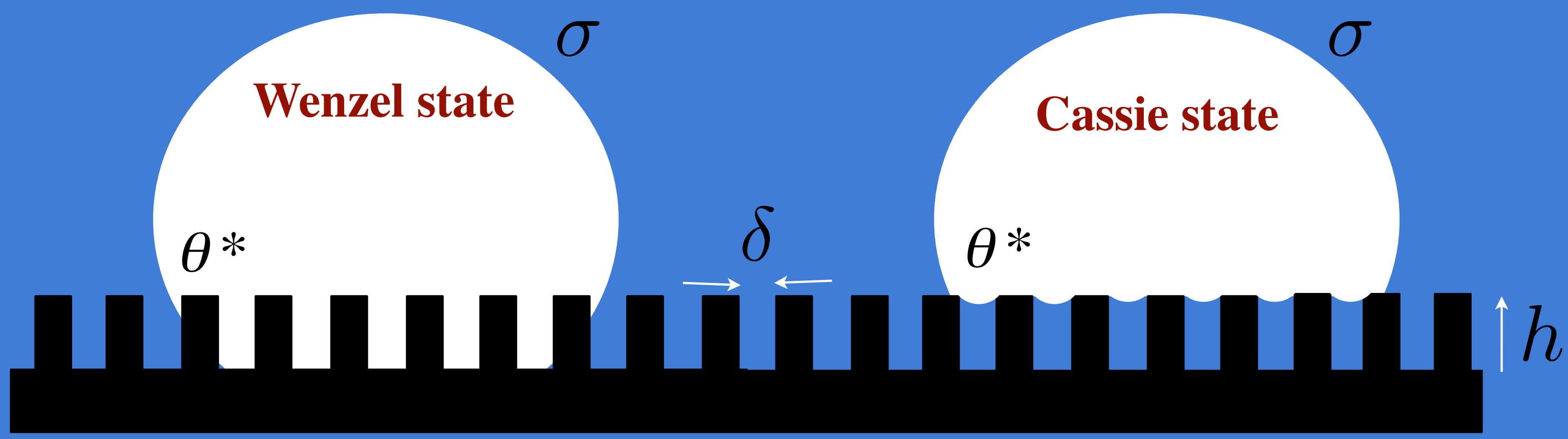
"One who performs his duty without attachment, surrendering the results unto the Supreme Being, is unaffected by sinful action, as the lotus leaf is untouched by water."
Bhagavad Gita 5.10



Feng et al. (2004)

- the lotus leaf is superhydrophobic and self-cleaning by virtue of its waxy surface roughness

Wetting of a rough hydrophobic surface: Wenzel vs. Cassie



$$dW = r\,dx\,(\sigma_{SG} - \sigma_{SL}) - dx\,\sigma\,cos\theta^*$$

$$\boxed{\cos\theta^* = r\,\cos\theta}$$

where r is total/planar area

θ^* INCREASES, but $\Delta\theta$ INCREASES

$$\boxed{\cos\theta^* = -1 + f_s + f_s\cos\theta}$$

where f_s is exposed/planar area

θ^* INCREASES

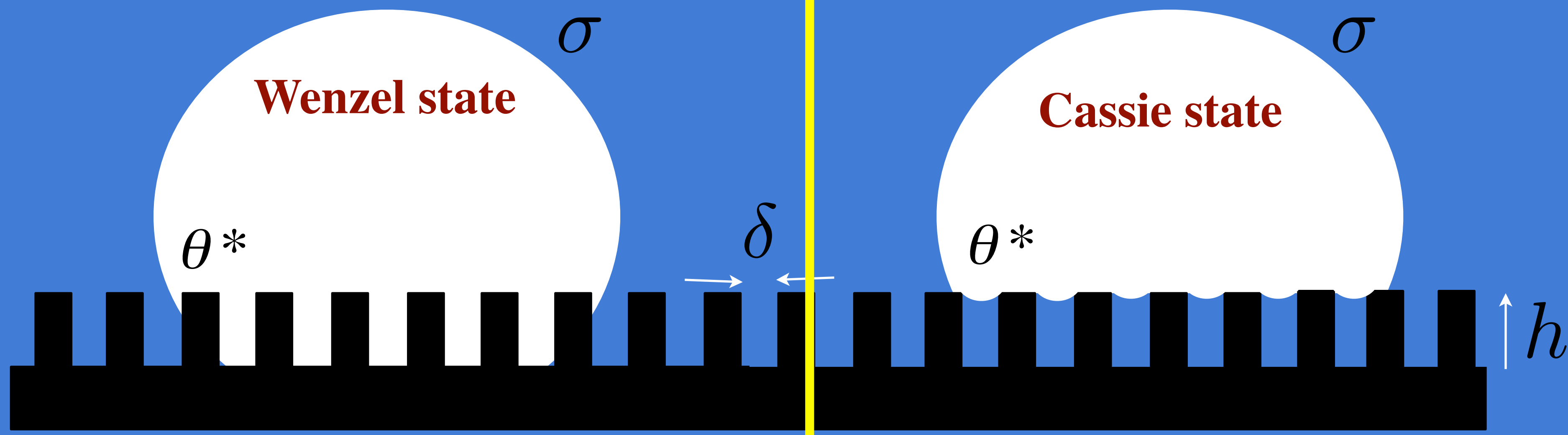
$\Delta\theta$ DECREASES

Water-repellency: requires the maintenance of a Cassie state

$$\rightarrow \quad \boxed{P_{applied} < \sigma\left(\frac{1}{\delta}\,,\ \frac{h}{\delta^2}\right)}$$

Bartolo et al. (2006)
Reyssat et al. (2007)

Wetting of a rough hydrophobic surface: Wenzel vs. Cassie



$$dW = r\ dx\ (\sigma_{SG} - \sigma_{SL}) - dx\ \sigma\ cos\theta^*$$

$$\boxed{\cos\theta^* = r\ \cos\theta}$$

where r is total/planar area

θ^* INCREASES, but $\Delta\theta$ INCREASES

$$\boxed{\cos\theta^* = -1 + f_s + f_s \cos\theta}$$

where f_s is exposed/planar area

θ^* INCREASES

$\Delta\theta$ DECREASES

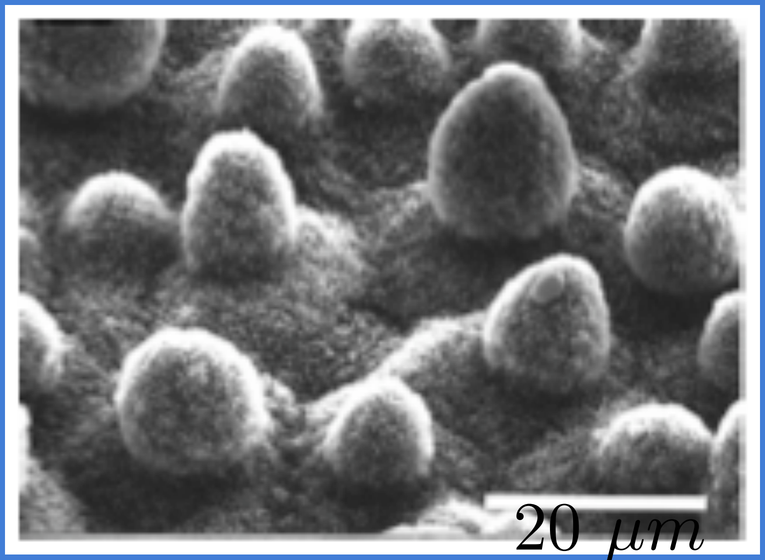
Water-repellency: requires the maintenance of a Cassie state

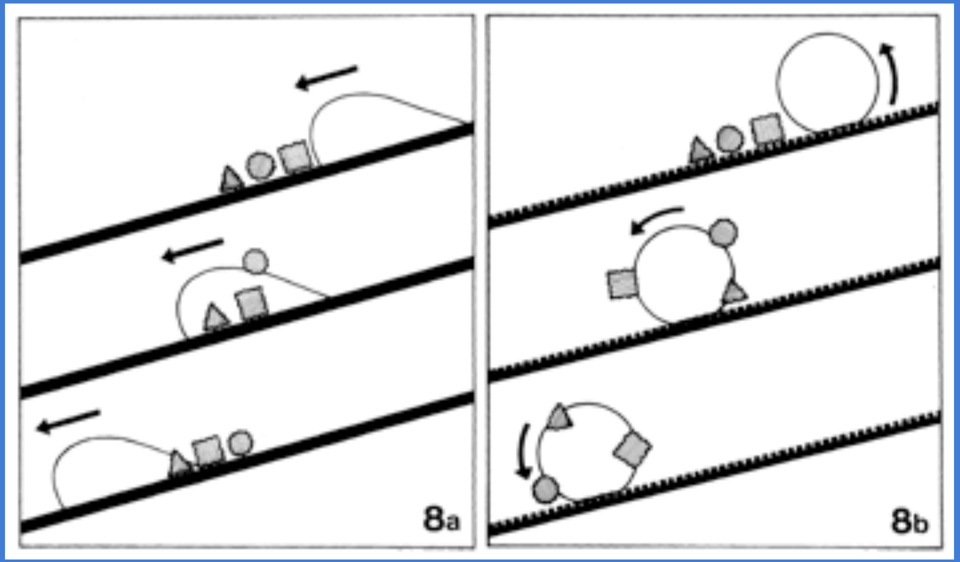
$$\rightarrow \boxed{P_{applied} < \sigma \left(\frac{1}{\delta}\ ,\ \frac{h}{\delta^2} \right)}$$

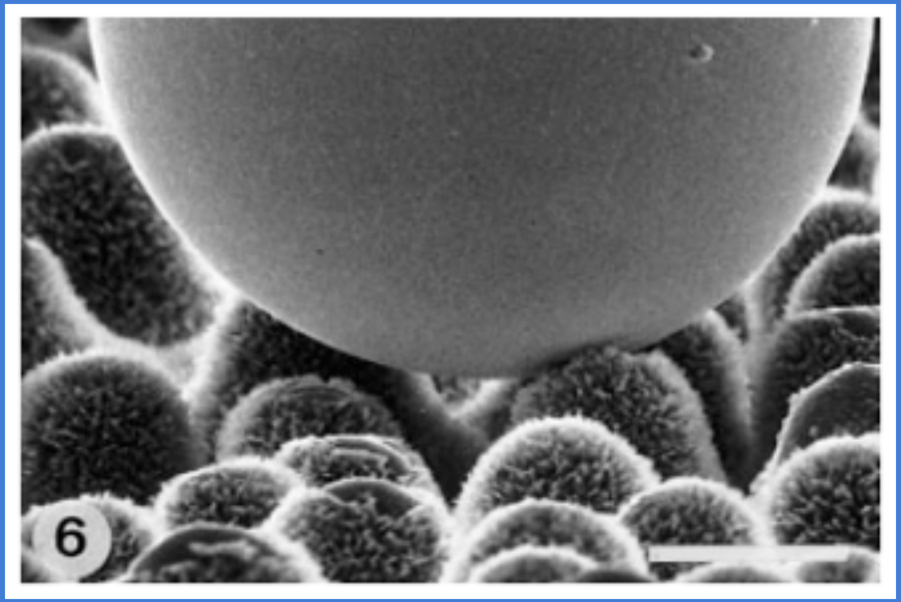
Bartolo et al. (2006)
Reyssat et al. (2007)

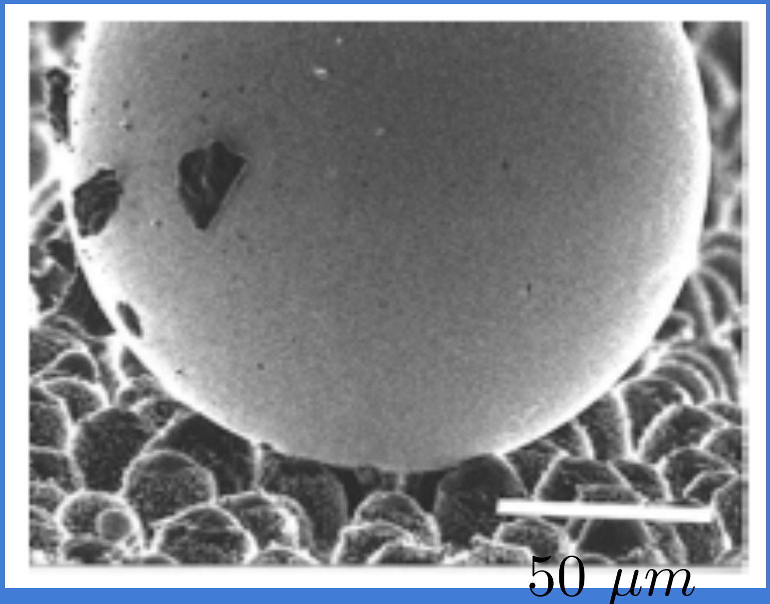
The lotus leaf

Barthlott & Neinhuis (1997)



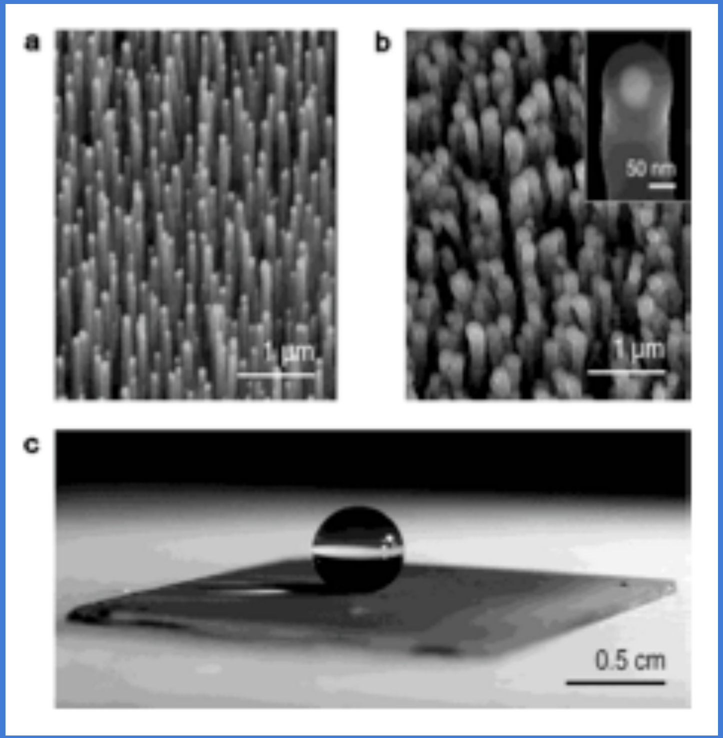




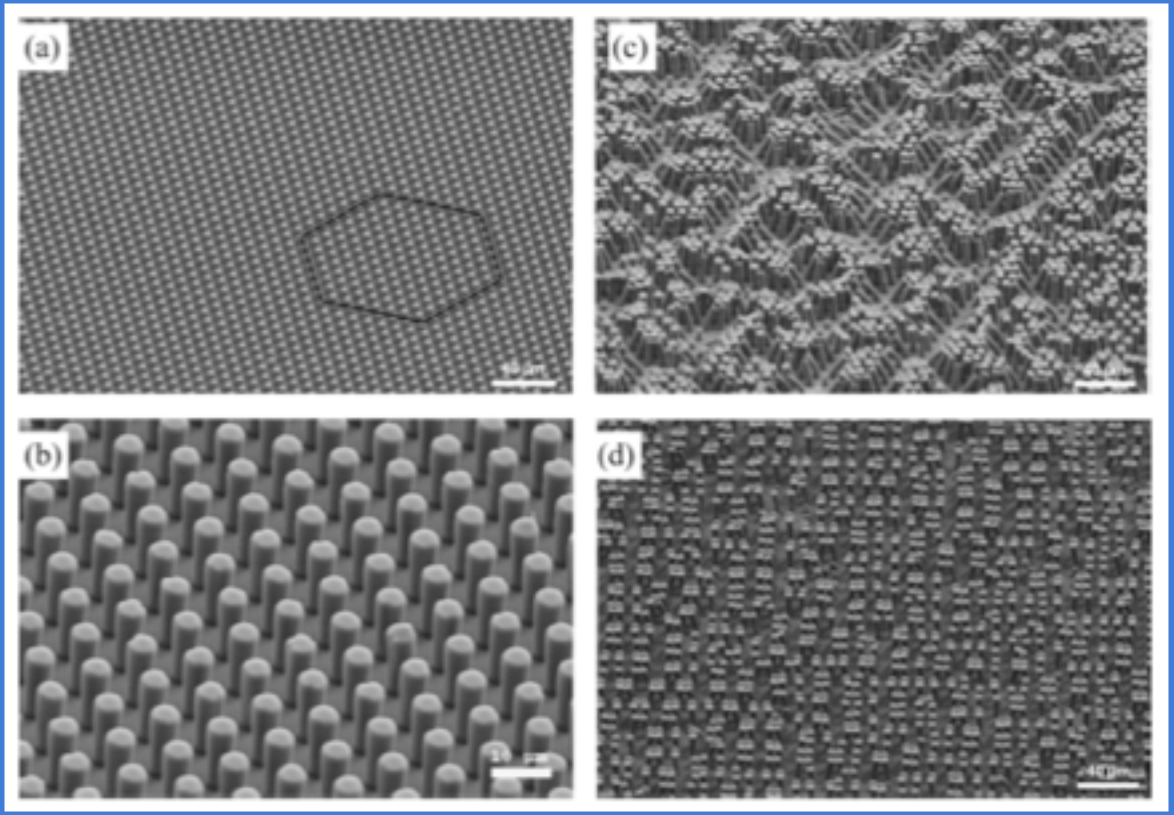


- water-repellent: isotropic surface roughness maintains Cassie state
  - contact forces on droplets minimized
- self-cleaning: surface impurities (e.g. dust) adhere to droplets

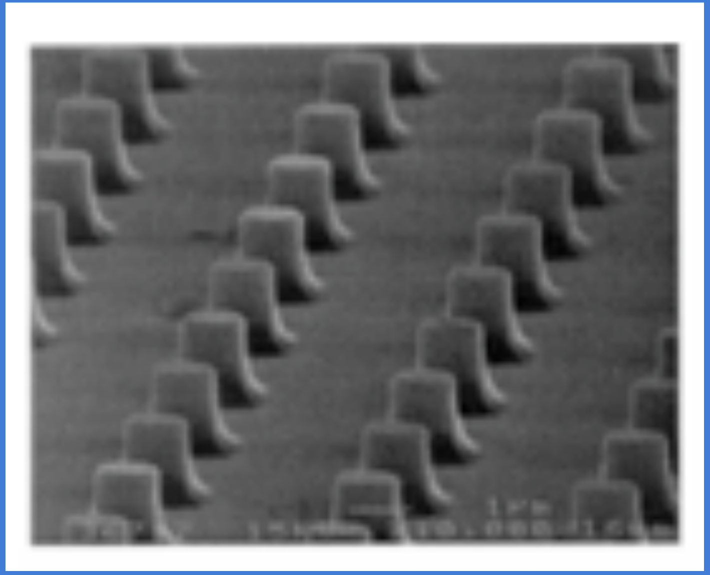
Biomimetic water-repellent surfaces: viable with new microfab techniques



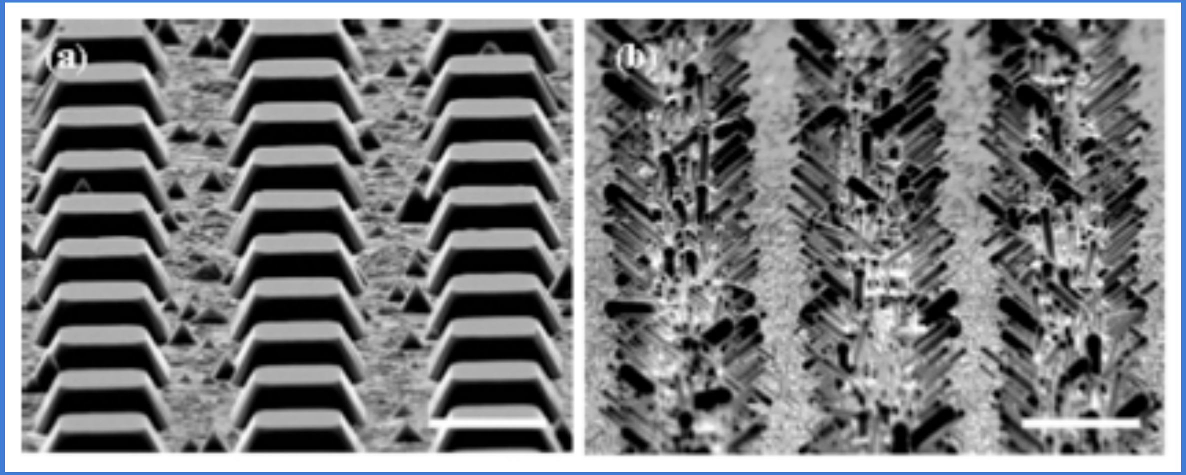
Lau et al. (2003)



Greiner et al. (2007)



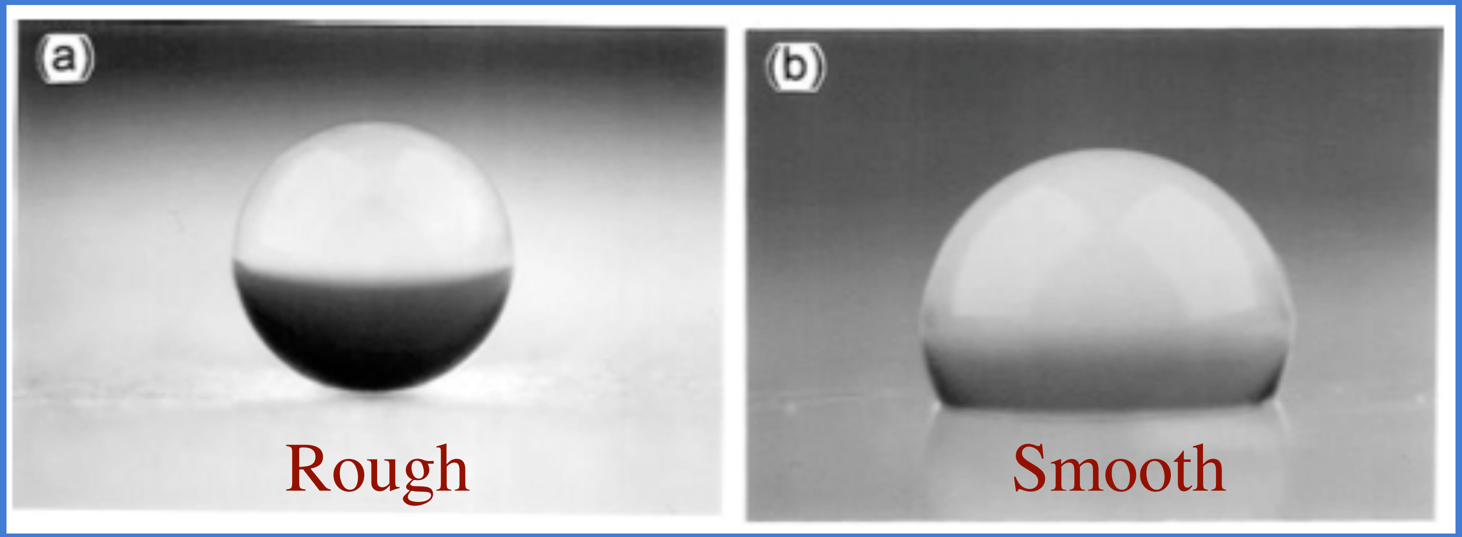
Bico et al. (1999)

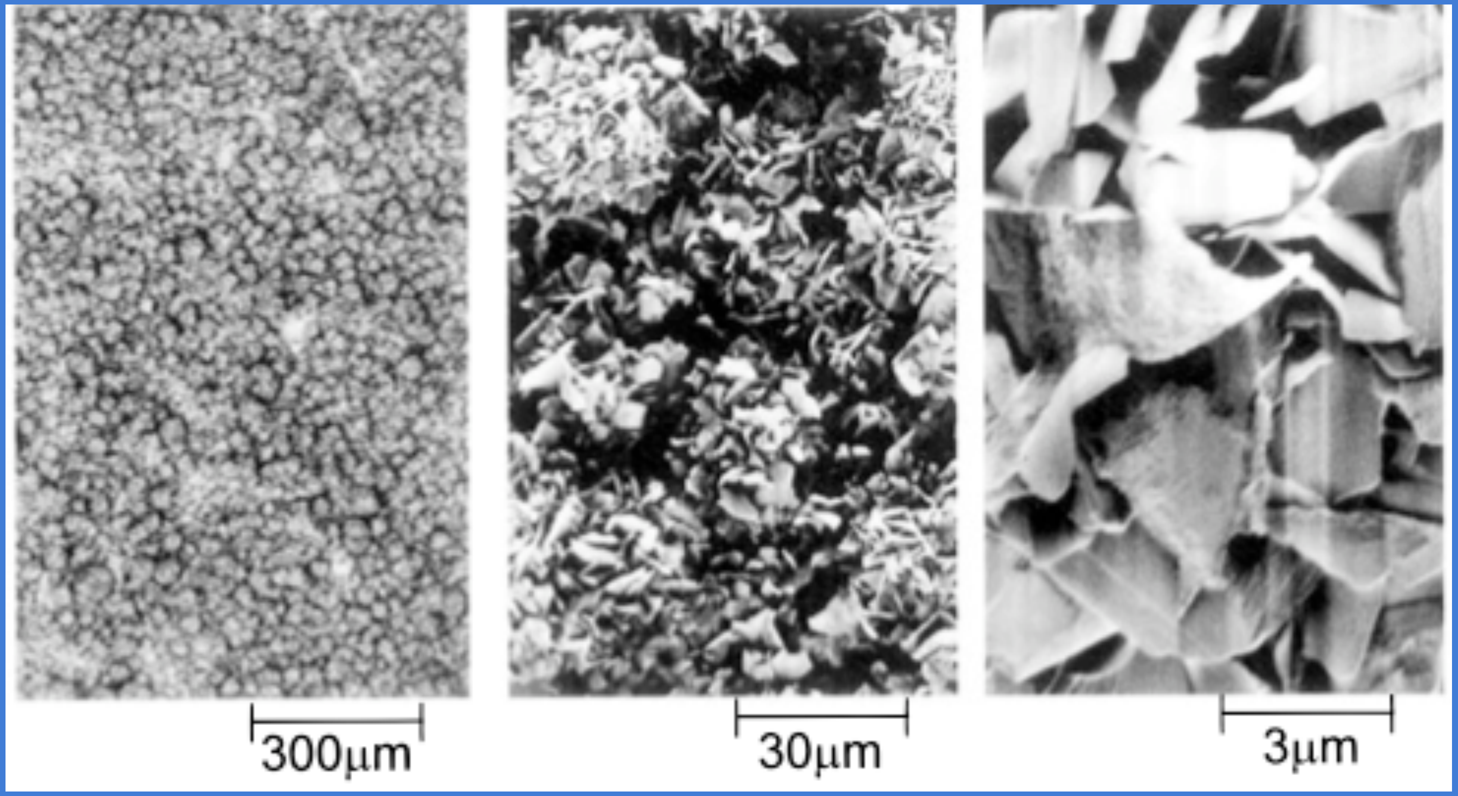


Cao et al. (2007)

