A Bibliography of Vortex Dynamics 1858-1956

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Every great man of the first rank is unique. Each has his own office and his own place in the historic procession of the sages. That office did not exist even in the imagination, till he came to fill it, and none can succeed to his place when he has passed away. Others may gain distinction by adapting the exposition of science to the varying language of each generation of students, but their true function is not so much didactic as pedagogic – not to teach the use of phrases which enable us to persuade ourselves that we understand a science, but to bring the student into living contact with the two main sources of mental growth, the fathers of the sciences, for whose personal influence over the opening mind there is no substitute, and the material things to which their labours first gave a meaning.

J. C. Maxwell [640]

1. Introduction

The subject of vortex dynamics[3] can fairly be said to have been initiated by the seminal paper [393] of Hermann Ludwig Ferdinand Helmholtz (1821-1894) now nearly 150 years ago[4]. In this paper Helmholtz established his three "laws" of vortex motion in roughly the form they are found today in textbooks on fluid mechanics (often without attribution). His motivations for taking up this new research interest remain unclear, for at that time Helmholtz was professor of physiology and anatomy at the University of Bonn, and the memoir appeared in the year of his coming to the University of Heidelberg as a professor of physiology. One motivation seems to have been his interest in frictional phenomena [969], carried over from his interest in energetics; another was his growing awareness of the power of Green's theorem in hydrodynamics. In a speech [399] at a banquet on the occasion of his 70th birthday – an event that brought together 260 friends and admirers at Kaiserhof on November 2, 1891 – Helmholtz gave the following expanded account:

I have also been in a position to solve several problems in mathematical physics, some of which the great mathematicians since the time of Euler had worked on in vain — for example, problems concerning vortex motion and the discontinuity of motion in fluids, the problem of the motion of sound waves at the open ends of organ pipes, and so on. But the pride which I might have felt about the final result of these investigations was considerably lessened by my knowledge that I had only succeeded in solving such

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[3]For an accessible, widely-ranging introduction to the entire "universe" of vortex motions across an immense range of scales and physical phenomena we recommend the book by Lugt [606].

[4]Helmholtz was ennobled and added 'von' to his name in 1882. As this occurred later, he appears throughout this review simply as Helmholtz. The life and scientific work of this outstandinding natural philosopher of 19th century – upon his death obituary notices appeared in more than 50 scientific journals all over the world – has been praised by his contemporaries [270, 274, 507, 641, 816, 887] and by modern scientists [109, 188, 520, 969]. There exist many books in many languages devoted to Helmholtz, e.g. [239, 508, 557, 558, 652, 1016].