Superhydrophobic surfaces achieved with fractal texturing

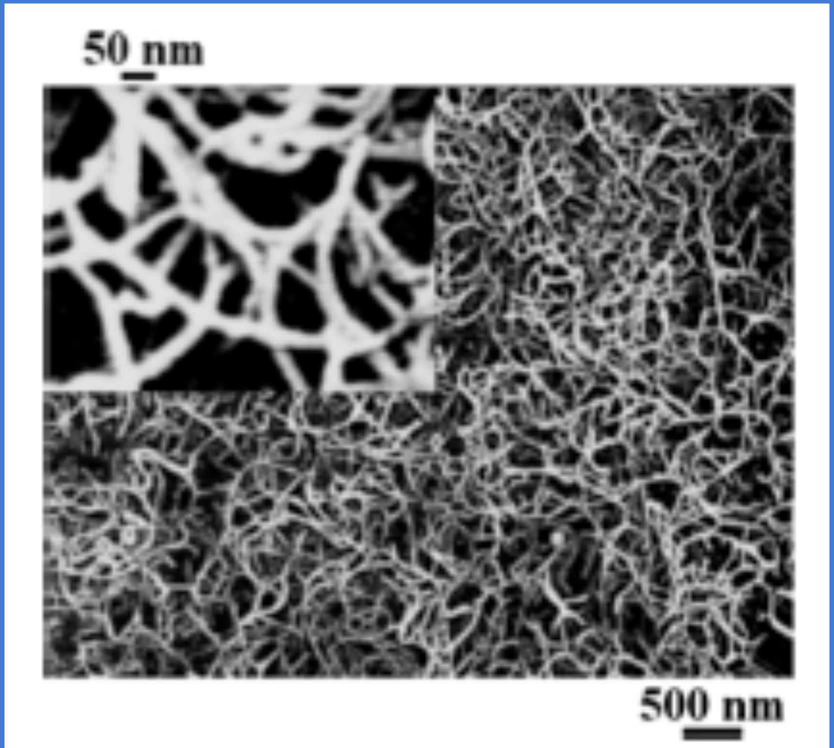
Shibuichi et al. (1996), Onda et al. (1997), Herminghaus (2000)





A perfectly hydrophobic surface

Gao & McCarthy (2006)



“The Lichao surface”

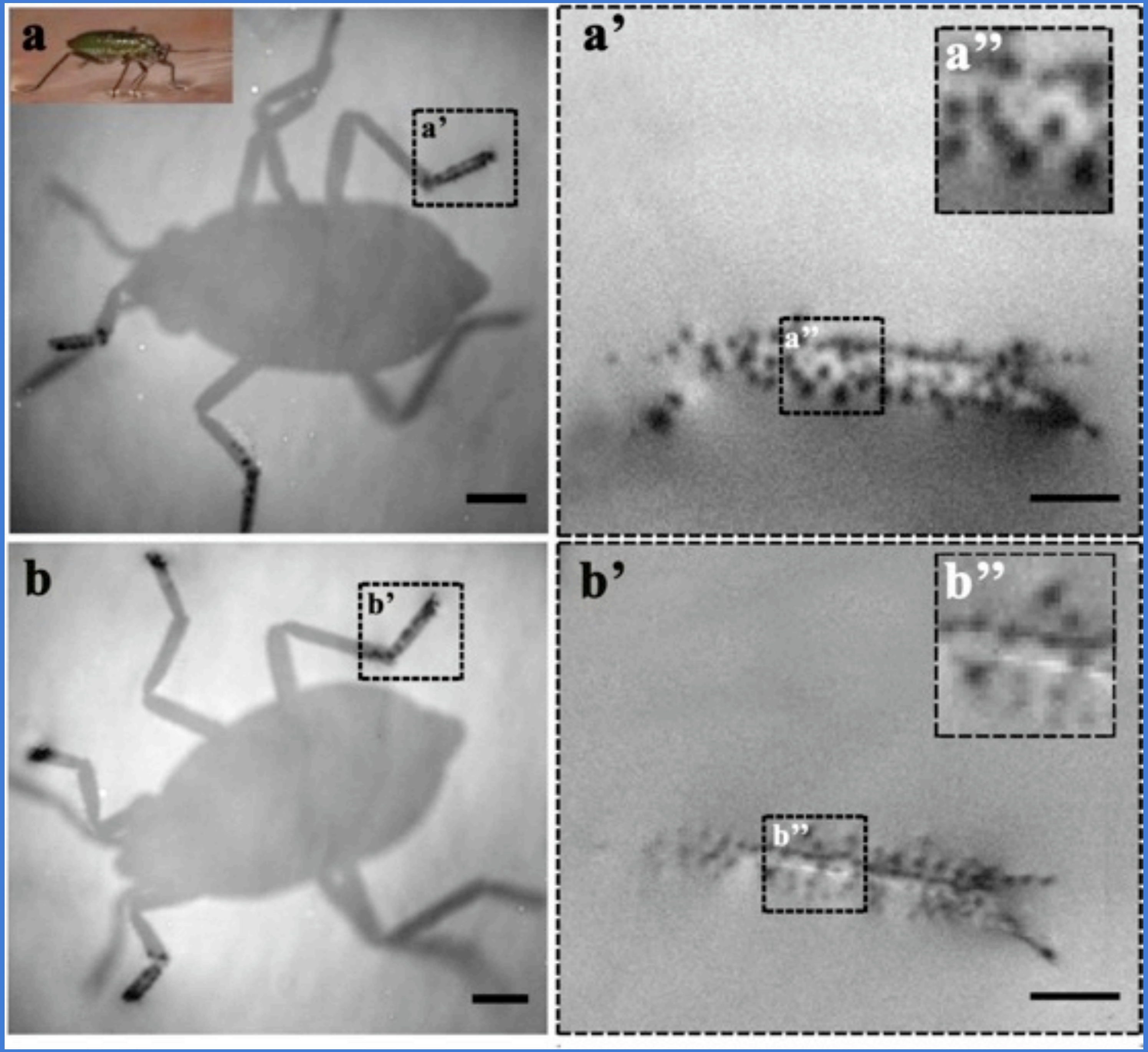
$$\boxed{\theta = \theta_A = \theta_R = 180^\circ}$$

The integument of water-walking insects and spiders

- body and legs covered in dense mat of fine hairs: "the Lotus Effect"

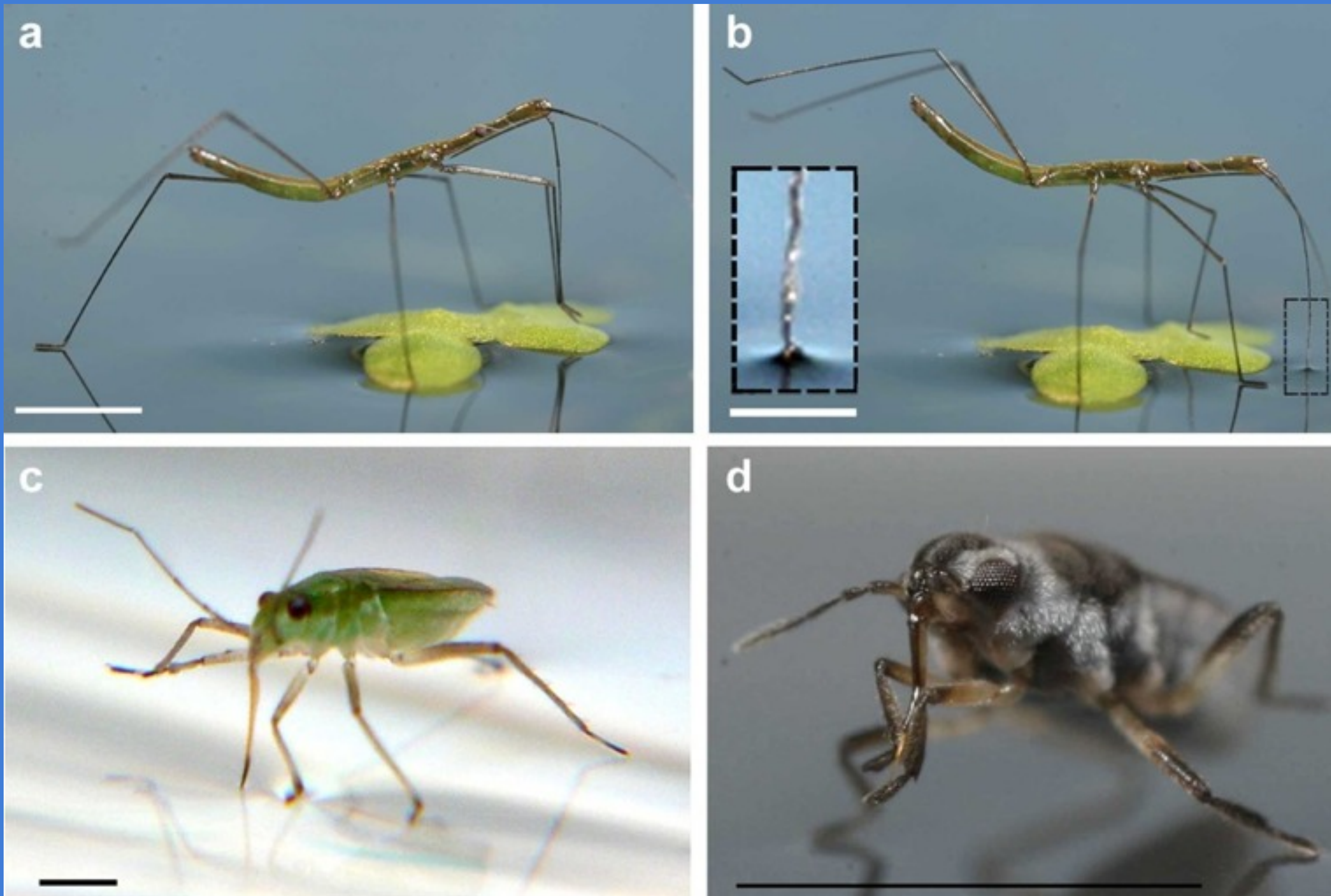
- hair layer increases surface area and so energetic cost of wetting
- hair mat thus discourages wetting, enhances water-repellency

Water-walking arthropods: in a Cassie state

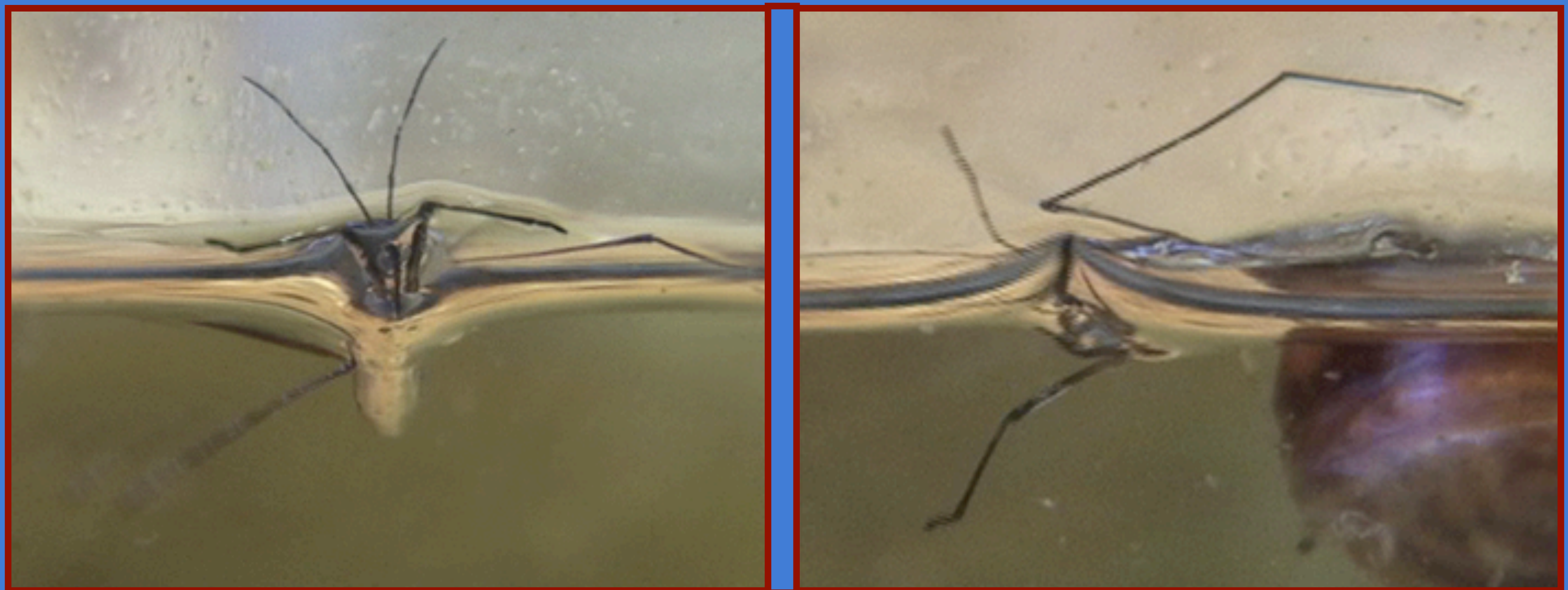


Mesovelia

Maintenance of their Cassie state prompts frequent grooming sessions.



The struggle of a partially submerged water strider (in soapy water)



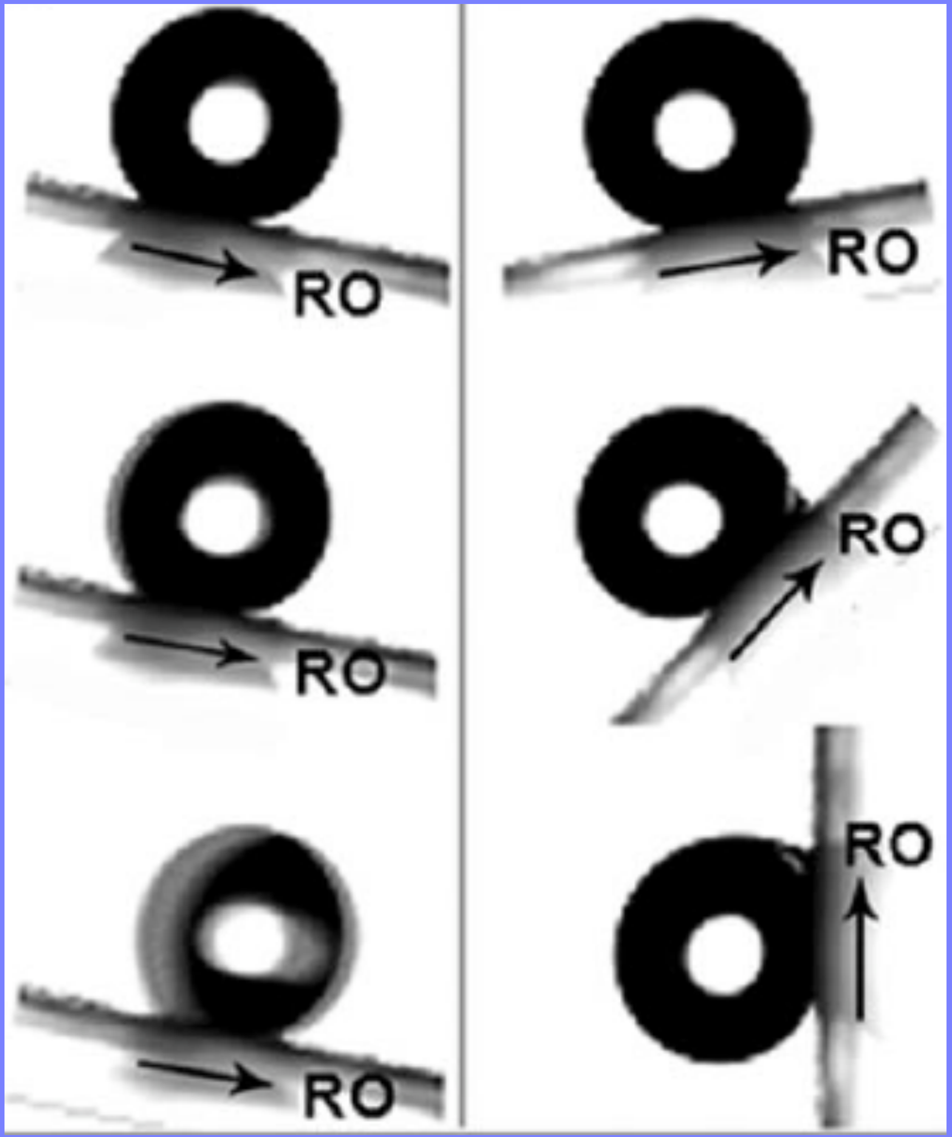
σ = 70 dynes/cm

- body weight: W = M g ~ 5 dynes ;
- total contact perimeter (leg plus body length): P ~ 7 cm
- force required to cross the interface: $\boxed{\sigma\ P \sim 500 \text{ dynes} \sim 100\ W}$
- soap has destroyed the water-repellency of their integument

Unidirectional adhesion

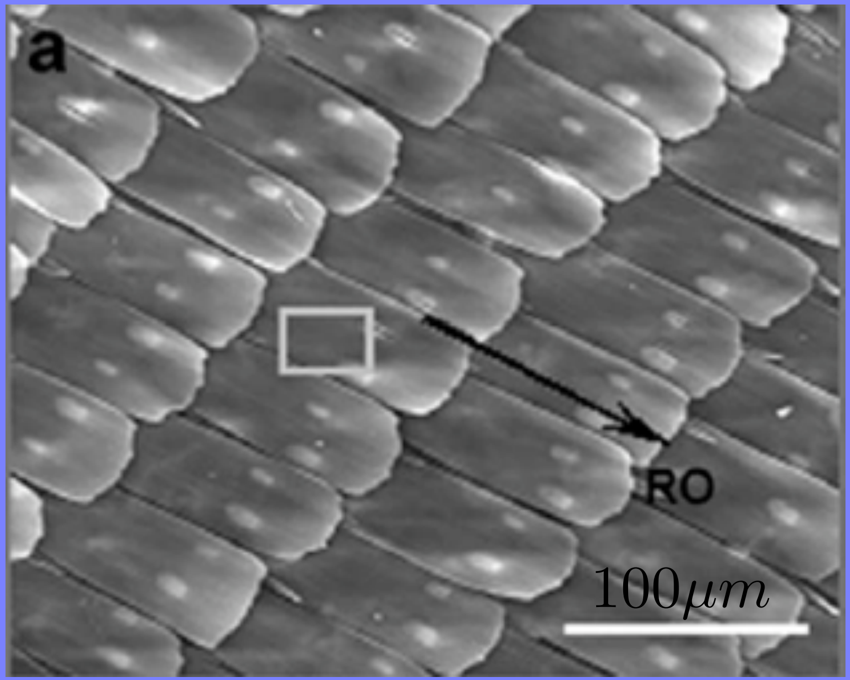
on the butterfly wing

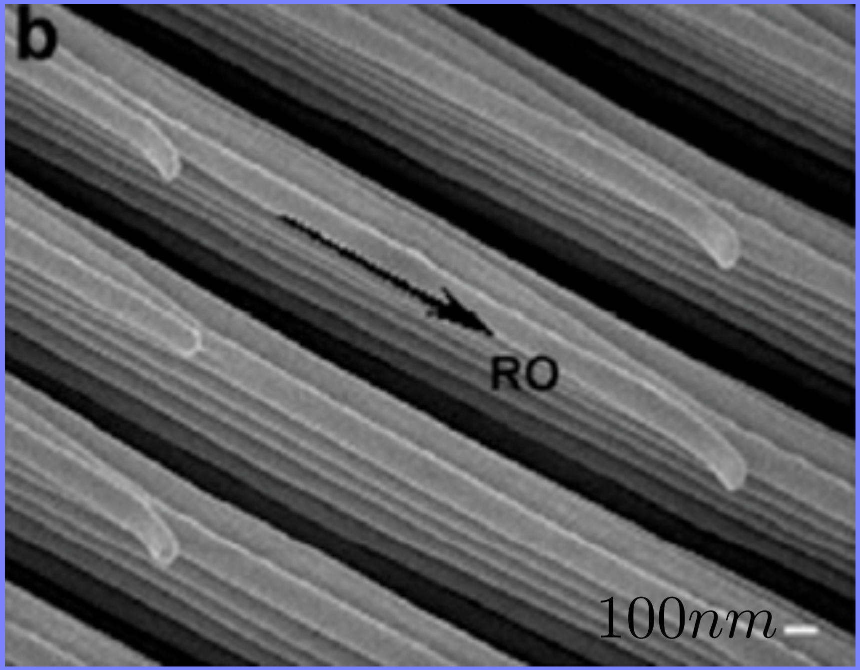
Zheng et al. (2007)

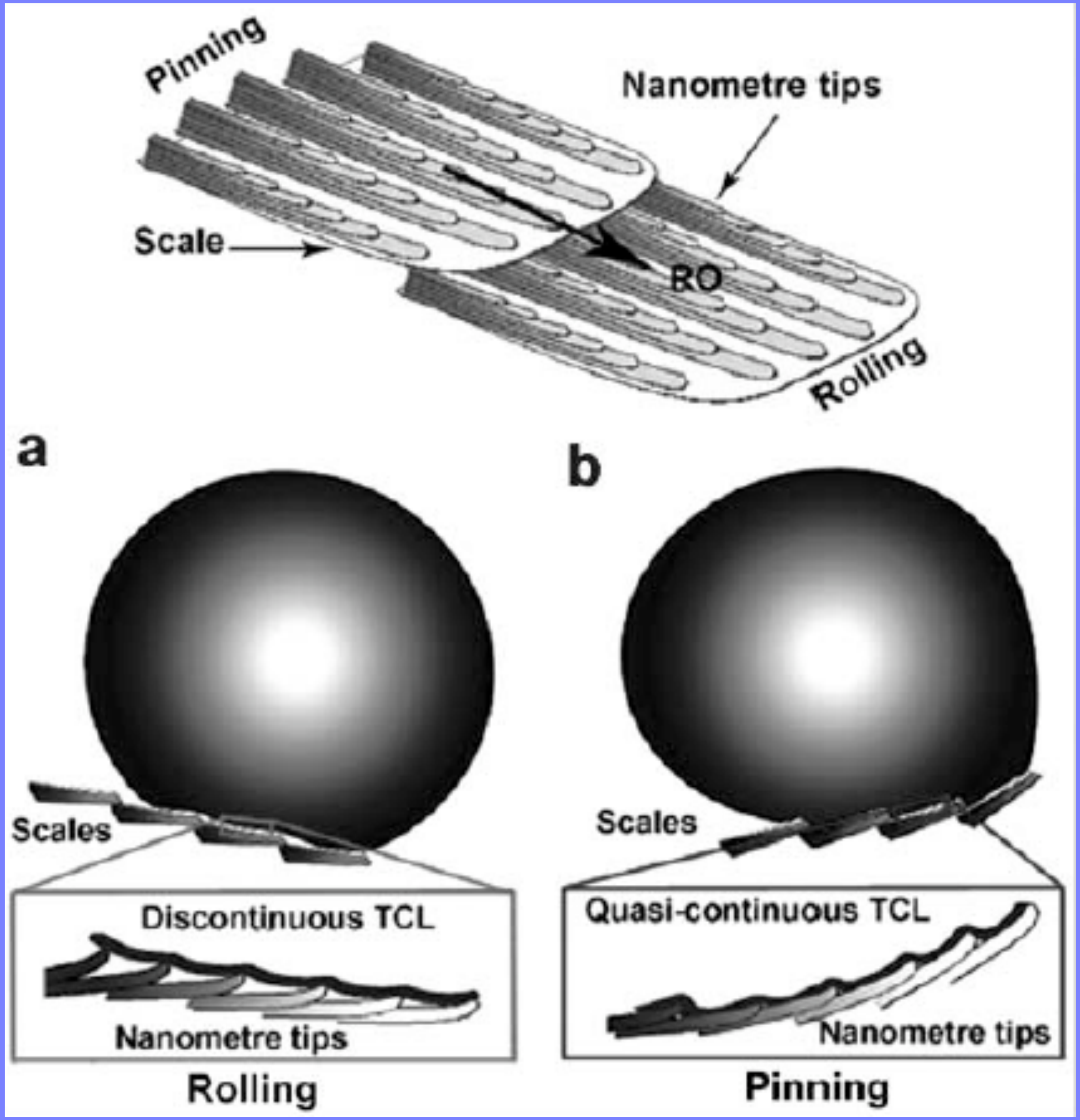


Unidirectional adhesion

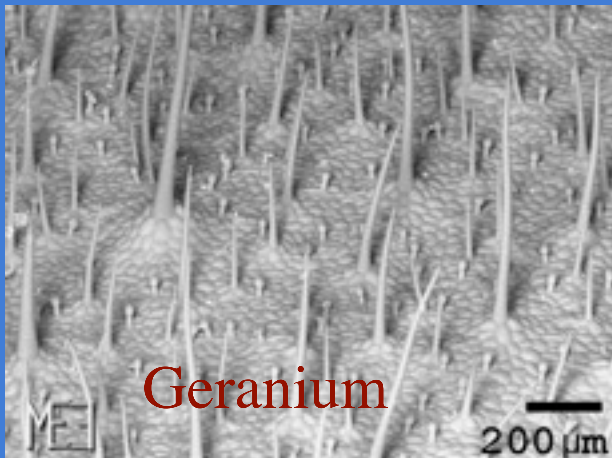
Zheng et al. (2007)

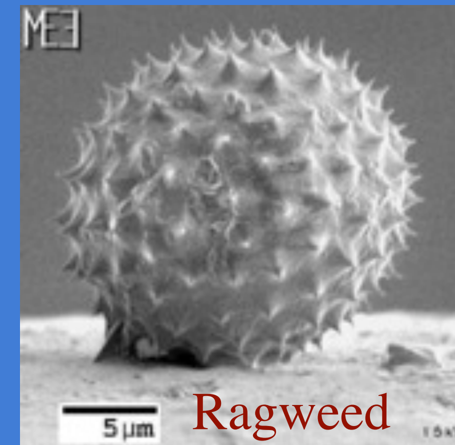






Plants are bumpy: isotropic roughness provides water-repellency

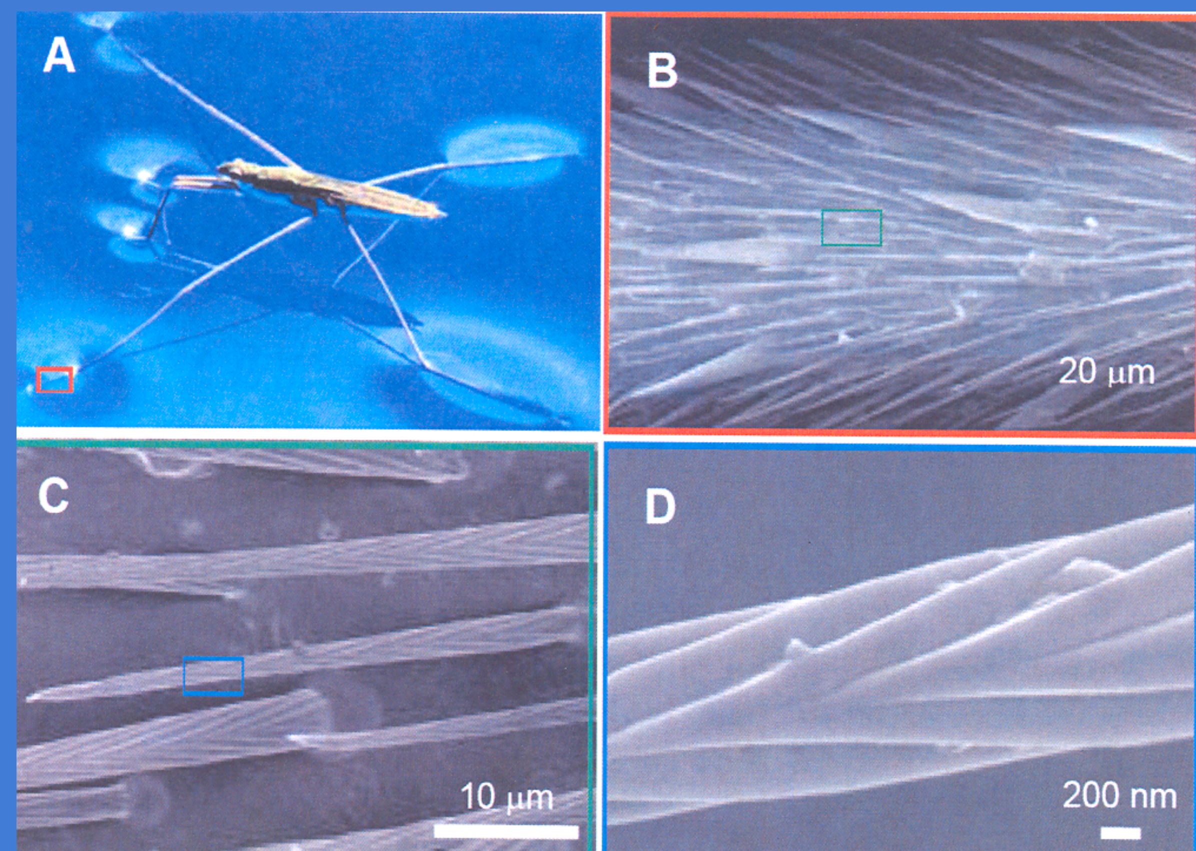




Water-walking bugs are hairy

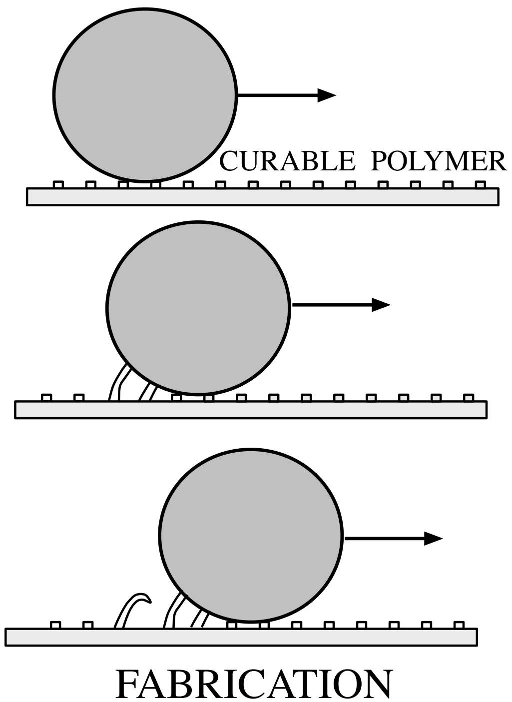
- roughness provides water-repellency
- driving leg exhibits unidirectional adhesion
- anisotropic roughness facilitates **propulsion**

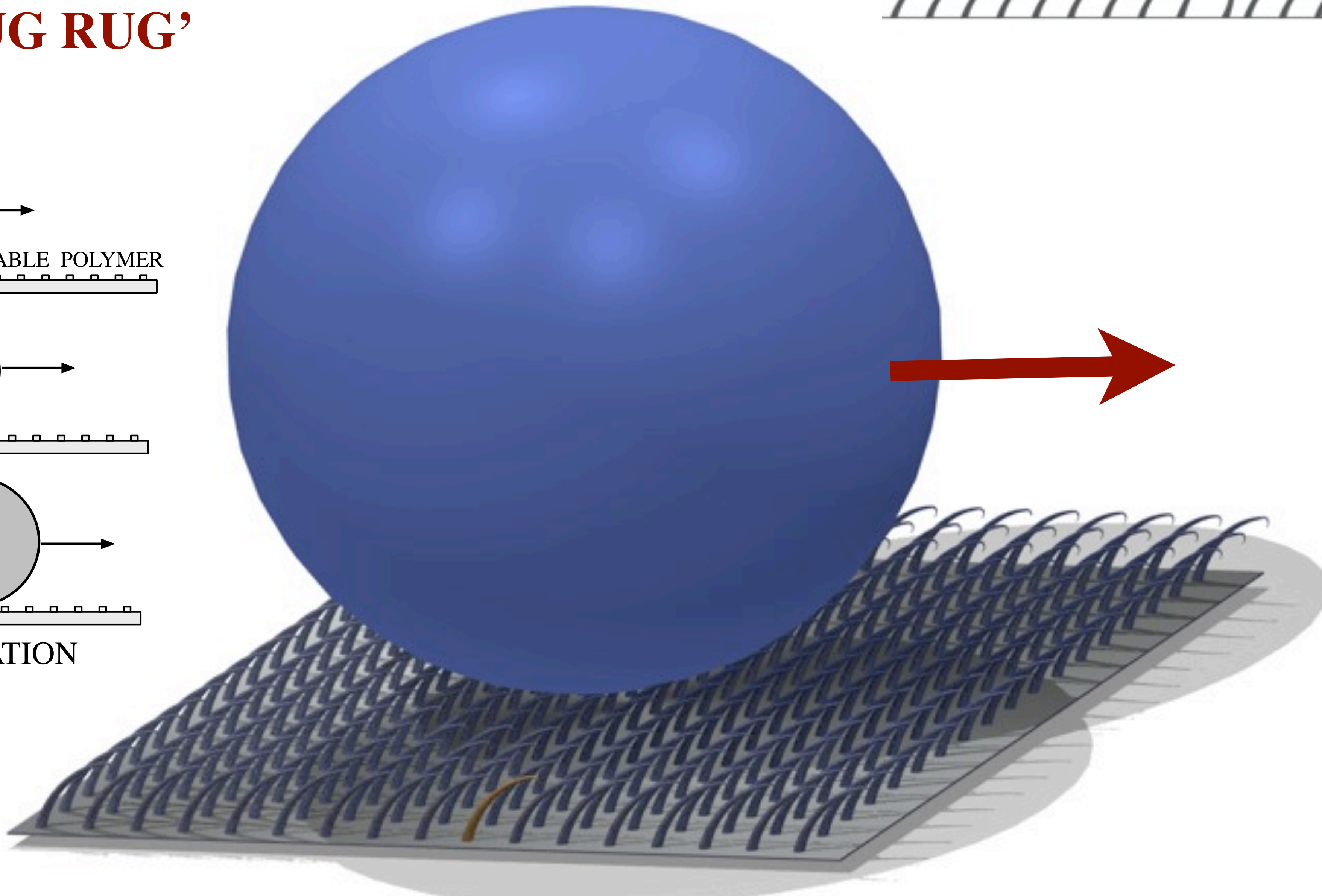
(Prakash & Bush 2011)



Biomimetic unidirectional surface

`THE BUG RUG'



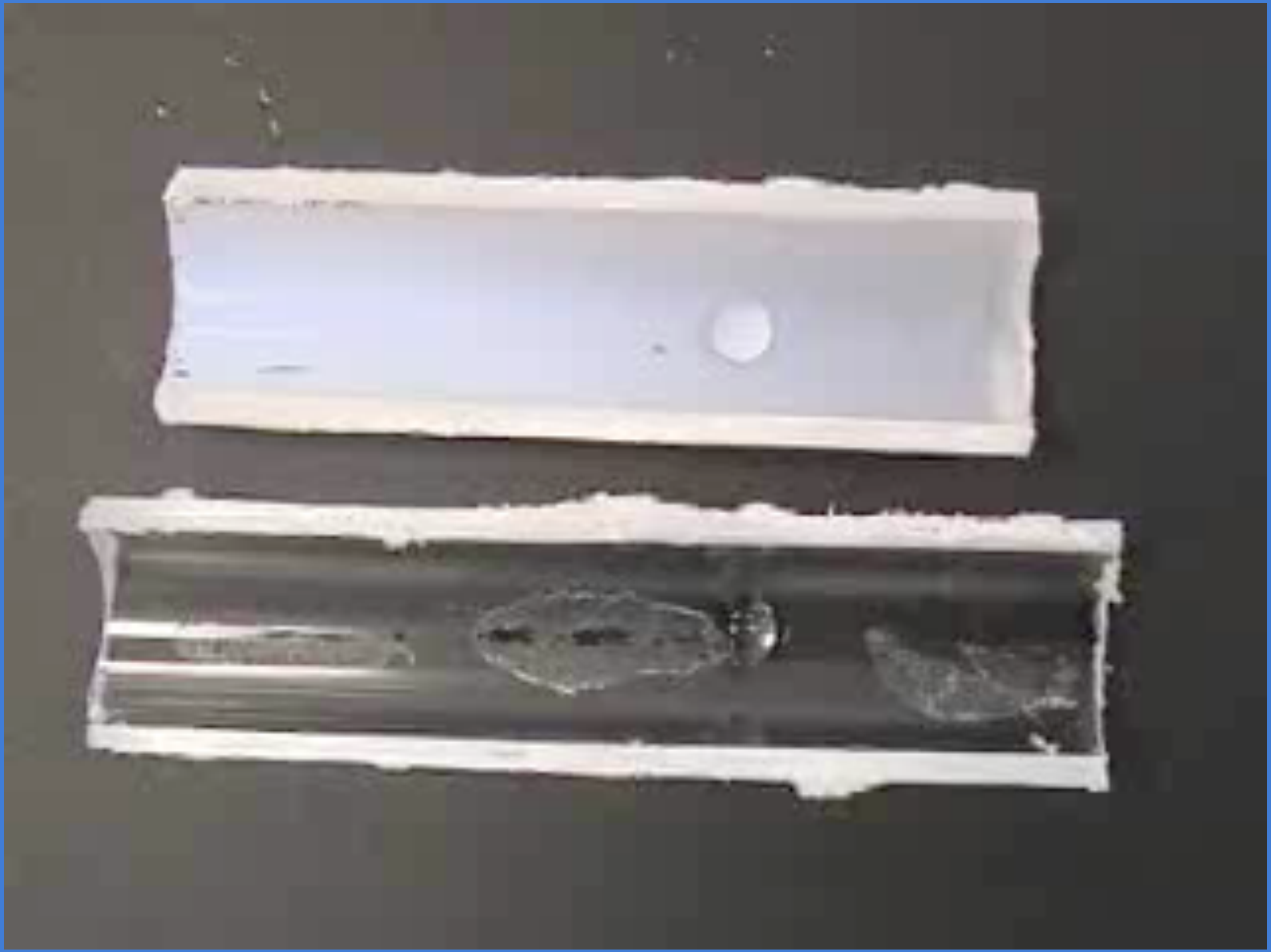


- permits drop motion in only one direction
- applications in directional draining, microfluidics

Vibration-induced drop motion on a textured substrate

Textured

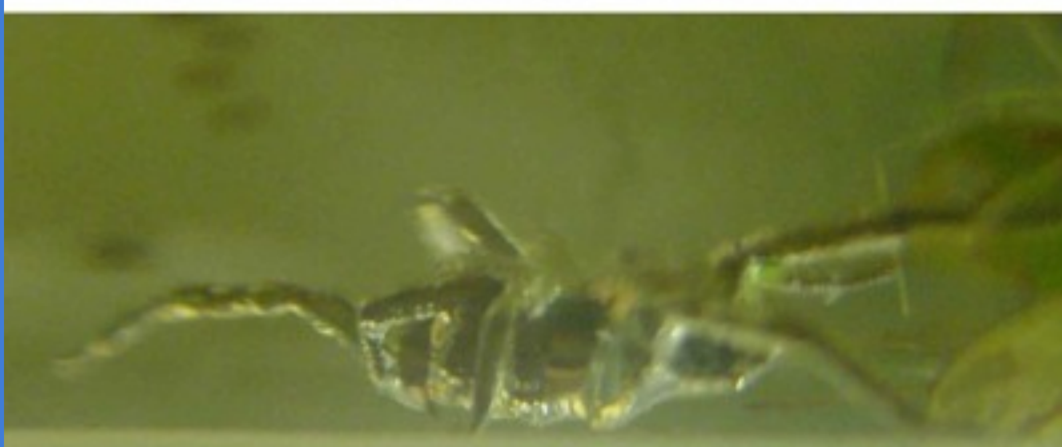
Smooth



Demirel and Hancock (2010)

Underwater breathing via water-repellency

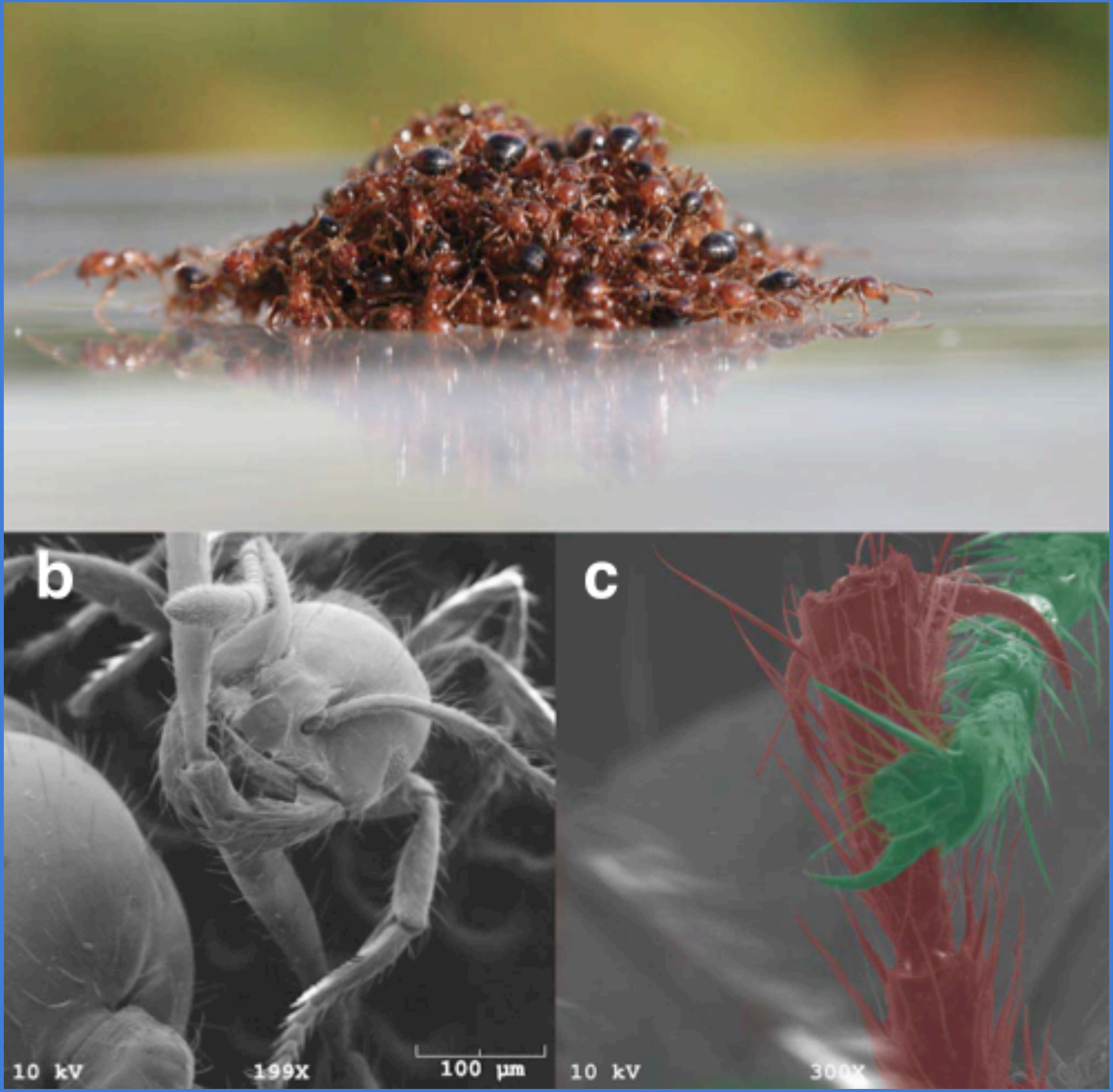
- thin air layer, termed the `plastron', trapped on body surface



- plastron serves as external gill
- oxygen diffuses into plastron, enabling extended dives
- may sustain bug indefinitely

Flynn & Bush (2007)

The ant raft: a self-assembling superhydrophobic surface

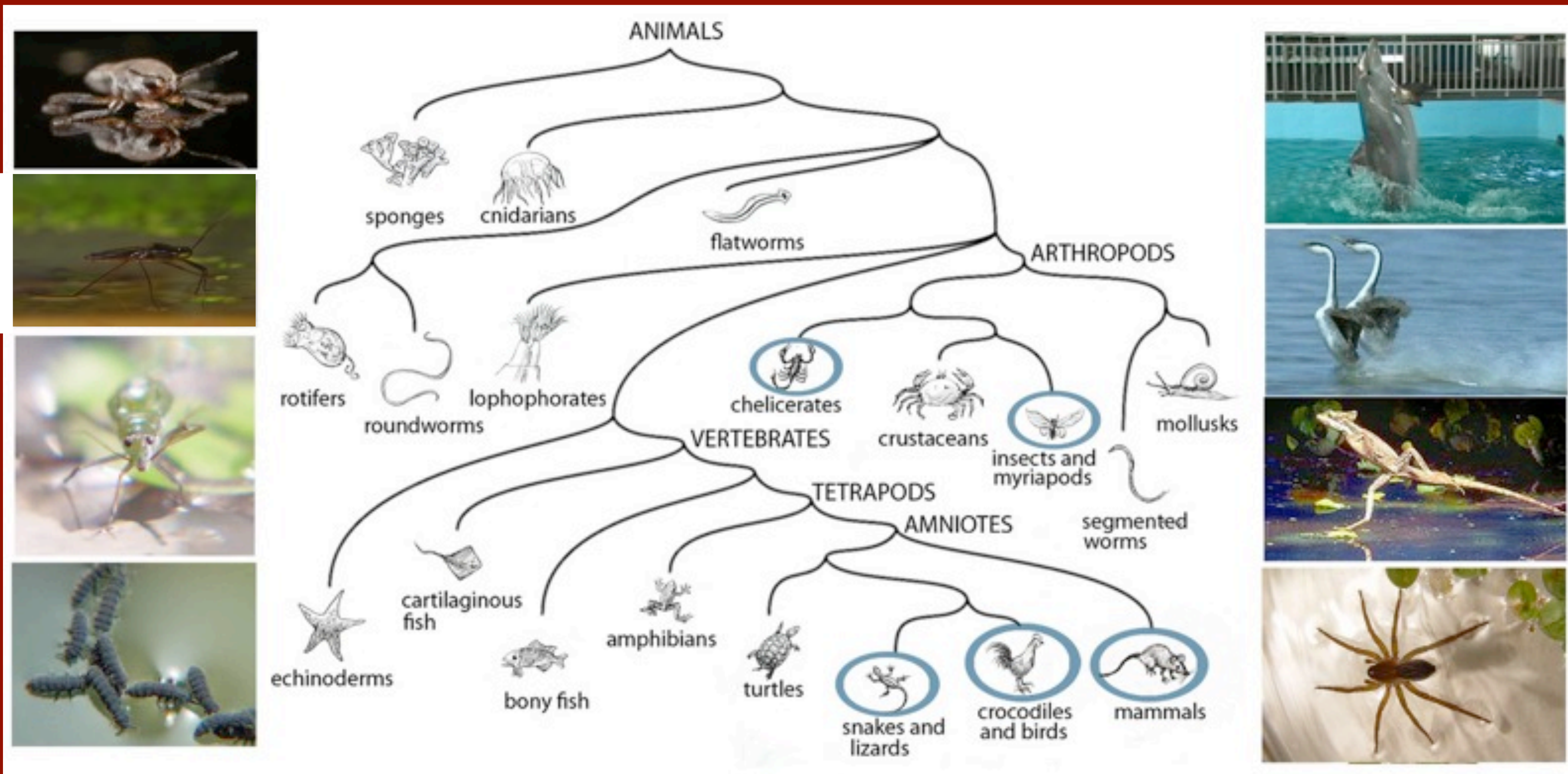


Mlot et al. (2011)

“The whale does not sing because it has an answer:
it sings because it has a song.” Gregory Colbert


“Choose only one master - Nature.” --- Rembrandt

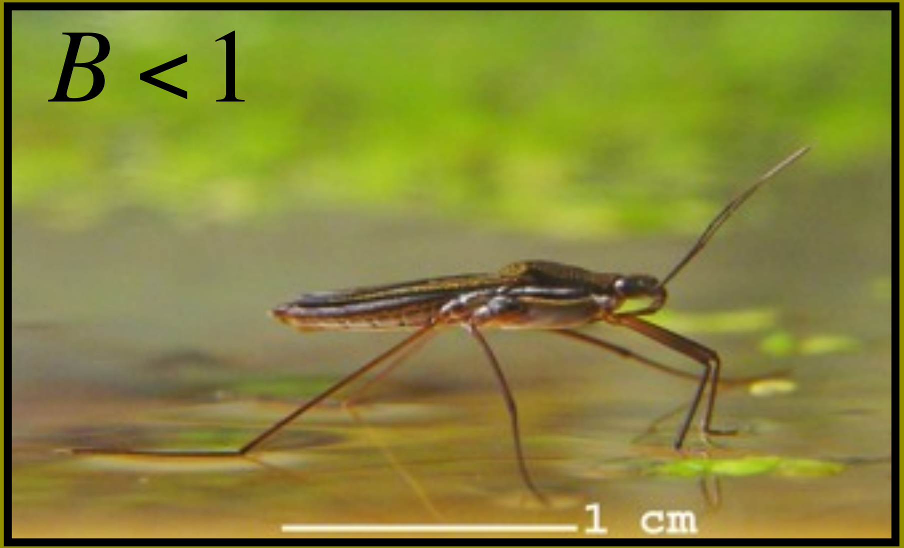
Water-walkers in the tree of life (over 1200 species)



Motivation: foraging on water surface, avoidance of predators

Two modes of weight-support:

$$B = \frac{Mg}{\sigma P} = \frac{\text{weight}}{\text{surface tension force}}$$

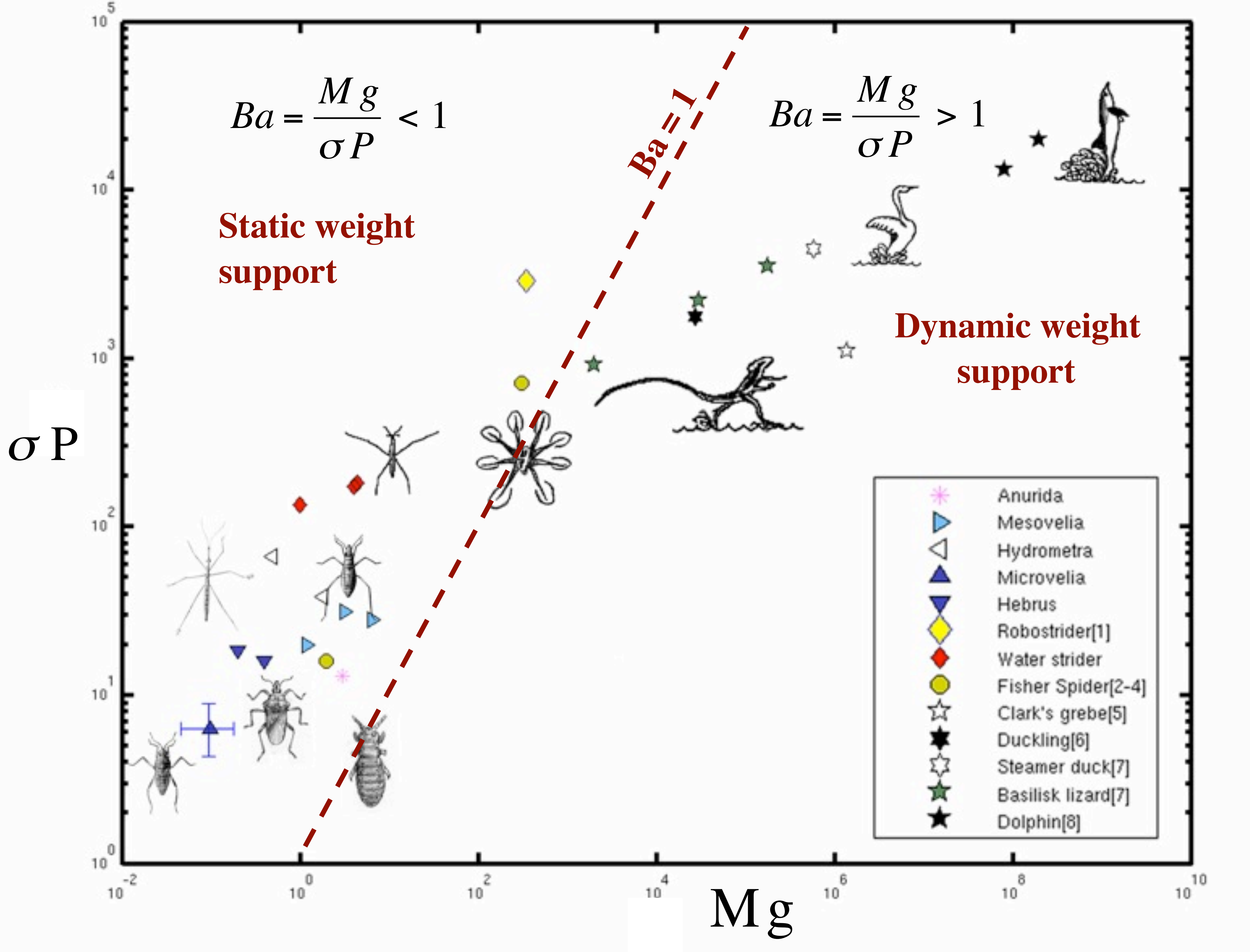


- static weight support by σ

$F \sim 2\sigma L$

Glasheen & McMahon (1996)

- dynamic weight support
- vertical hydrodynamic forces generated by slapping



$Ba = \frac{Mg}{\sigma P} < 1$
$Ba = \frac{Mg}{\sigma P} > 1$
Ba = 1
Static weight support
Dynamic weight support
σP
Mg
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
10^{8}
10^{10}
Anurida
Mesovelia
Hydrometra
Microvelia
Hebrus
Robostrider[1]
Water strider
Fisher Spider[2-4]
Clark's grebe[5]
Duckling[6]
Steamer duck[7]
Basilisk lizard[7]
Dolphin[8]

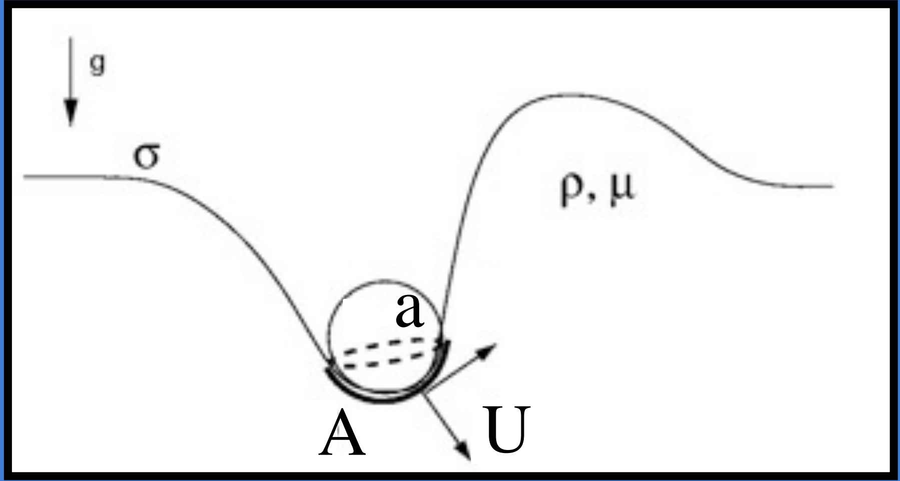
Biological classification

- made along evolutionary grounds

Dynamic classification

- group according to propulsion mechanism
- evaluate relative magnitudes of hydrodynamic forces

Dynamic classification of water-walkers



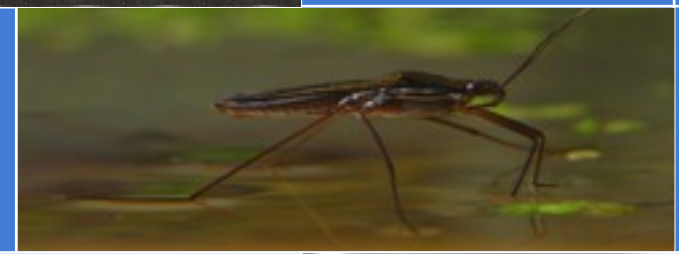
$$F_H \sim \rho g V_s + \rho U^2 A + \rho V \frac{dU}{dt} + \rho \nu U a + \sigma(\underline{\nabla} \cdot \underline{n}) A - \underline{\nabla}\sigma A$$

buoyancy | form drag | acceleration reaction | viscous drag | curvature | Marangoni

Mathematician: which terms can I discard to get a tractable equation?

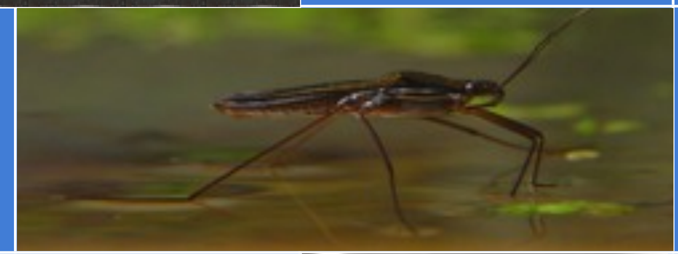
Physicist: which forces are used by which creatures?

→ every force is used by some creature

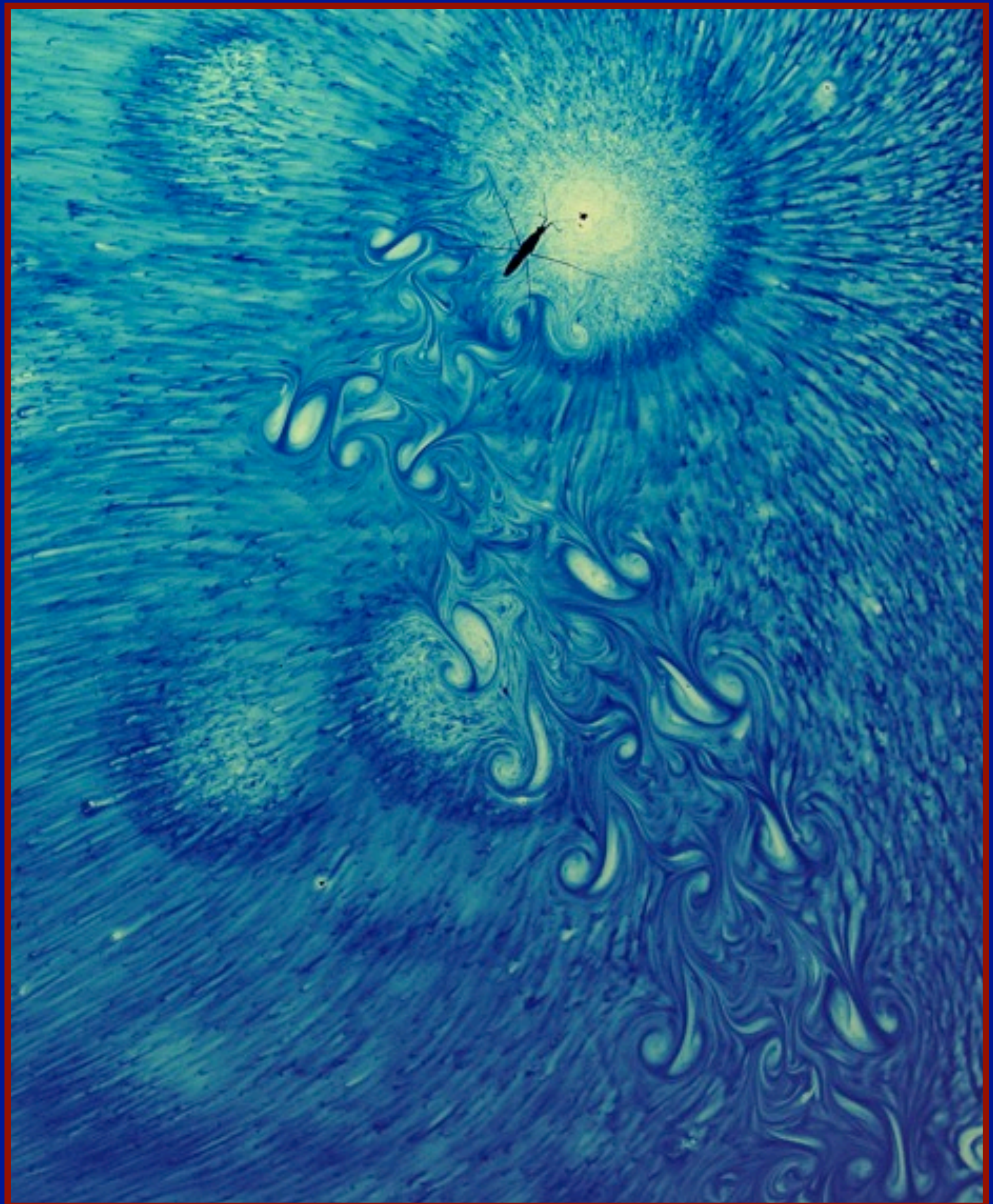
	$\rho g z\, A$	$\rho V dU/dt$	$\rho U^2 A$	$\sigma \nabla\cdot \underline{n}\, A$	$\underline{\nabla}\sigma\, A$
Surface slapping					
Rowing & walking					
Surface distortion					
Marangoni propulsion					

Clark’s Grebe: clip courtesy of “Winged Migration”

Courtesy of National Geographic

	$\rho g z\, A$	$\rho V dU/dt$	$\rho U^2 A$	$\sigma \nabla \cdot \underline{n}\, A$	$\underline{\nabla} \sigma\, A$
Surface slapping					
Rowing & walking					
Surface distortion					
Marangoni propulsion					

$\underbrace{\hspace{6em}}_{\text{quasi – static propulsion}}$

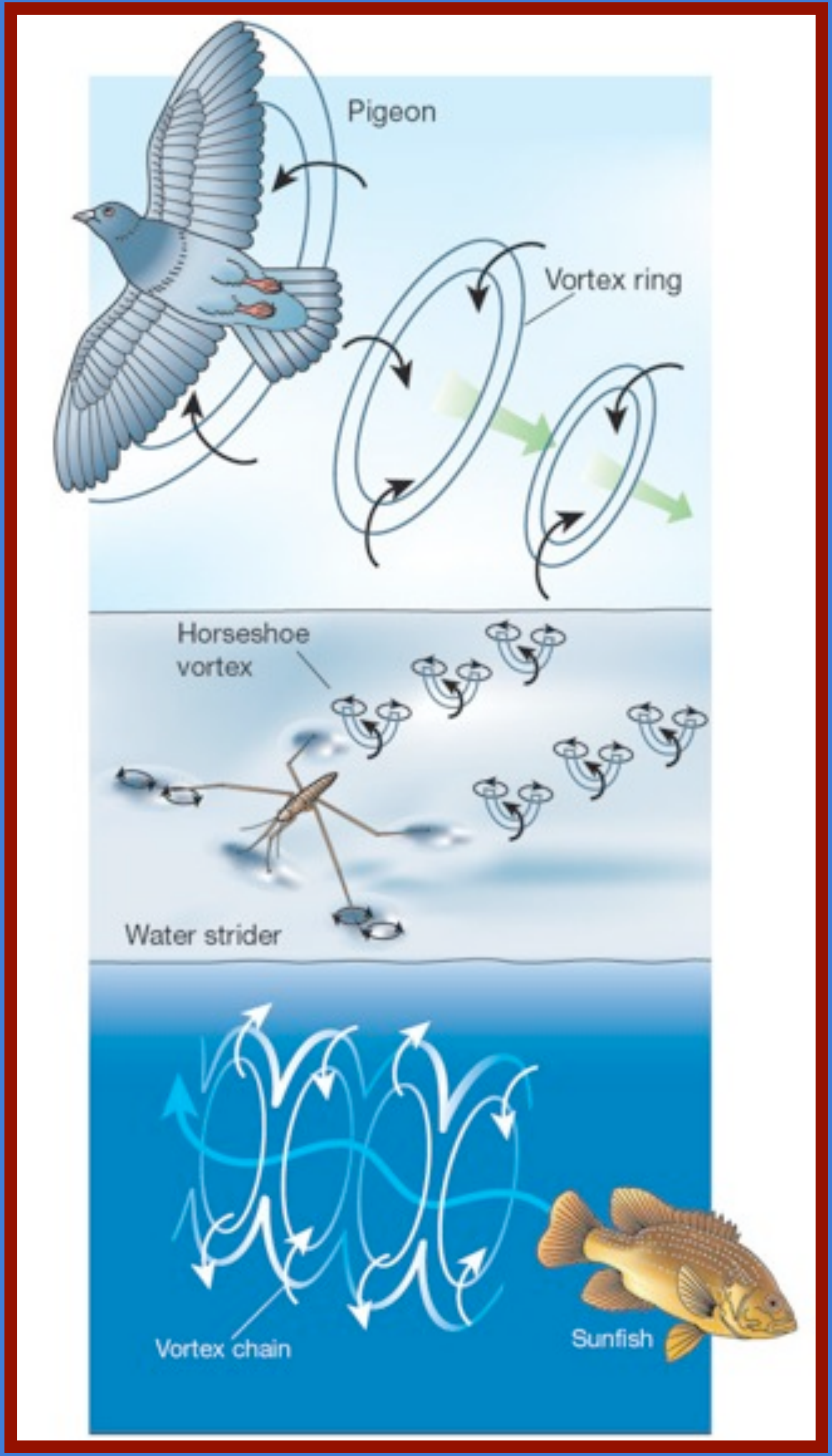



Dickinson (2003)

Flying

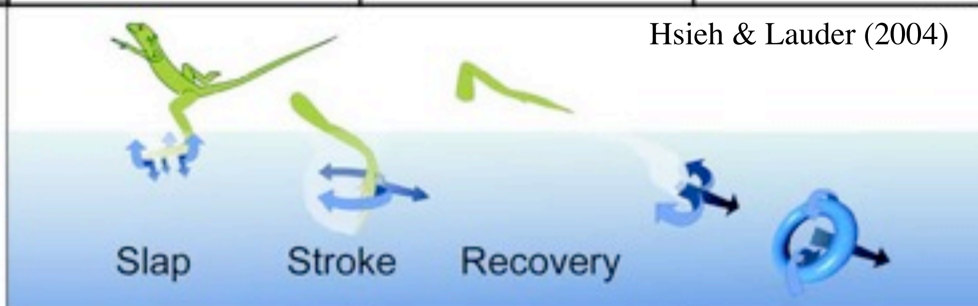
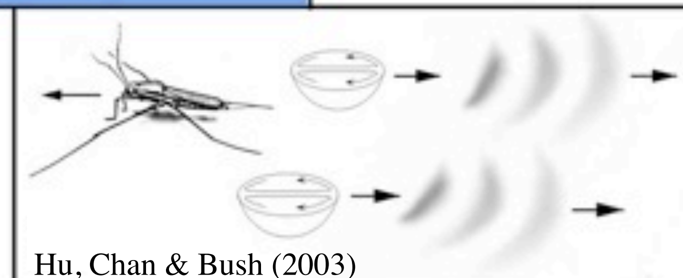
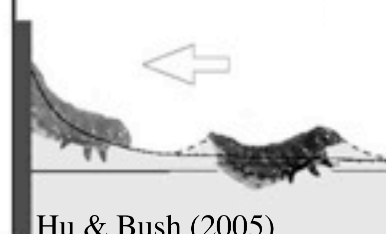
Rowing

Swimming



SUMMARY

Bush & Hu (2006)

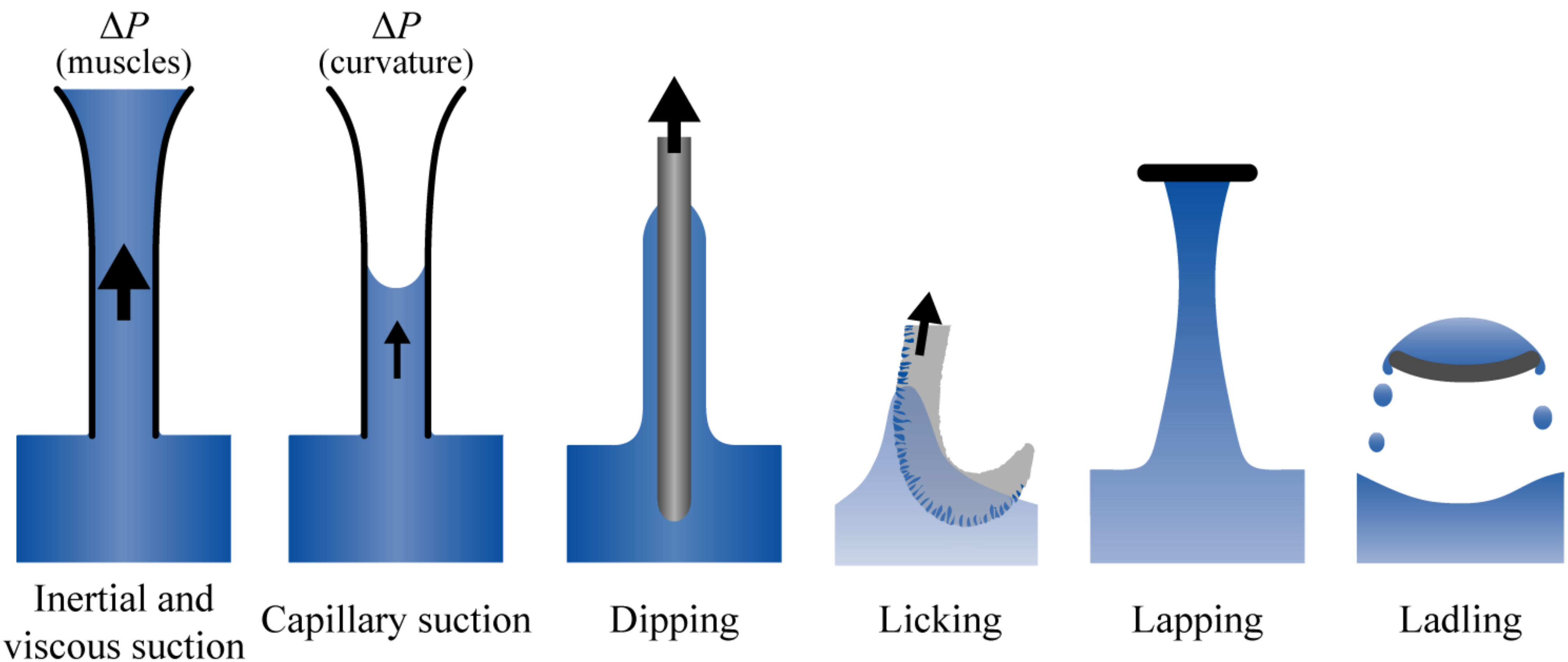
	Buoyancy	Added mass	Inertia	Curvature	Marangoni
Surface slapping	 Hsieh & Lauder (2004)				
Rowing & walking			 Hu, Chan & Bush (2003)		
Meniscus climbing				 Hu & Bush (2005)	
Marangoni propulsion					

Drinking strategies in nature

with Wonjung Kim (see Poster)

Various drinking techniques in nature

Classification according to dominant driving and resistive forces



Goal: classify and rationalize all drinking styles

Scales of forces in drinking

Scales of forces in drinking

fluid properties (ρ, μ), flow speed (u), mouth size (L), applied pressure (ΔP), and gravitational acceleration (g)

$F_{pressure} \sim \Delta P\, L^2$

$F_{inertia} \sim \rho u^2 L^2$

$F_{viscous} \sim \mu u L$

$F_{gravity} \sim \rho g L^3$

$\Delta P_{muscle} \sim 10$ kPa

$\Delta P_{curvature} \sim \sigma / L$

$F_{max} \sim l^2$, $\Delta P_{max} \sim F_{max} / l^2 \sim l^0$

e.g., 10 kPa for mosquitoes, humans, and elephants

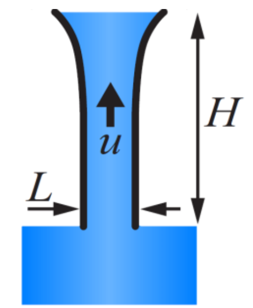
Relative magnitudes of hydrodynamic forces

$Bo = \rho g L^2 / \sigma$ ~ curvature to hydrostatic pressure

$Re = \rho u L / \mu$ ~ inertia to viscous forces

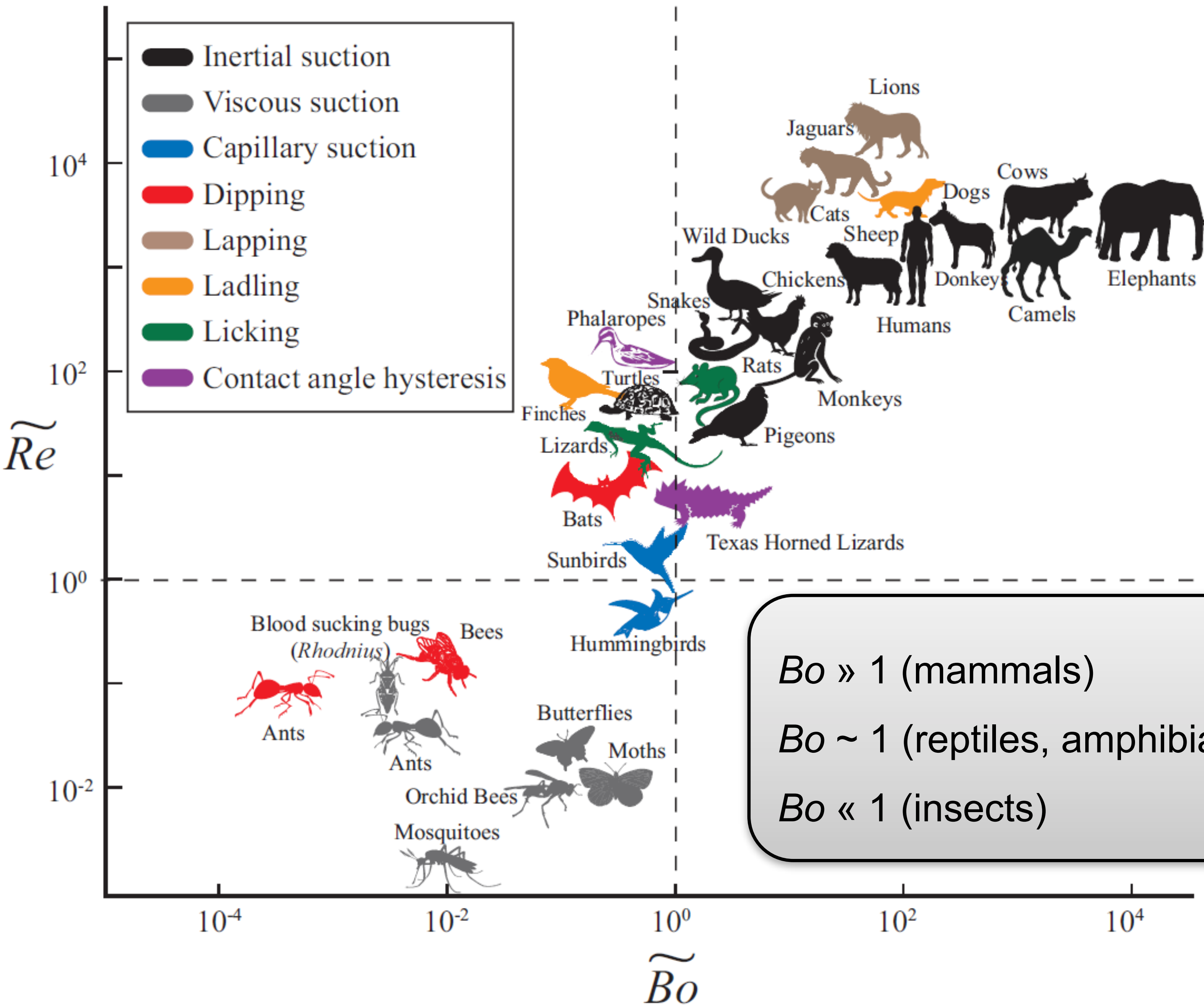
$\widetilde{Bo} = \rho g L^2 / \sigma \cdot (H/L)$

$\widetilde{Re} = \rho u L / \cdot (L/H)$



Assessment of these dimensionless numbers ⇒ dominant forces in drinking

Re and *Bo* in drinking of various creatures



Bo » 1 (mammals)

Bo ~ 1 (reptiles, amphibians, birds)

Bo « 1 (insects)

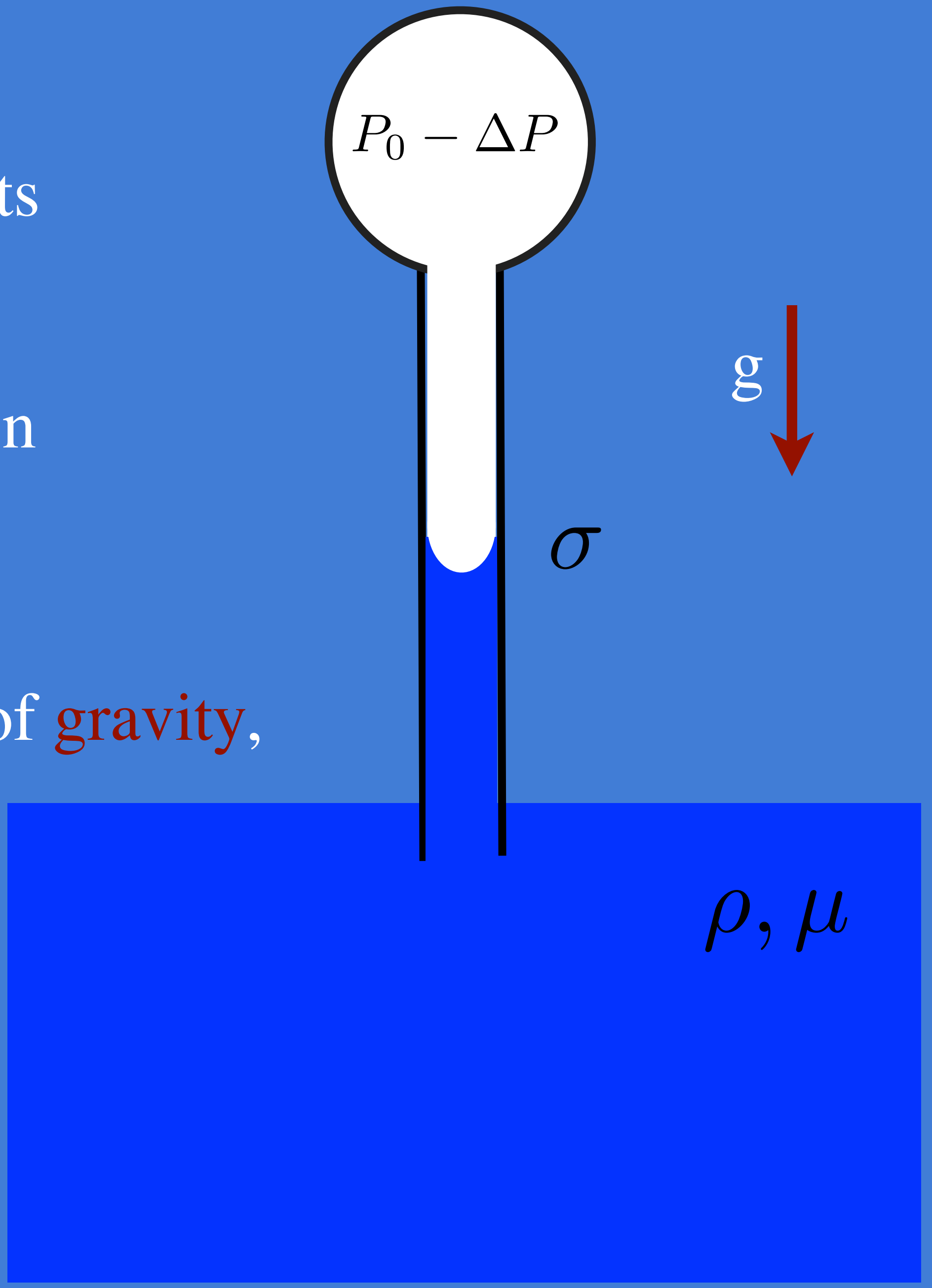
On drinking through a tube

from mosquitoes to elephants

- fluid rises through some combination of applied suction and capillarity

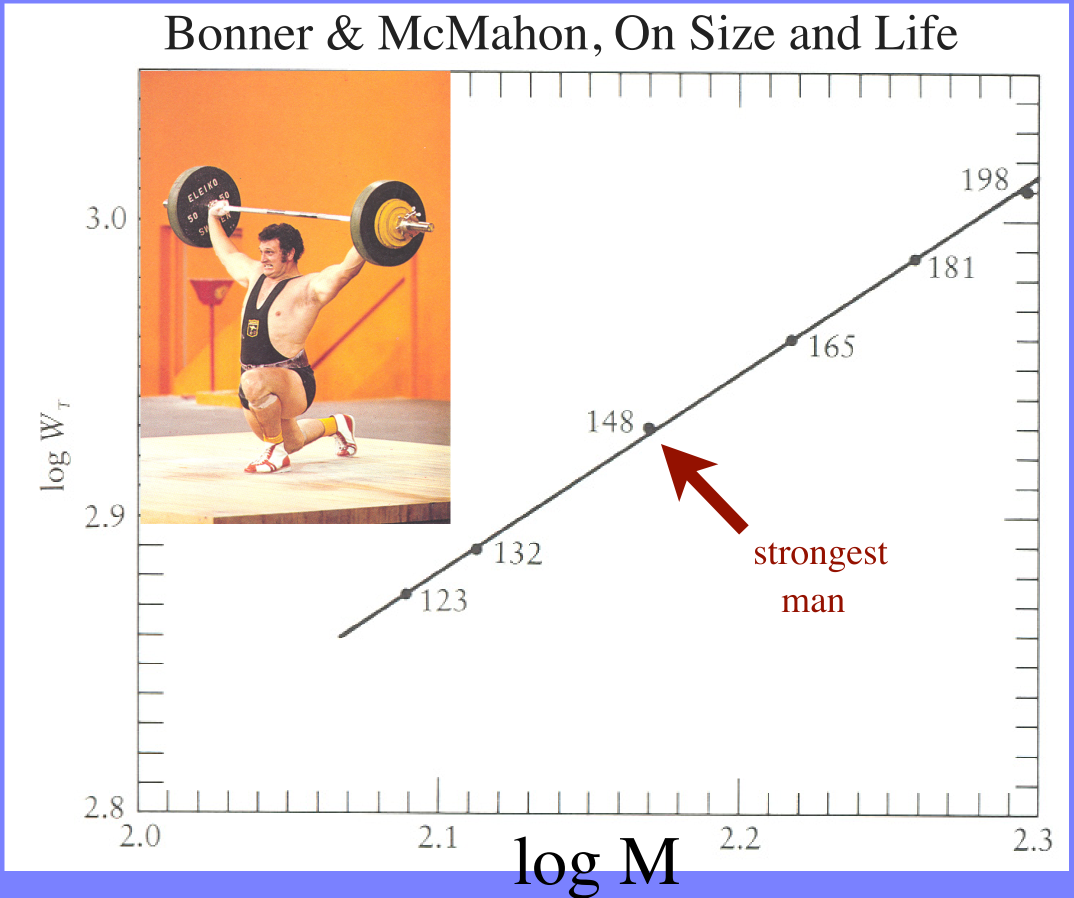
- rise resisted by some combination of gravity, inertia and viscosity

The relative magnitudes of these forces will be prescribed by the scale of the drinker.



A creature of length L and mass M can generate a force:

$$W_T \sim M^{2/3} \sim L^2$$



The suction pressure generated by a creature should thus $\sim L^2/L^2$, and so be INDEPENDENT OF SIZE.

IS THIS TRUE?

e.g. elephants, bees	$\Delta P \sim 20\ kPa$
mosquitoes	$\Delta P \sim 8\ kPa$
humans	$\Delta P \sim 4\ kPa$
butterflies	$\Delta P \sim 40\ kPa$
bedbugs	$\Delta P \sim 80\ kPa$

$$\Delta P \sim 0.1 - 0.5\ atm$$

On drinking through a tube

Driving force

$$\frac{suction}{capillarity} \sim \frac{a\Delta P}{\sigma}$$

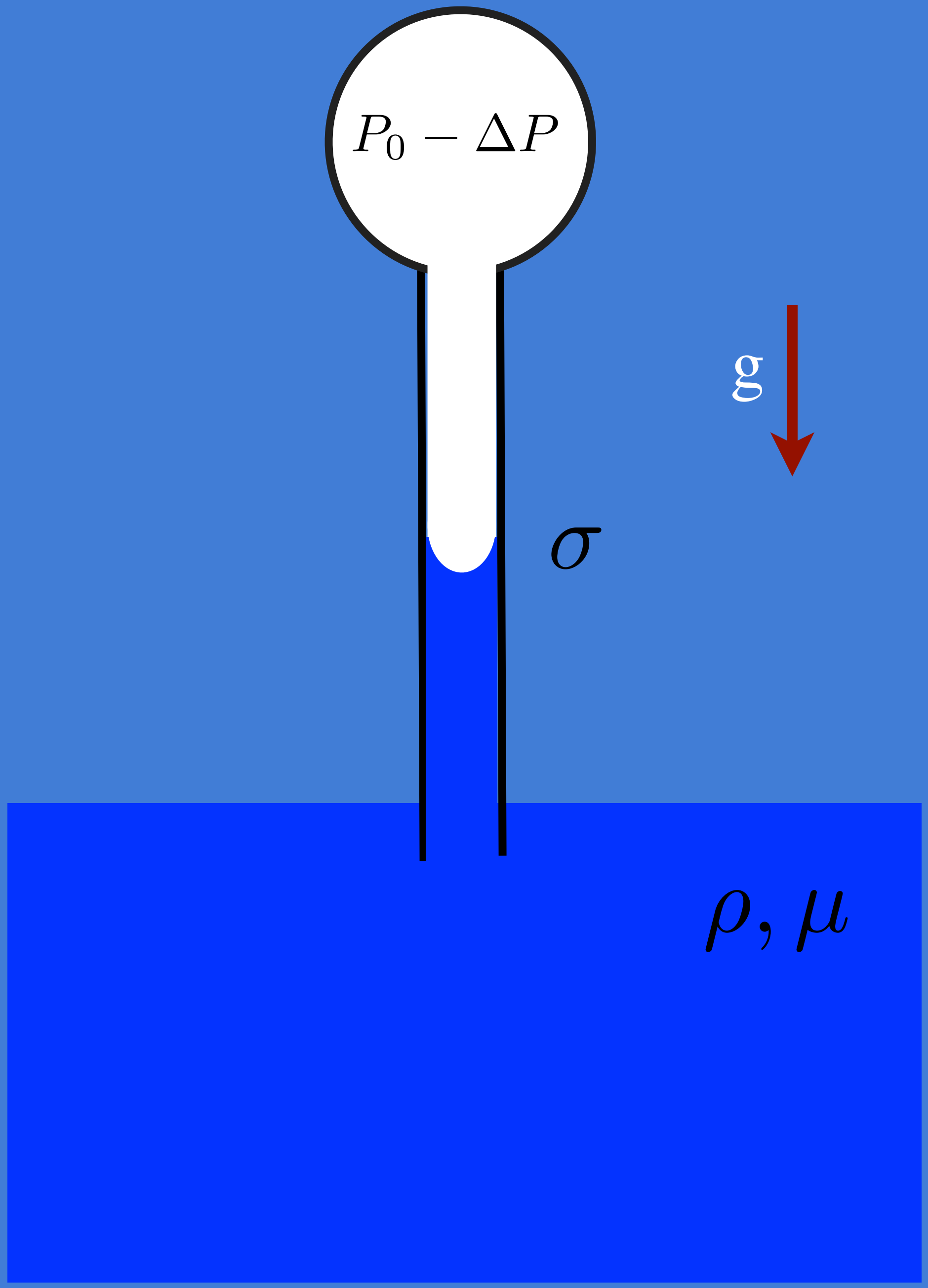
Scale at which capillarity wins:

$$a < \frac{\sigma}{\Delta P} \sim \frac{70 \text{ dynes/cm}}{10^5 \text{ dynes/cm}^2} \sim 10\mu m$$

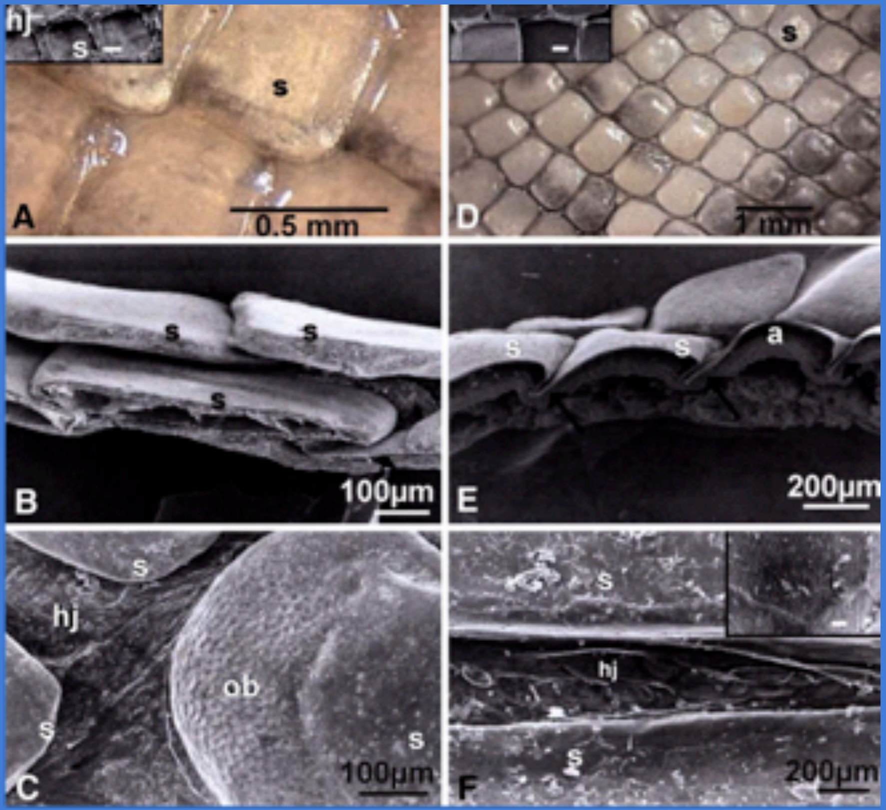
Resisting force

$$\frac{inertia}{viscosity} \sim \frac{Ua}{\nu}$$

$$\frac{gravity}{viscosity} \sim \frac{gLa}{\nu U}$$



Dermal capillary drinking by the Thorny Devil Lizard



(Sherbrooke et al. 2007)

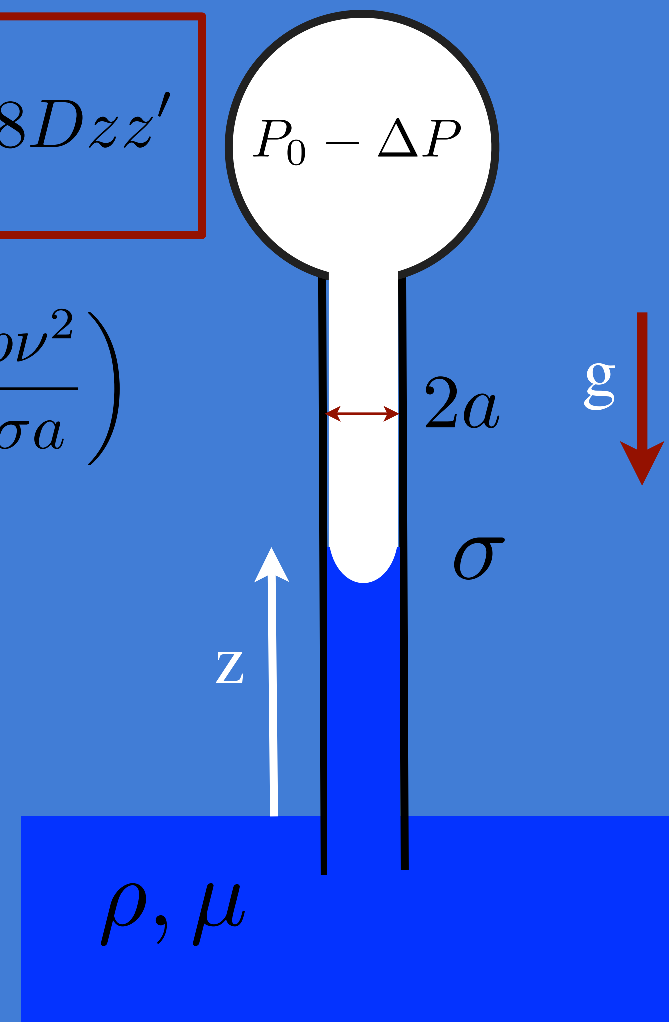
Force balance

$$\Delta P \;+\; 2\,\frac{\sigma}{a}\cos\theta \;=\; \frac{1}{2}\,\rho\, z'^2 \;+\; \rho\,(z+\frac{7}{6}a)\,z'' \;+\; \rho g z \;+\; 8\,\frac{\mu}{a^2}\,zz'$$

SUCTION CAPILLARITY INERTIA GRAVITY VISCOSITY

Nondimensionalize: radius a , height L , time $L\left(\frac{\rho a}{\sigma}\right)^{1/2}$

$$\boxed{P^* + 2\cos\theta \;=\; \frac{1}{2}z'^2 \;+\; (z+\frac{7}{6})z'' \;+\; B_o z \;+\; 8Dzz'}$$



where $P^* = \frac{a\Delta P}{\sigma}$, $B_o = \frac{\rho g a L}{\sigma}$, $D = \frac{L}{a}\left(\frac{\rho \nu^2}{\sigma a}\right)$

Approach:

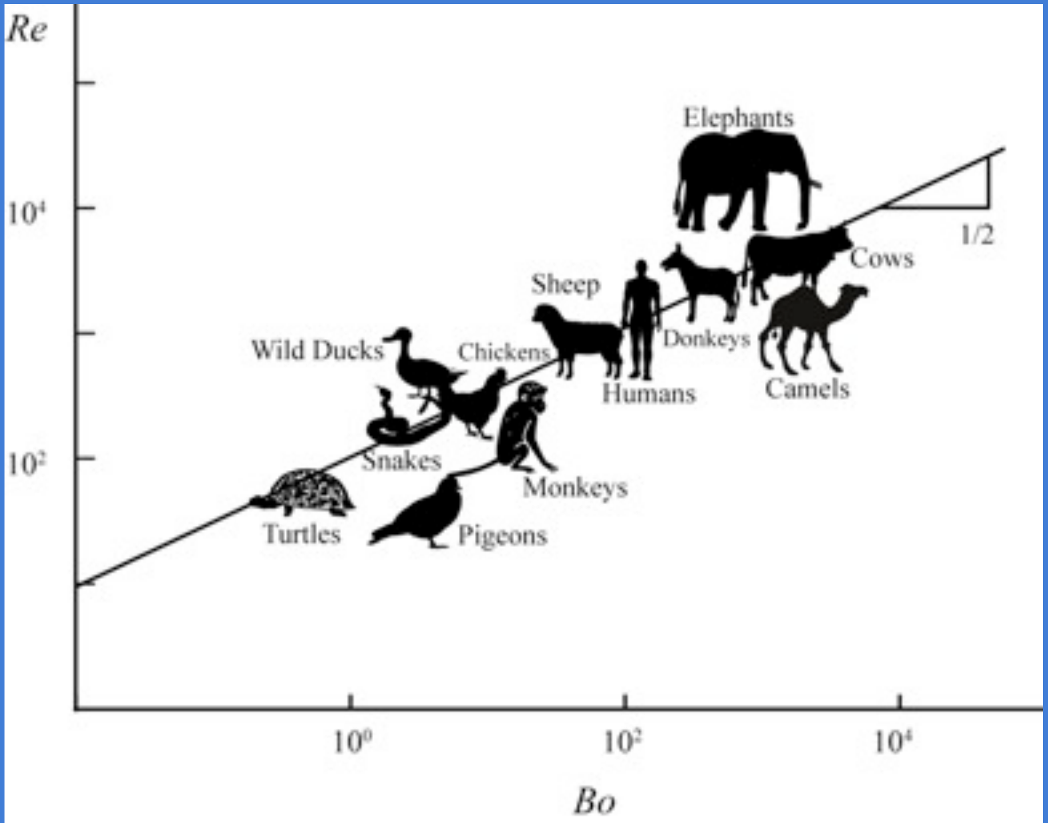
assess values of these dimensionless groups
to elucidate dominant physics

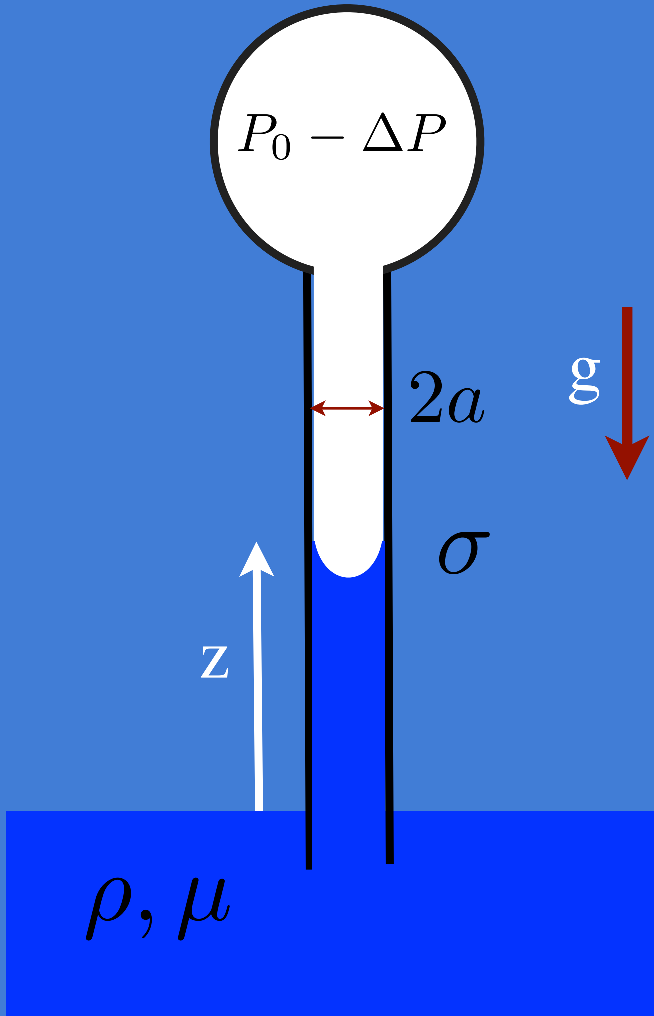
Inertial suction

$$\boxed{P^* + \cancel{2\cos\theta} = \frac{1}{2}z'^2 + \cancel{(z+\frac{7}{6})}z'' + \cancel{B_o}z + 8\cancel{Dz}z'}$$

where $P^* = \dfrac{a\Delta P}{\sigma}$, $B_o = \dfrac{\rho g a L}{\sigma}$, $D = \dfrac{L}{a}\left(\dfrac{\rho\nu^2}{\sigma a}\right)$

$$\rightarrow \quad \boxed{u \sim (\Delta P/\rho)^{1/2}}$$



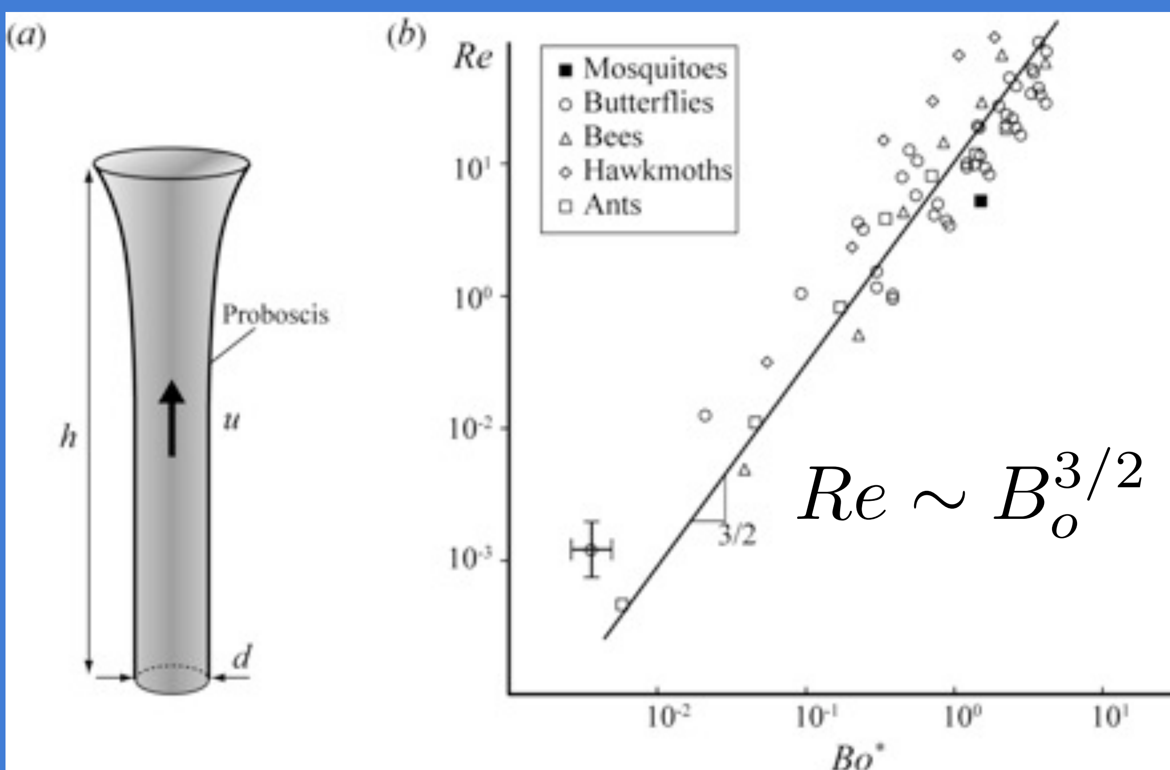


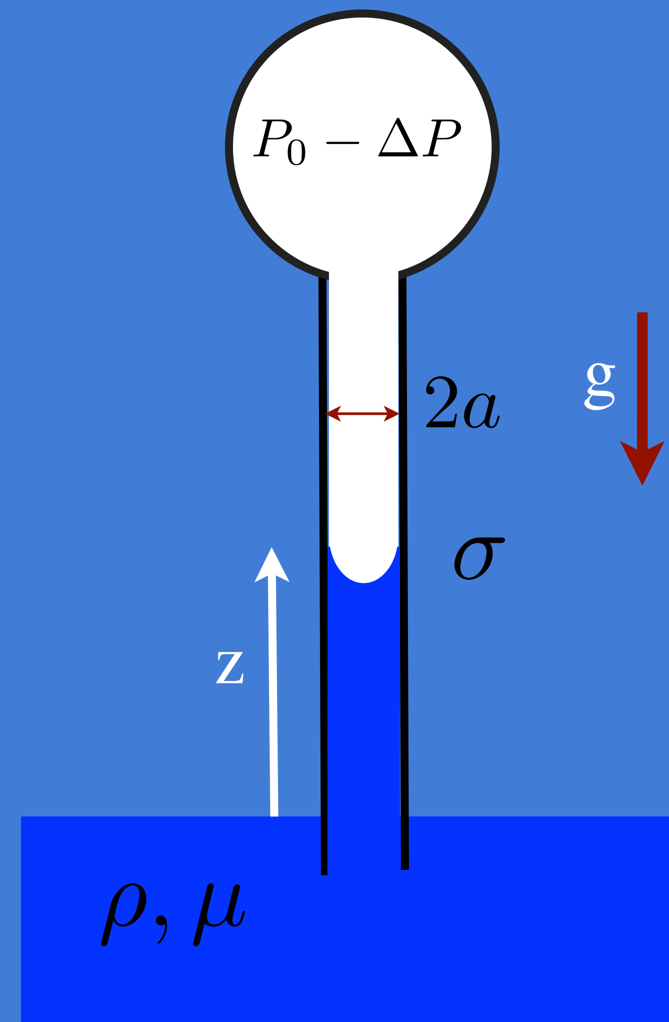
Viscous suction

$$\boxed{P^* + \cancel{2\cos}\,\theta \;=\; \cancel{\frac{1}{2}z'^2} \;+\; \cancel{(z+\frac{7}{6})}z'' \;+\; \cancel{B_o}z \;+\; 8Dzz'}$$

where $P^* = \dfrac{a\Delta P}{\sigma}$, $B_o = \dfrac{\rho g a L}{\sigma}$, $D = \dfrac{L}{a}\left(\dfrac{\rho\nu^2}{\sigma a}\right)$

$$\longrightarrow \quad \boxed{u \sim a^2\Delta P/(\mu L)}$$





Capillary suction

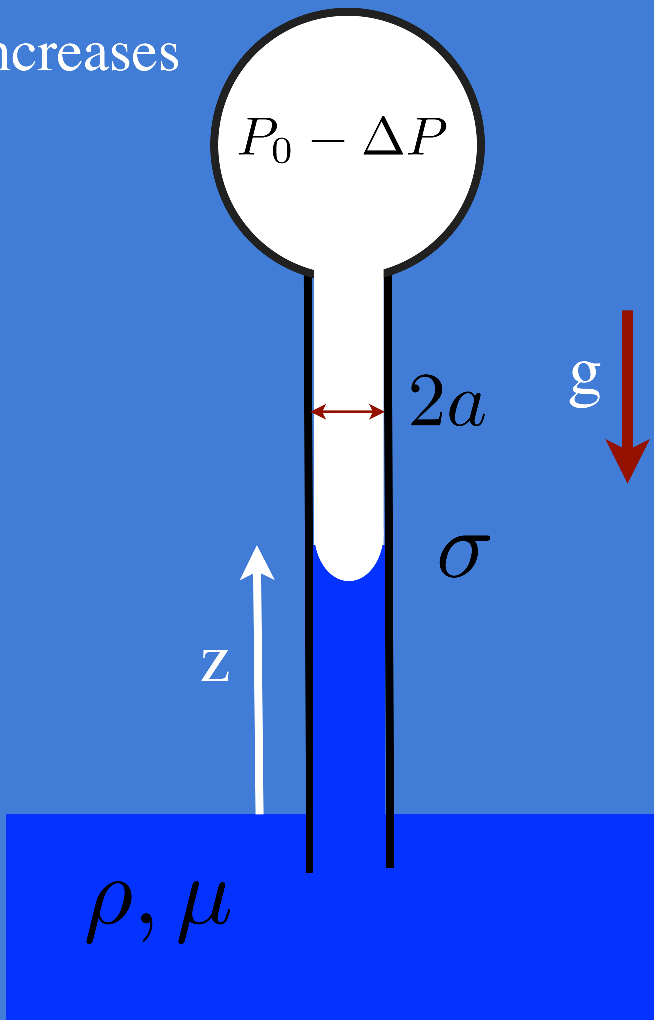
$$\boxed{\cancel{P^*} + 2\cos\theta \;=\; \cancel{\frac{1}{2}z'^2} \;+\; \cancel{\left(z + \frac{7}{6}\right)z''} \;+\; \cancel{B_o z} \;+\; 8Dzz'}$$

where $P^* = \dfrac{a\Delta P}{\sigma}$, $B_o = \dfrac{\rho g a L}{\sigma}$, $D = \dfrac{L}{a}\left(\dfrac{\rho\nu^2}{\sigma a}\right)$

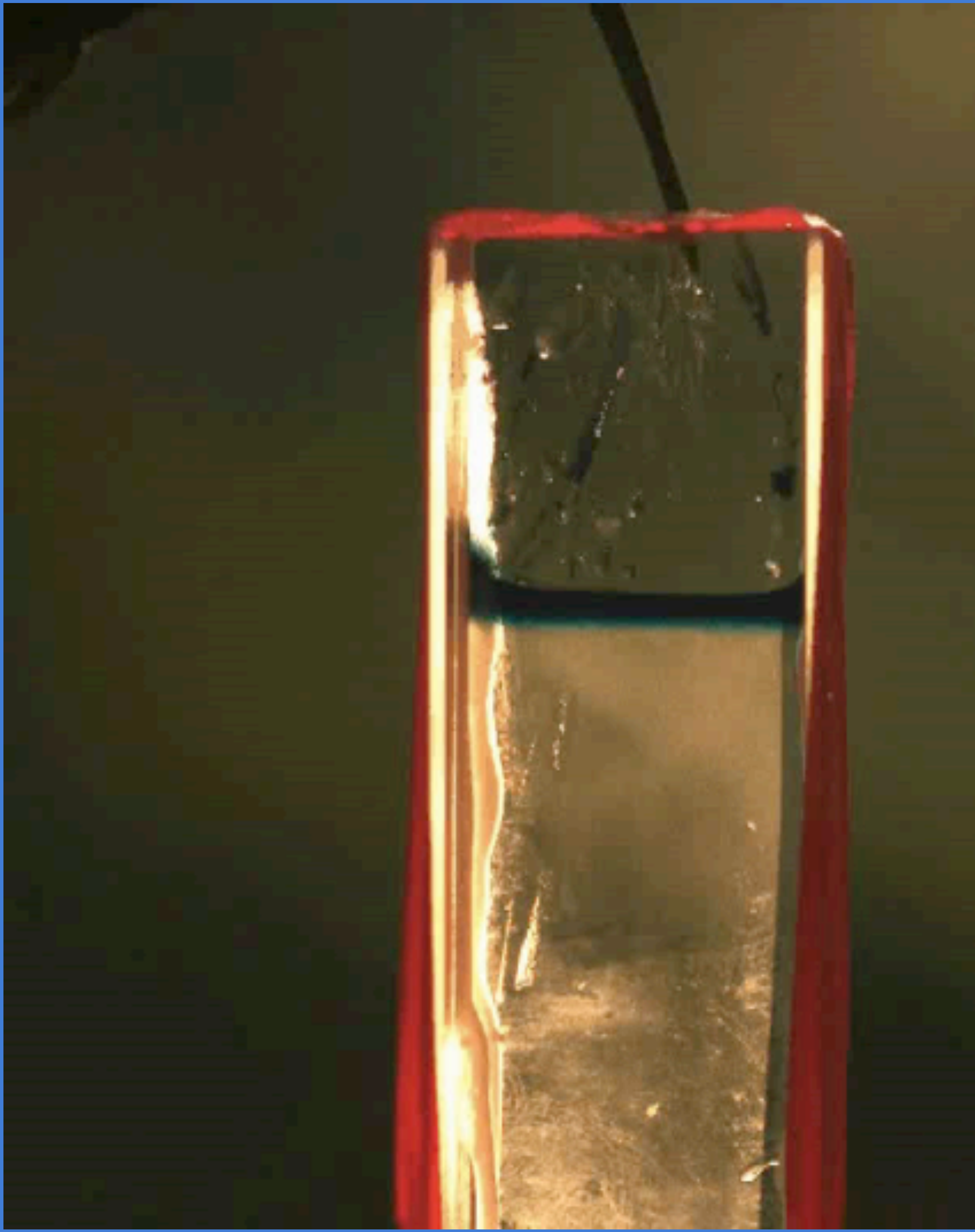
- applied pressure remains constant while length of column increases
- front advances according to Washburn's Law:

$$\boxed{z(t) = \left(\frac{\sigma a}{4\mu}\right)^{1/2} t^{1/2}}$$

- front speed decreases with time



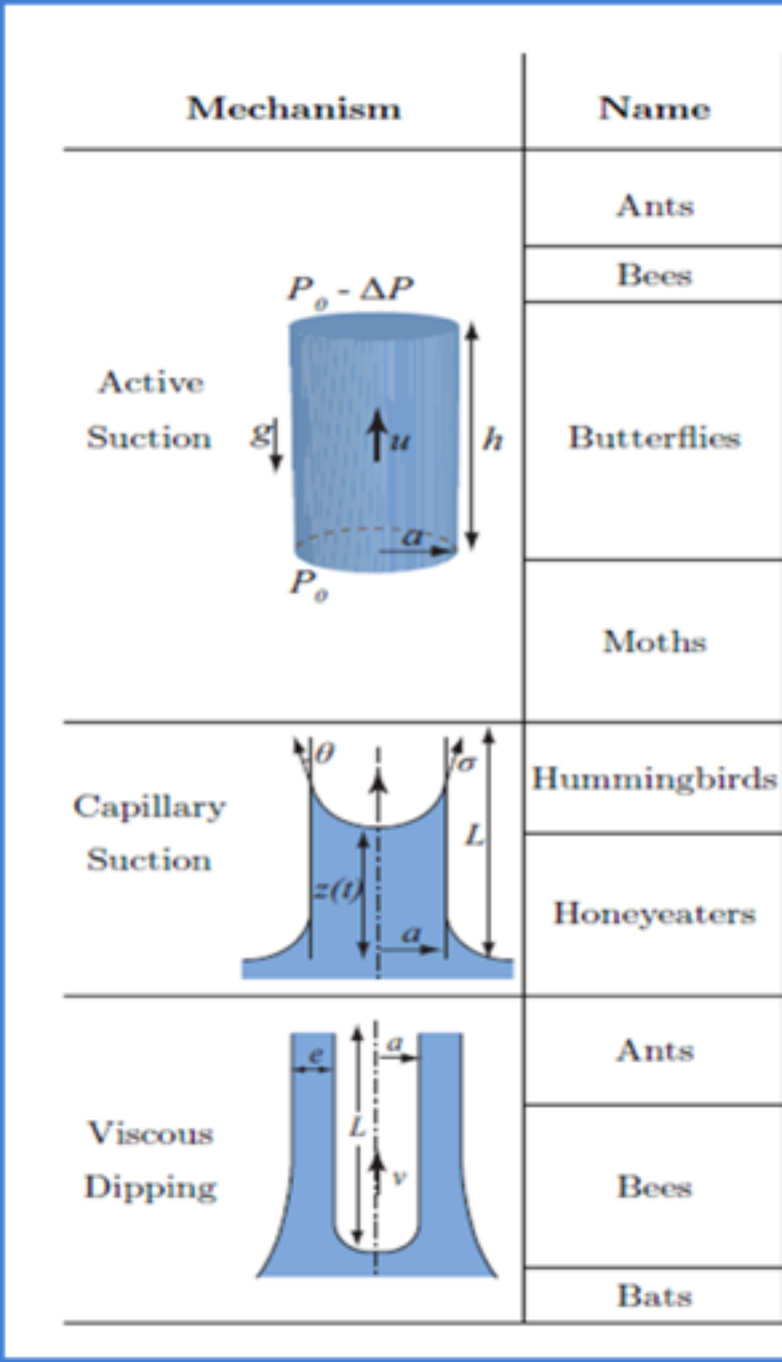
Capillary feeding by the hummingbird



Nectar drinking

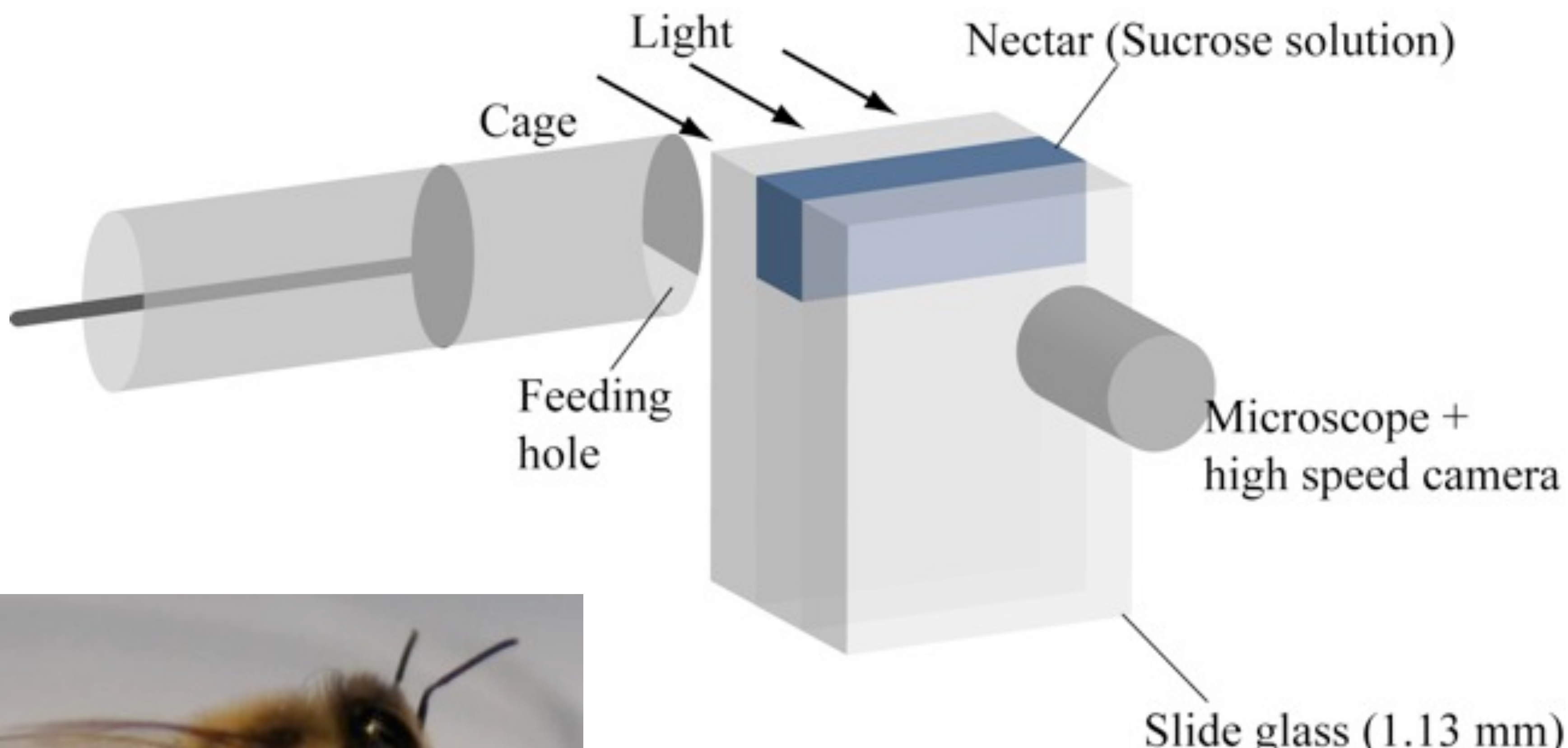
- drinking through a tube, or viscous dipping

with Wonjung Kim and Tristan Gilet

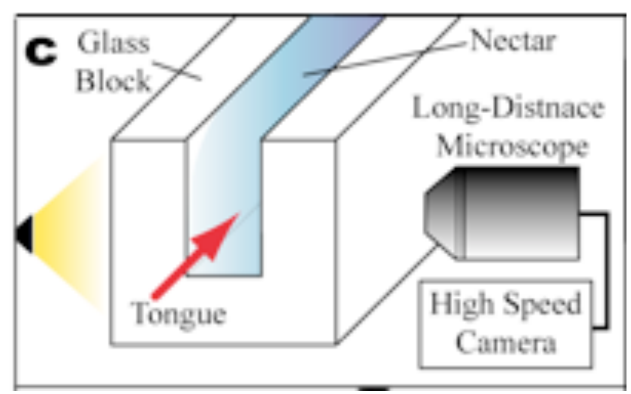


Mechanism	Name
Active Suction	Ants
	Bees
	Butterflies
	Moths
Capillary Suction	Hummingbirds
	Honeyeaters
Viscous Dipping	Ants
	Bees
	Bats

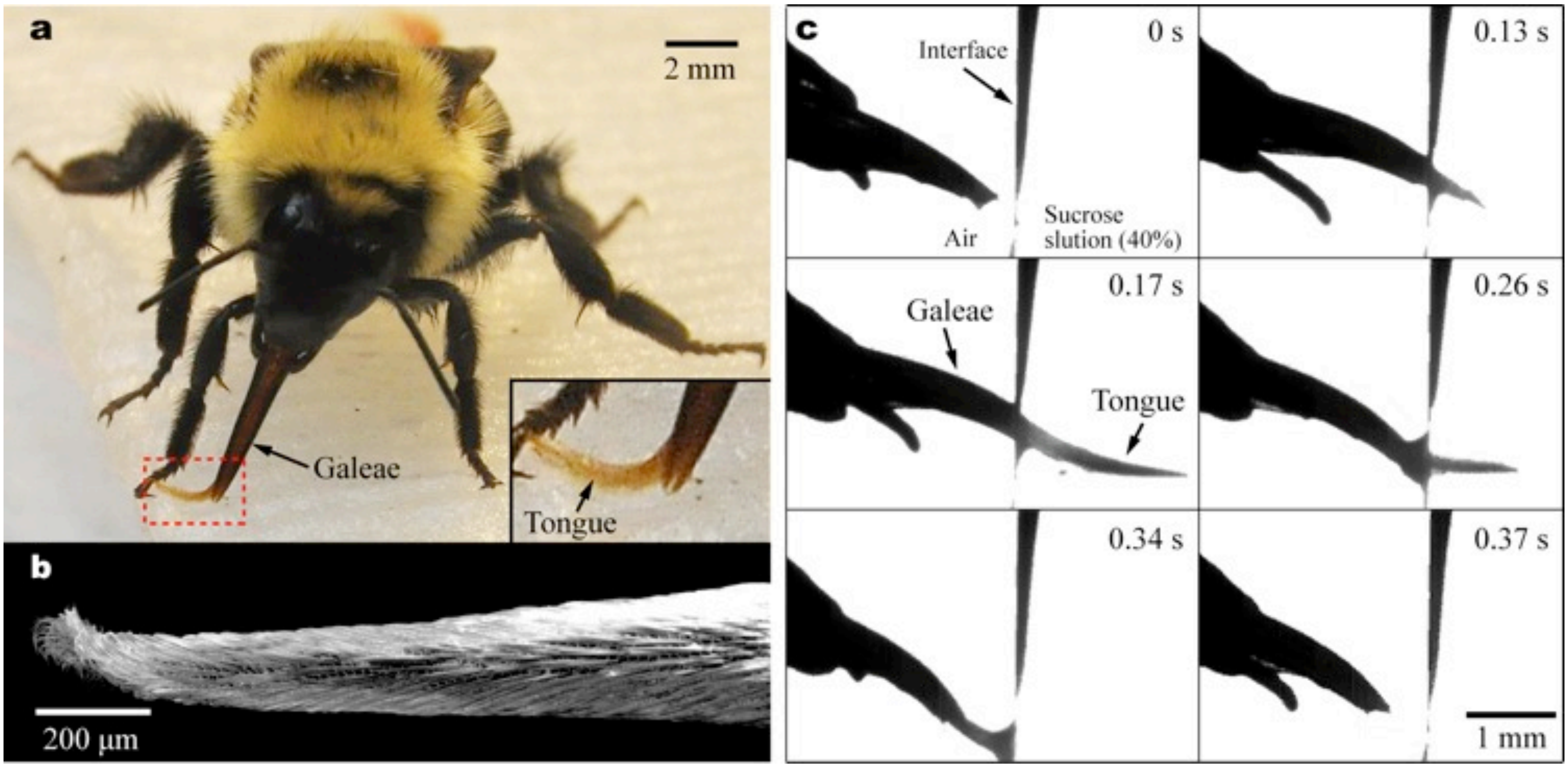
Experimental study of viscous dipping



Honey bees (*Apis*)

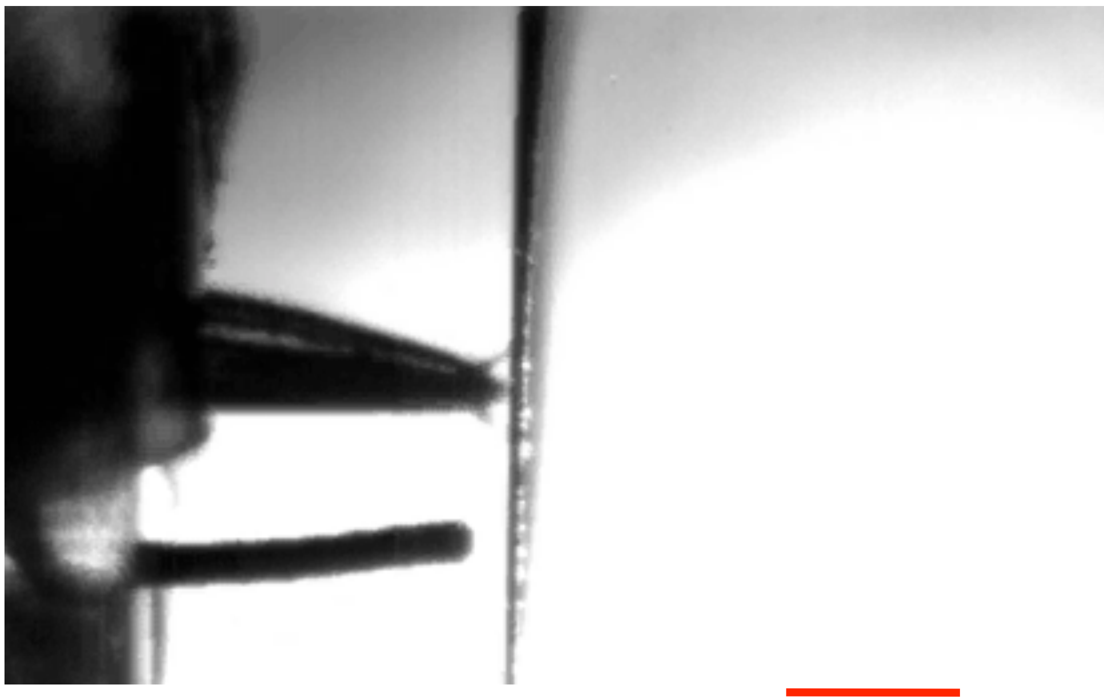


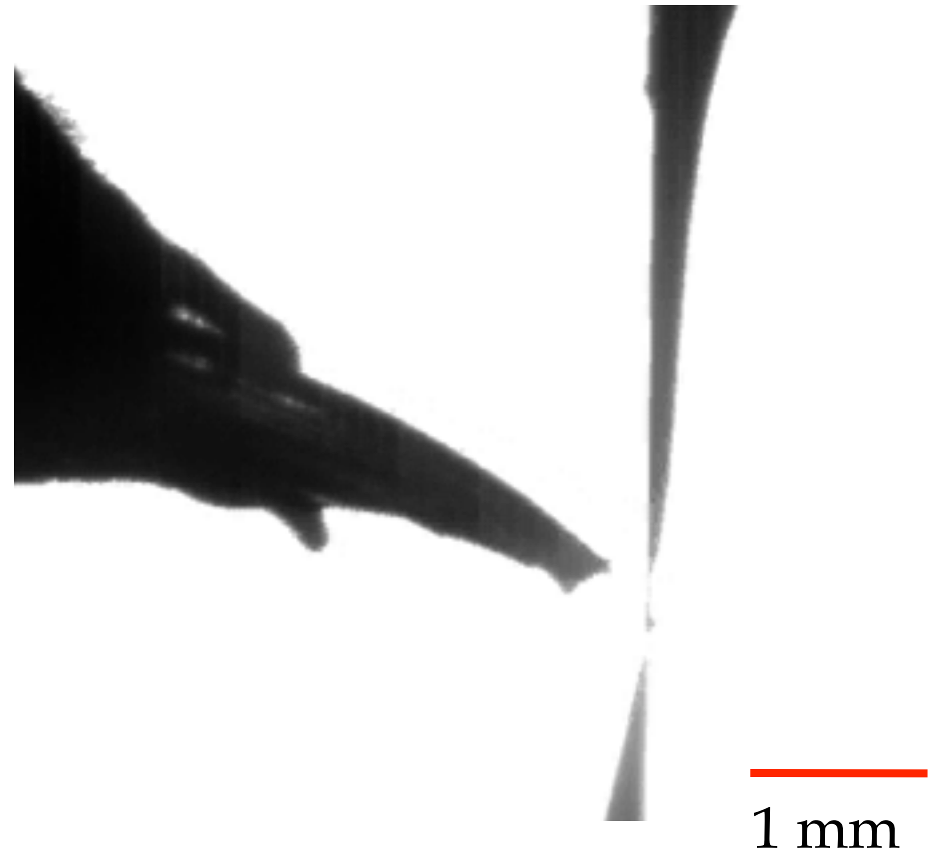
Viscous dipping



Viscous dipping

Videos slowed by factor of 8

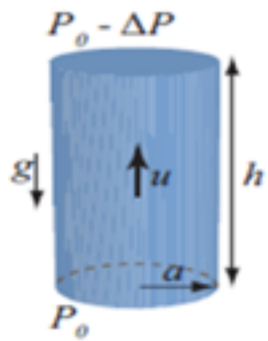
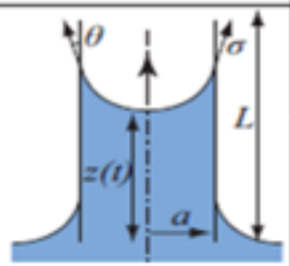
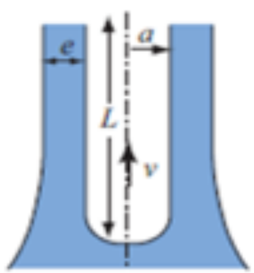




20% glucose

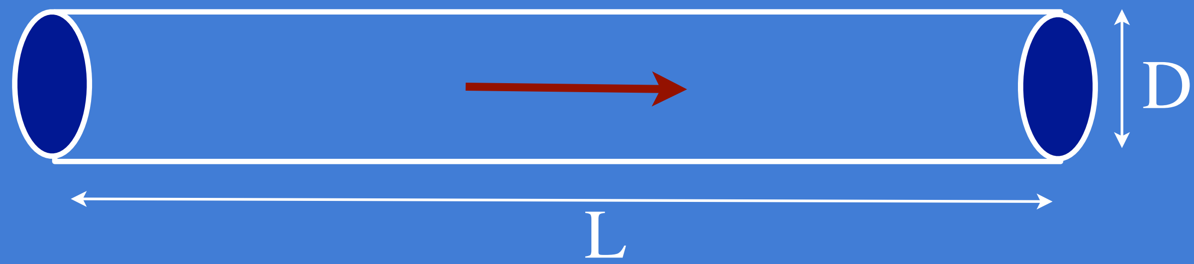
40% glucose

Nectar drinking

Mechanism	Name	Genus	Optimal (%)
Active Suction 	Ants	*Atta*[23] *Camponotus*[23]	30 40
	Bees	*Euglossa*[3]	35
	Butterflies	*Agraulis*[19] *Phoebis*[19] *Speryeria*[2] *Thymelicus*[24] *Vanessa*[9]	40 35 35 40 40
	Moths	*Pseudaletia*[24] *Macroglossum*[15] *Manduca*[31]	40 35 30
Capillary Suction 	Hummingbirds	*Selasphorus*[27] *Selasphorus*[33]	35 40-45
	Honeyeaters	*Anthochaera*[21] *Phylidonyris*[21] *Acanthorhynchus*[21]	50 40 30
Viscous Dipping 	Ants	*Pachycondyla*[23] *Rhytidoponera*[23]	50 50
	Bees	*Bombus*[12] *Apis*[30] *Melipona*[30]	55 55 60
	Bats	*Glossophaga*[29]	60

- optimal sugar concentration depends on drinking style

Drinking nectar through a tube



Volume flux: $$Q = \frac{\pi D^4 \Delta P}{128 \mu L} \sim \frac{\Delta P}{\mu}$$

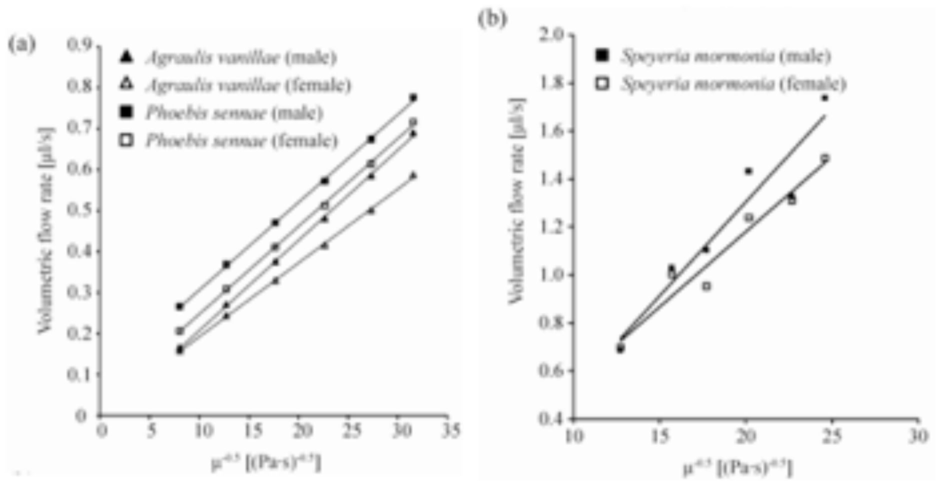
Energy flux: $$\dot{E} = \rho A Q S$$

Empirical relation:

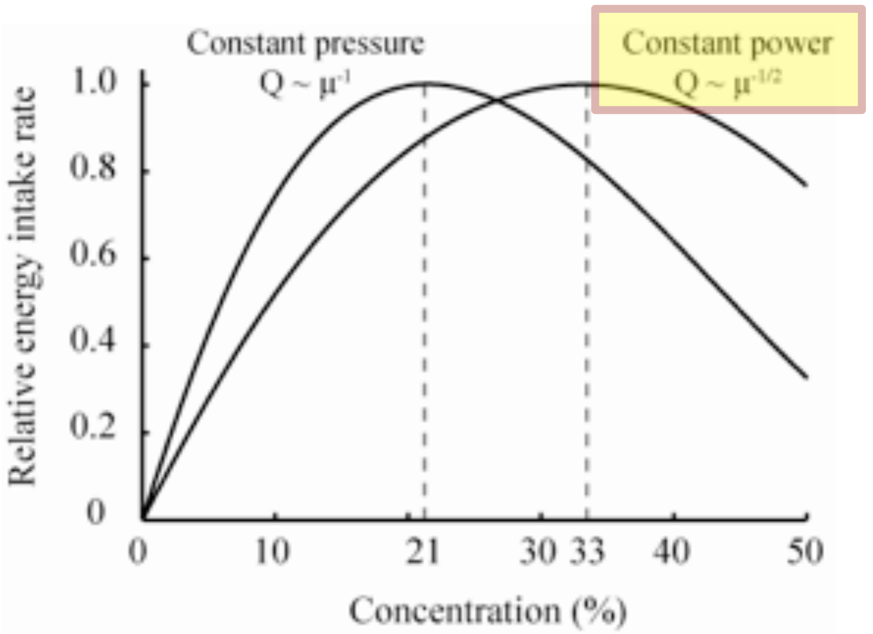
$$\mu(S) = e^{0.000000361 S^4 - .000303 S^3 + .00142 S^2 + .0131 S - 6.87}$$

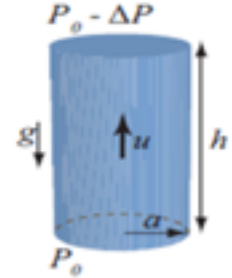
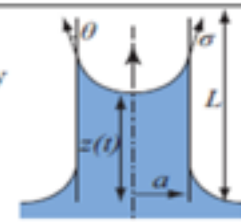
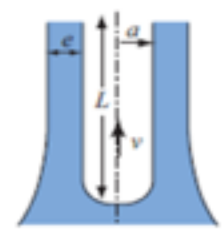
Assume constant	Q	$\dot{E}$	$S_{optimal}$
ΔP Kingsolver (1979)	$\sim \mu^{-1}$	$\sim S/\mu$	21%
$\dot{W} = Q \Delta P$	$\sim \mu^{-1/2}$	$\sim S/\mu^{1/2}$	33%

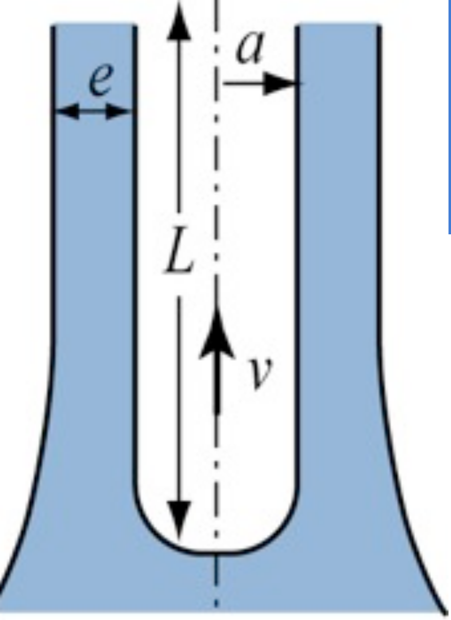
Simple model rationalizes data for active and capillary suction



P. G. May, *Oecologia* (1985) C. L. Boggs, *Funct. Ecol.* (1985)



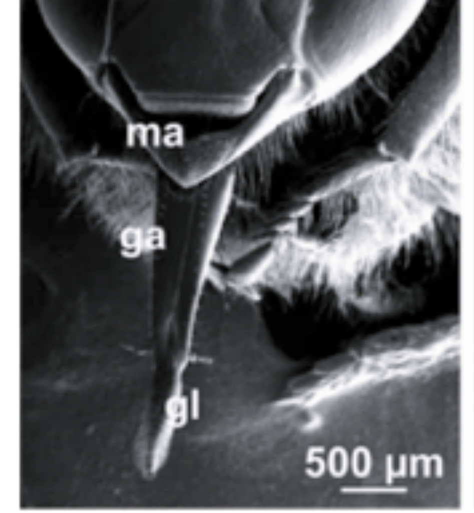
Mechanism	Name	Genus	Optimal (%)
Active Suction 	Ants	*Atta*[23]	30
		Camponotus[23]	40
	Bees	*Euglossa*[3]	35
	Butterflies	*Agraulis*[19]	40
		Phoebis[19]	35
		Speryeria[2]	35
		Thymelicus[24]	40
		Vanessa[9]	40
	Moths	*Pseudaletia*[24]	40
		Macroglossum[15]	35
		Manduca[31]	30
Capillary Suction 	Hummingbirds	*Selasphorus*[27]	35
		Selasphorus[33]	40-45
	Honeyeaters	*Anthochaera*[21]	50
		Phylidonyris[21]	40
		Acanthorhynchus[21]	30
Viscous Dipping 	Ants	*Pachycondyla*[23]	50
		Rhytidoponera[23]	50
	Bees	*Bombus*[12]	55
		Apis[30]	55
		Melipona[30]	60
	Bats	*Glossophaga*[29]	60



Viscous dipping

$$Ca = \frac{\mu U}{\sigma}$$

$We \ll 1$	$Bo \ll 1$	$e \approx 1.34 a \mathrm{Ca}^{2/3}$	$(\mathrm{Ca} < 0.1)$
		$e \approx \dfrac{1.34 a \mathrm{Ca}^{2/3}}{1 - 1.34 \mathrm{Ca}^{2/3}}$	$(0.1 < \mathrm{Ca} < 0.6)$
	$Bo \gg 1$	$e \approx 0.94 l_c \mathrm{Ca}^{2/3}$	



H. W. Krenn, *Arthropod Structure & D*

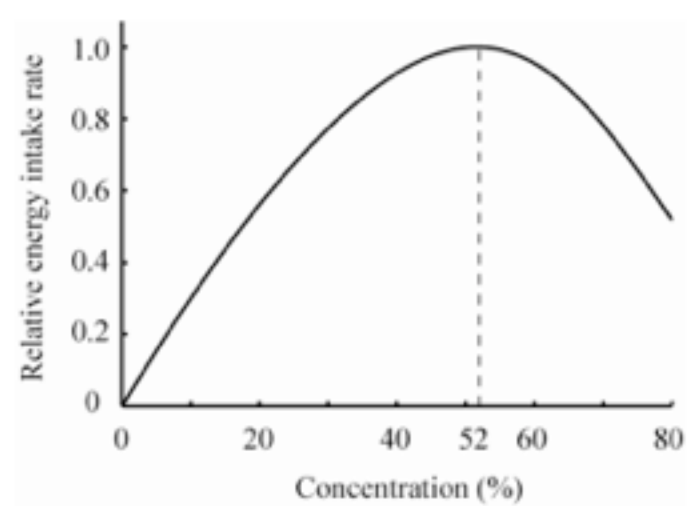
Power: $P \sim \mu L v^2 \longrightarrow v \sim \mu^{-1/2}$ at constant P

Volume flux: $\bar{Q} \sim aev \sim Ca^{2/3}\mu^{-1/2} \sim \mu^{1/6} v^{2/3} \sim \mu^{-1/6}$

Energy flux:

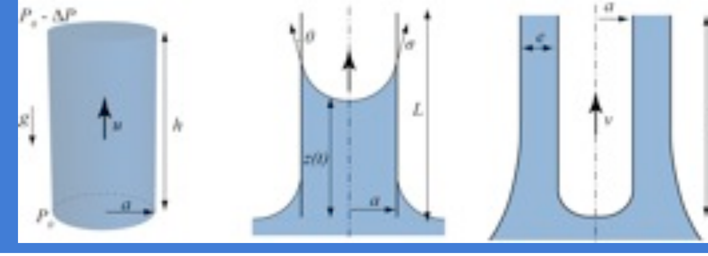
$$\dot{E} = \rho Q A S \sim S \mu^{-1/6}$$

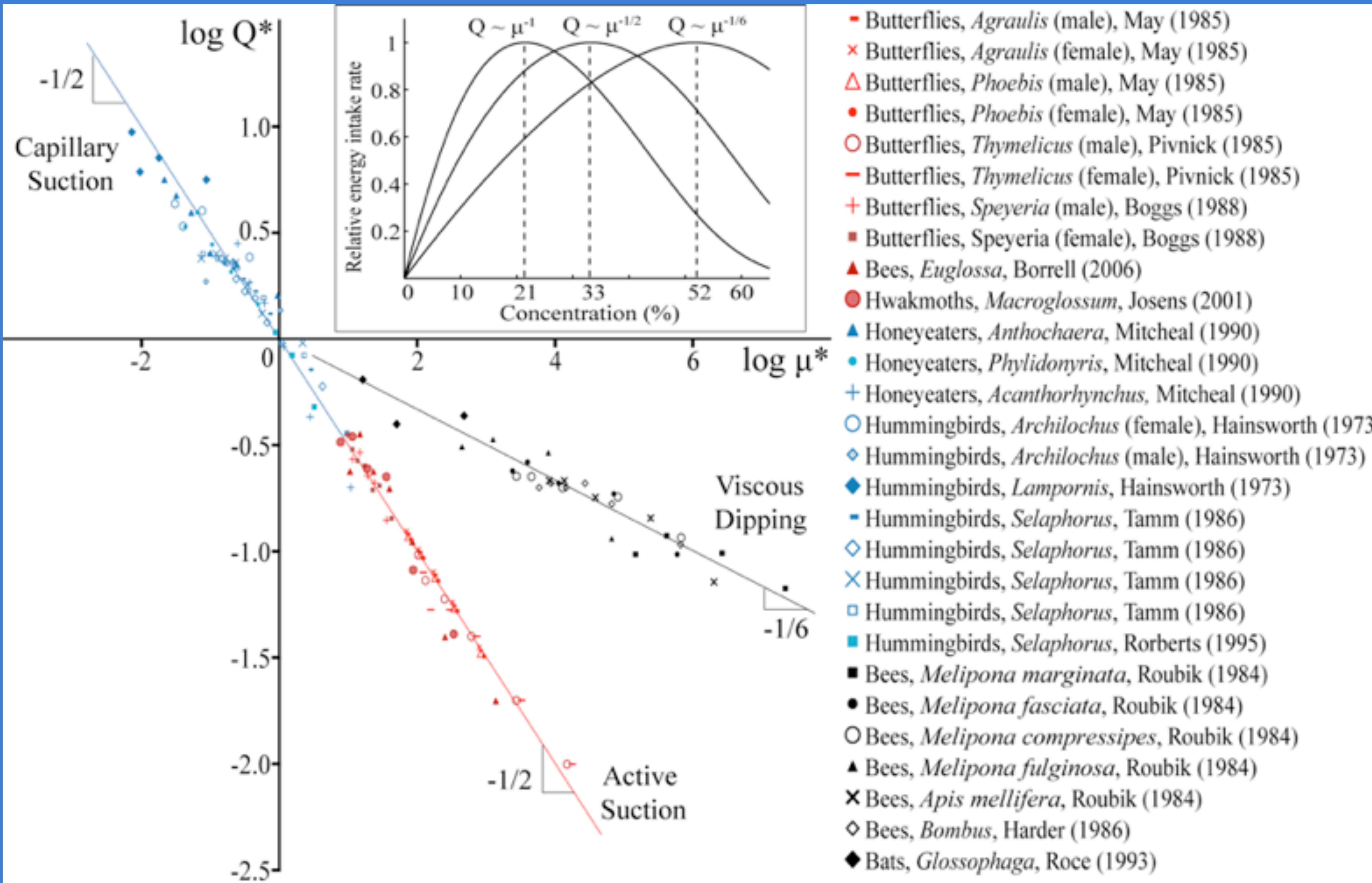
Optimal sugar concentration: 54%



Nectar drinking

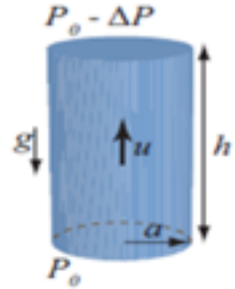
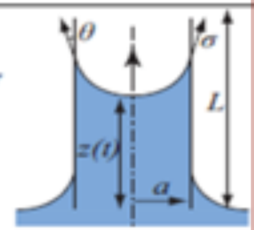
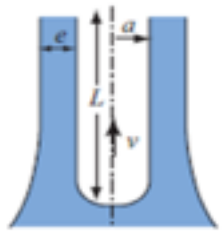
- observed fluxes consistent with constant power





Nectar drinking

- simple models allow for rationalization of optimal S

Mechanism	Name	Genus	Optimal (%)
Active Suction 	Ants	*Atta*[23]	30
		Camponotus[23]	40
	Bees	*Euglossa*[3]	35
	Butterflies	*Agraulis*[19]	40
		Phoebis[19]	35
		Speryeria[2]	35
		Thymelicus[24]	40
		Vanessa[9]	40
	Moths	*Pseudaletia*[24]	40
		Macroglossum[15]	35
		Manduca[31]	30
Capillary Suction 	Hummingbirds	*Selasphorus*[27]	35
		Selasphorus[33]	40-45
	Honeyeaters	*Anthochaera*[21]	50
		Phylidonyris[21]	40
		Acanthorhynchus[21]	30
Viscous Dipping 	Ants	*Pachycondyla*[23]	50
		Rhytidoponera[23]	50
	Bees	*Bombus*[12]	55
		Apis[30]	55
		Melipona[30]	60
	Bats	*Glossophaga*[29]	60

Suction

Dipping

- optimal S minimizes energy flux with constant power output

Namib Desert Beetle: drinking via refrigeration-free condensation

- the desert beetle has hydrophylic bumps to which 5 micron scale fog droplets stick, then grow by accretion until rolling through hydrophobic valleys and into their mouths

Parker & Lawrence (2001)

- inspired the development of superplastics for water gathering in the 3rd World

Zhai et al. (2006)

Capillary feeding in shorebirds

with Manu Prakash, David Quere

The Phalarope

- spinning motion sweeps preys to surface, like tea leaves in a swirling cup

Three ducks

Many ducks!

Question : how do they intake water?

Possibilities

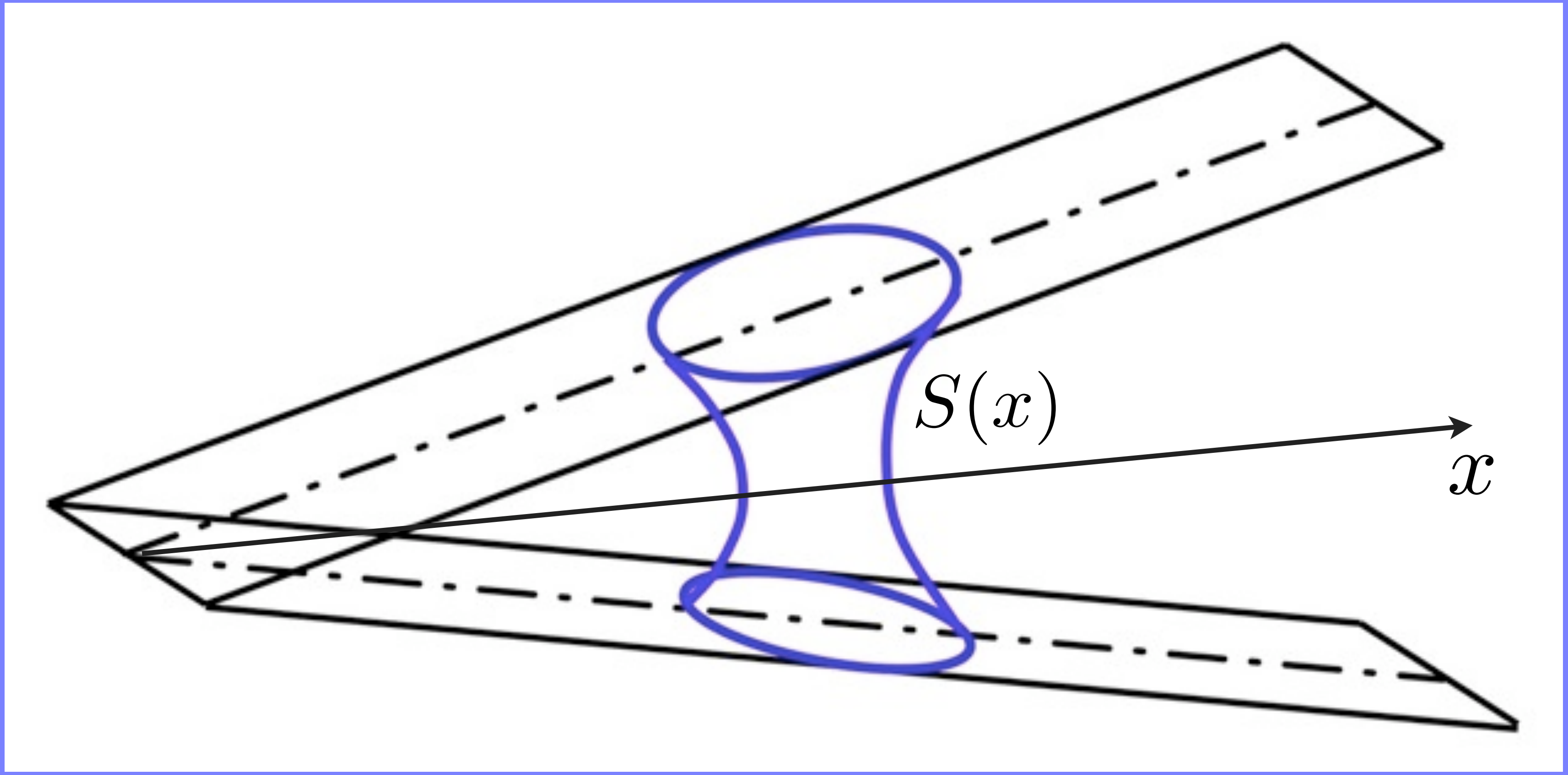
- suction: precluded by beak geometry
- gravity: requires head tilting
- capillarity

Observations

Rubega & Obst, 1993. Surface-tension feeding in phalaropes: a novel feeding mechanism, *The Auk*, **110**, 169-178.

- some shorebirds use capillary forces to draw water into their mouths
- plankton withdrawn from drop, then water expelled
- drops move at high speed ~ 30-50 cm/s
- pecking rates ~ 10 Hz; 2-3 mandibular spreading events per cycle

Toy Model: Catenoid between inclined plates



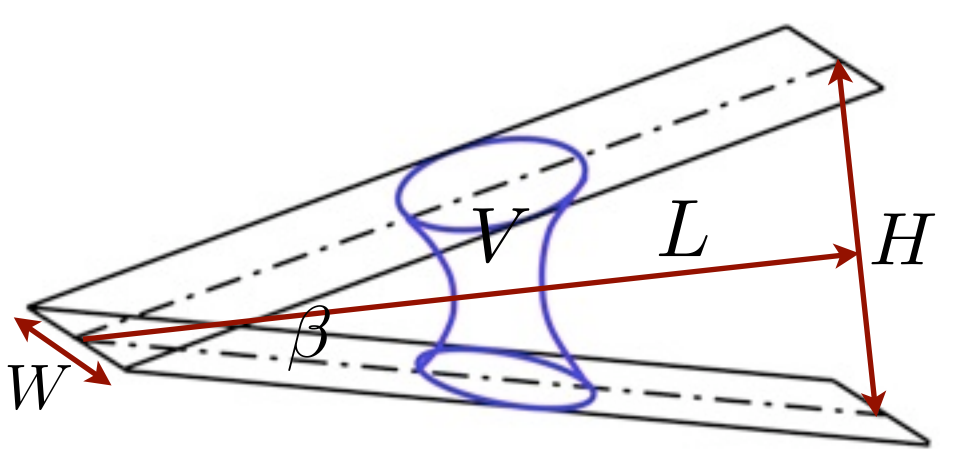
- neglect the influence of gravity
- isolate the influence of surface tension

Propulsive force:

$$F_c(x) = \sigma \frac{dS(x)}{dx}$$

Criterion for drop motion

$$V \; > \; \frac{2\pi}{3} W^2 H$$



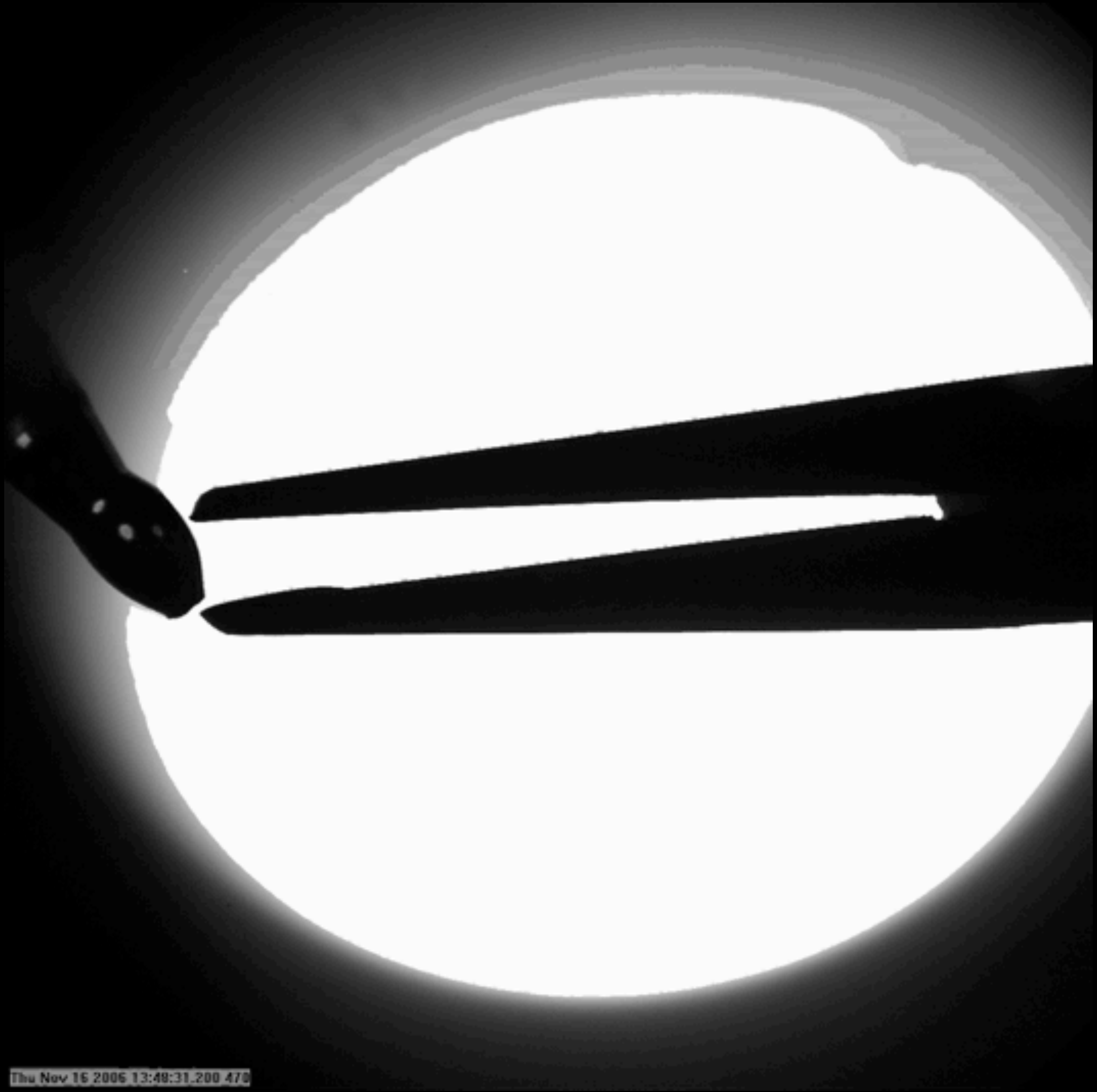
Criterion for drop stability

$$V \; > \; \frac{H^3}{\pi}$$

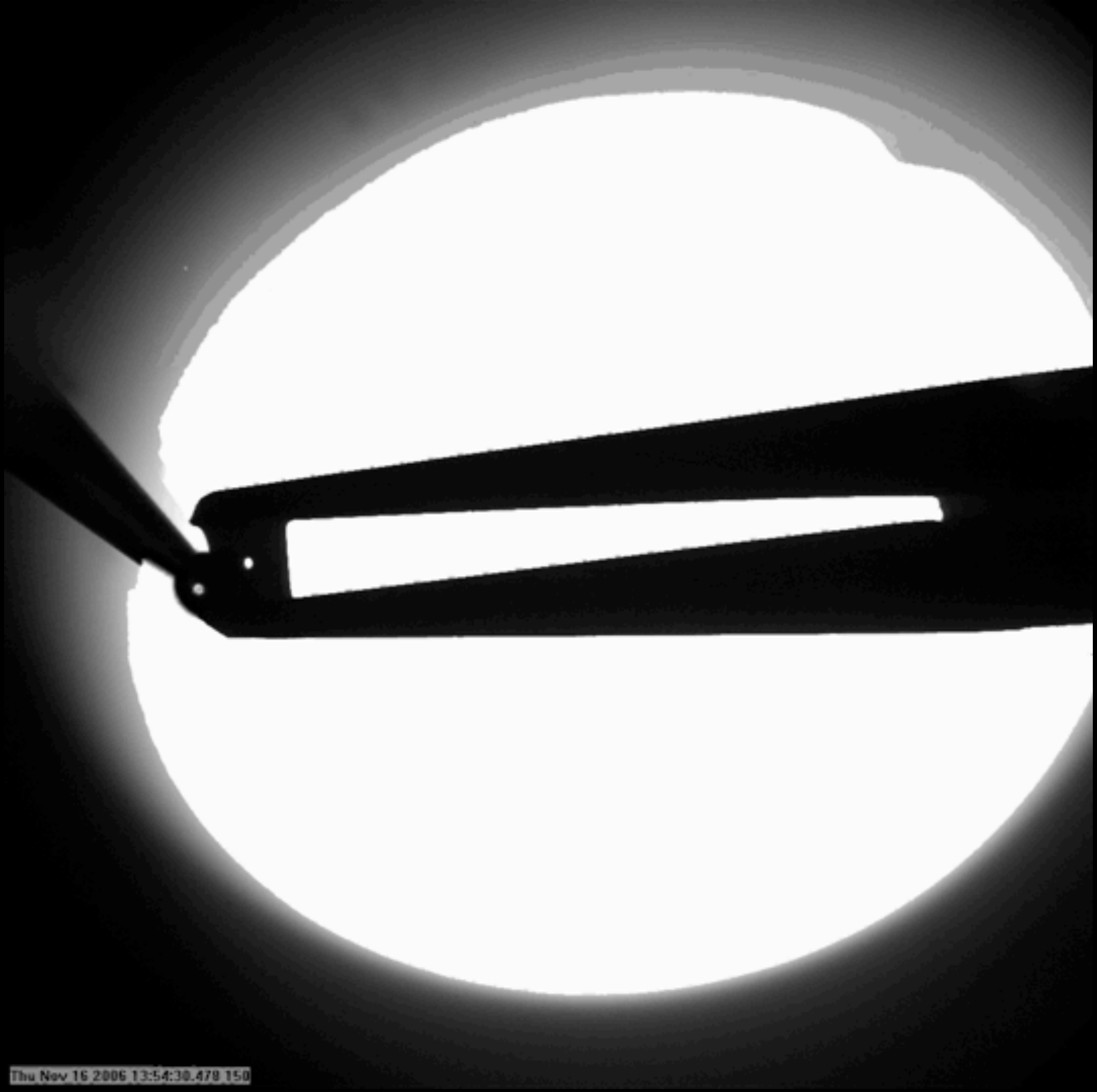
- these criteria may be satisfied simultaneously provided

$$\frac{H}{W} \; > \; \sqrt{2\pi} \quad \longrightarrow \quad \boxed{\frac{L}{W} \tan\frac{\beta}{2} \; > \; \sqrt{2\pi}}$$

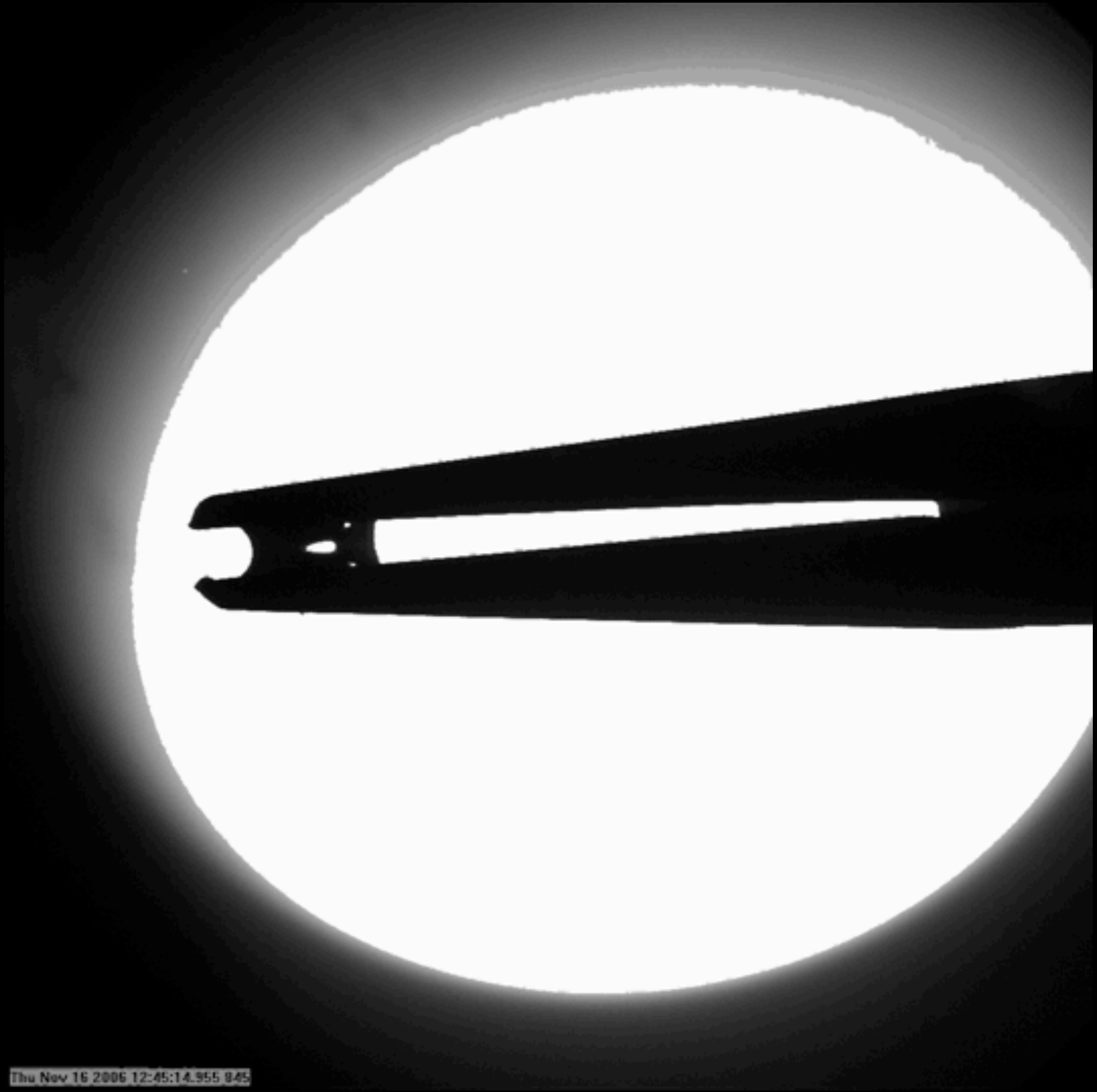
→ a meaningful constraint on the morphology of bird beaks?



- 5 cS silicon oil on stainless steel

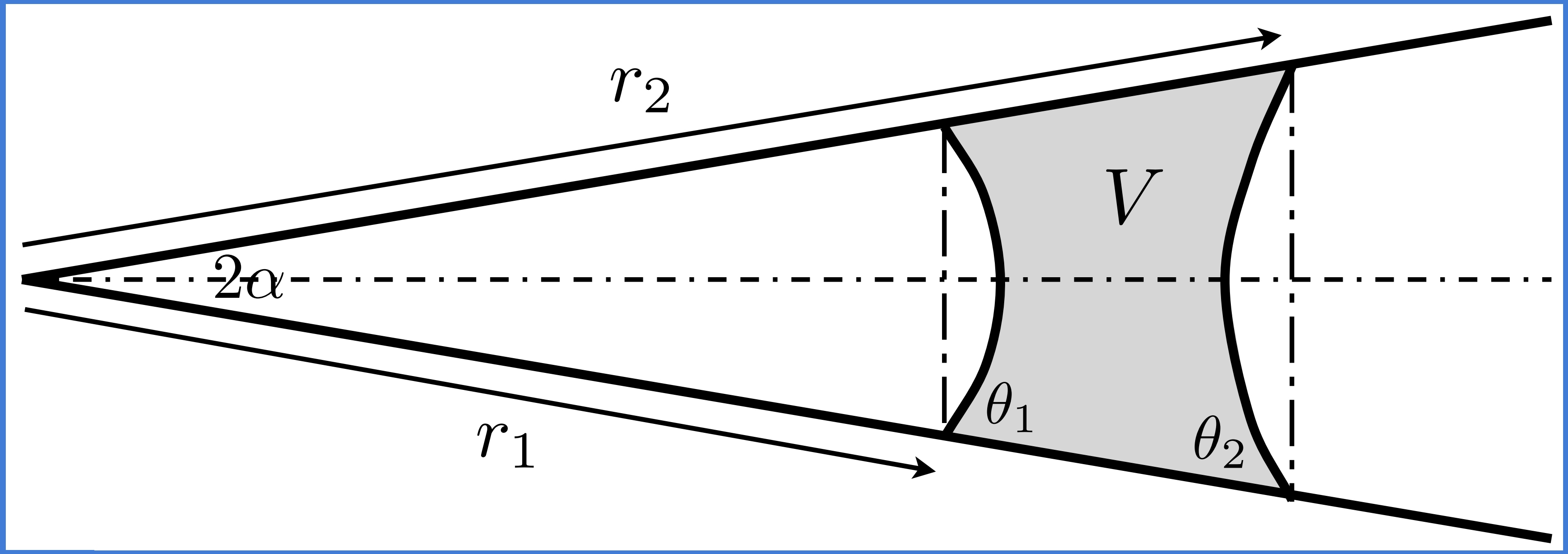


- water drop pinned by contact line



- water drop freed to move by oscillating boundaries

Capillary ratcheting: the non-wetting beak (2D)



Bounds on static contact angles: $\theta_a > (\theta_1, \theta_2) > \theta_r$

Lateral force balance on static drop: $\theta_1 - \theta_2 = 2\alpha$

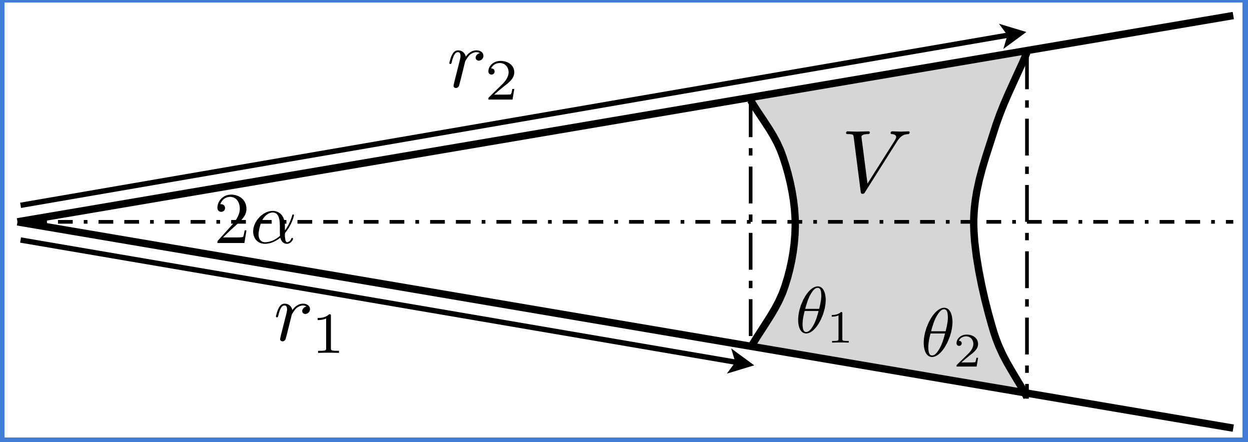
Continuity: $$\frac{V - \frac{1}{2}(r_2^2 - r_1^2)\sin 2\alpha}{(r_1^2 + r_2^2)\sin^2\alpha}\cos^2 x = \frac{\pi}{2} - x - \frac{1}{2}\sin x$$

where $x = \theta_1 - \alpha = \theta_2 + \alpha$

→ yields (θ_1, θ_2) in terms of (V, r_1, r_2, α)

Force balance requires:

$$\theta_1 - \theta_2 \;=\; 2\alpha$$

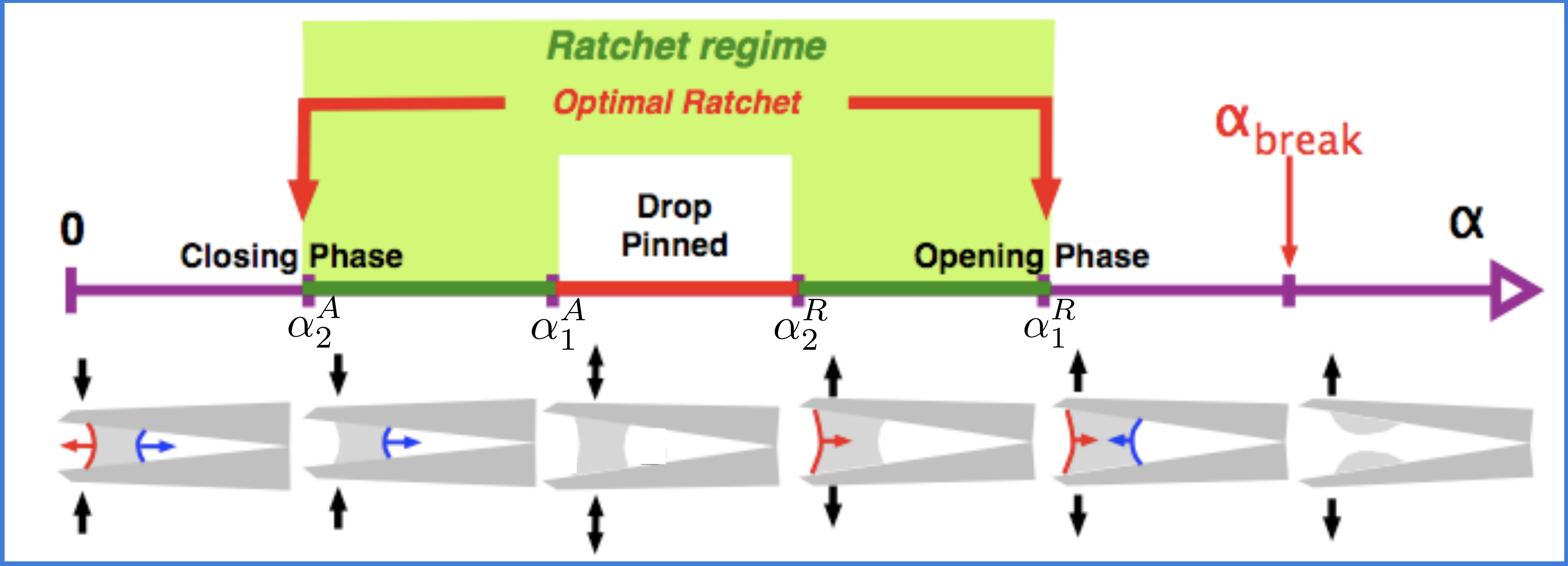


Closure phase: can deduce α_1^A at which $\theta_1 = \theta_A$

α_2^A at which $\theta_2 = \theta_A$

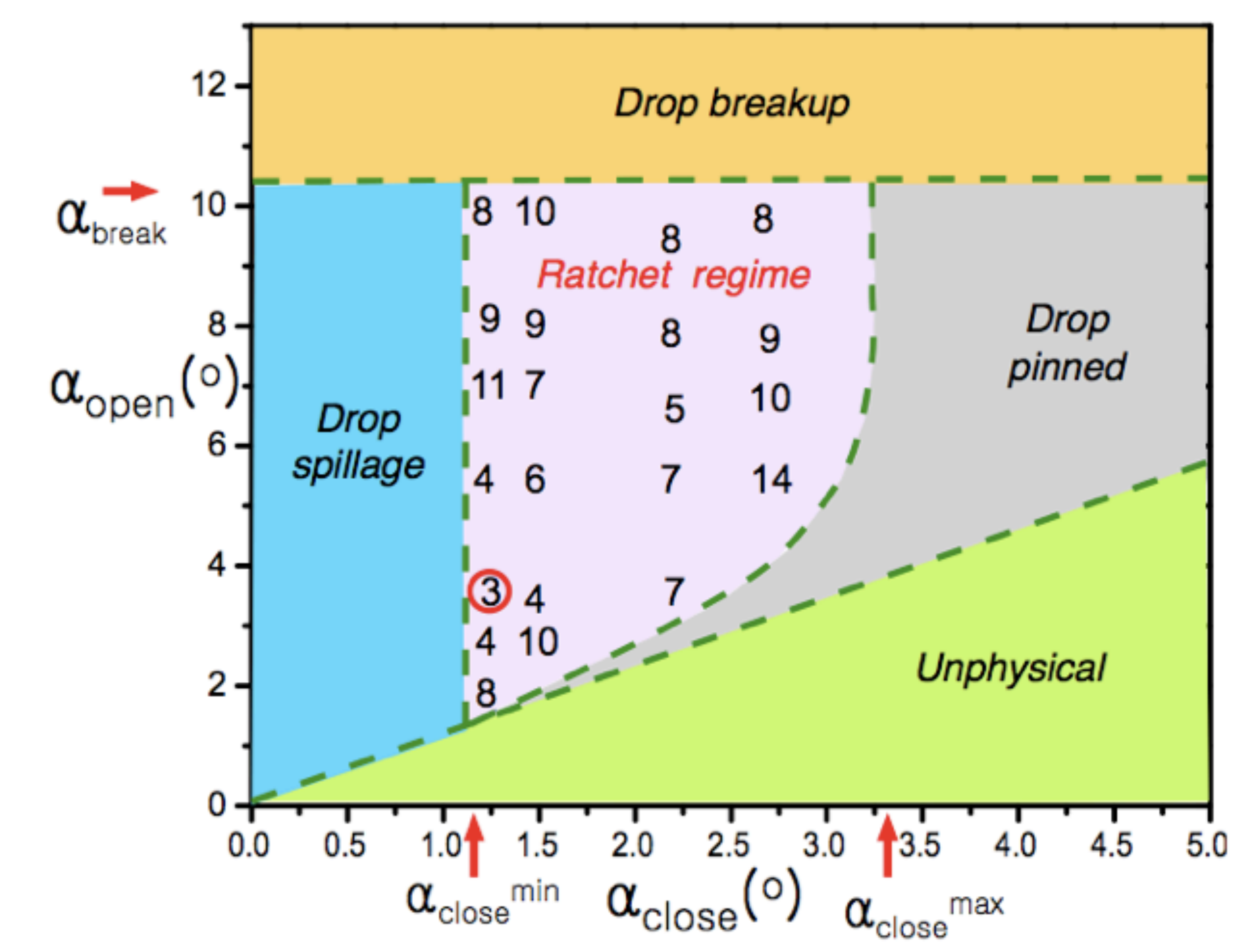
Opening phase: can deduce α_2^R at which $\theta_2 = \theta_R$

α_1^R at which $\theta_1 = \theta_R$



Tuning the capillary ratchet

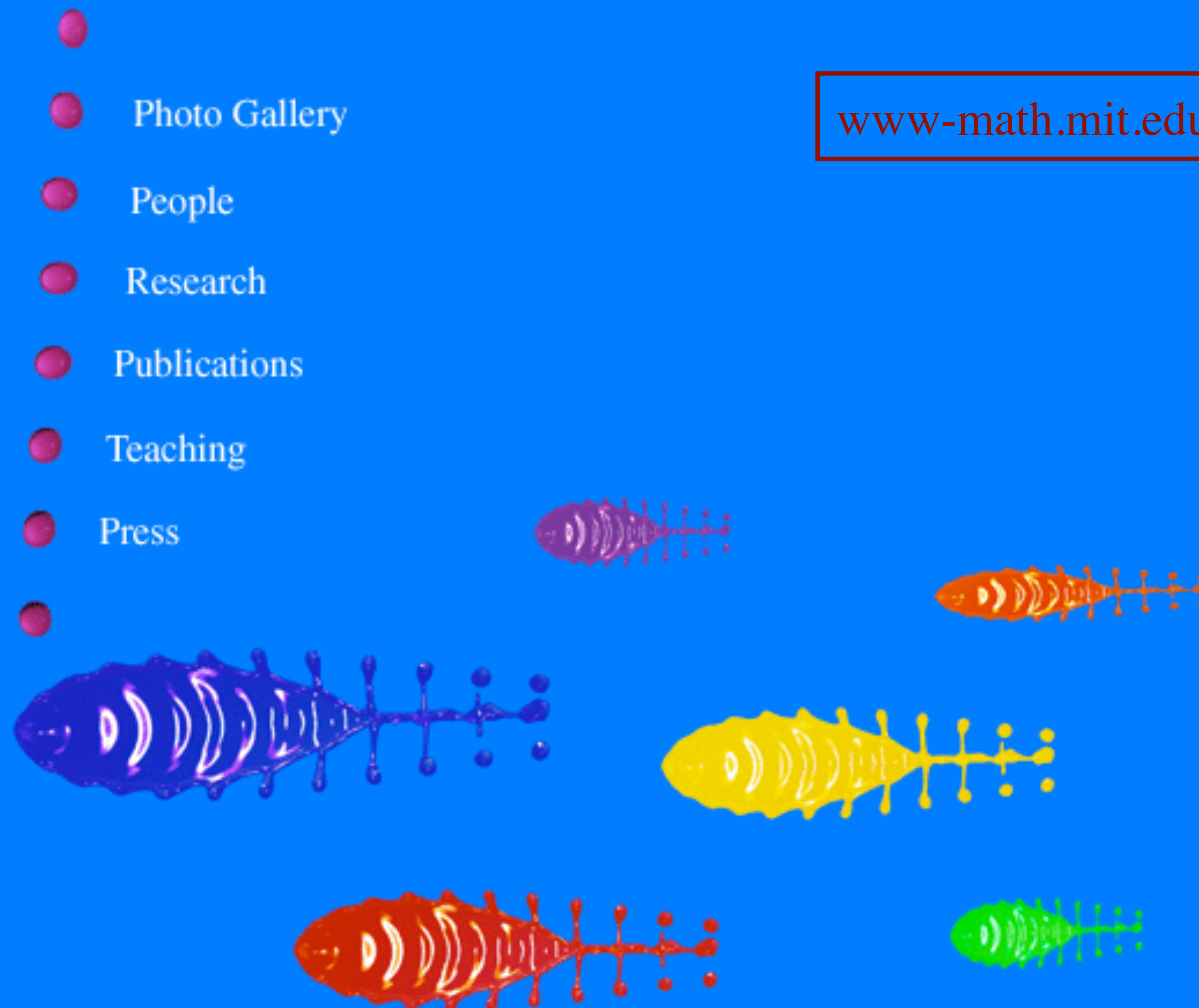
- fix drop volume



- the bird beak regime: 2-3 cycles per feeding event

Big picture

- wetting properties of beaks important to shorebirds: effect of oil spills?
- σ - dominated flows to be more prevalent at smaller scales
- similar mechanisms bound to exist in the insect world
- contact angle hysteresis coupled with geometry can drive motion
    - applications: discrete fluid transport in microfluidic devices



www-math.mit.edu/~bush

"As you dream amidst nature, extrapolate art from it." - Paul Gauguin

Thank you!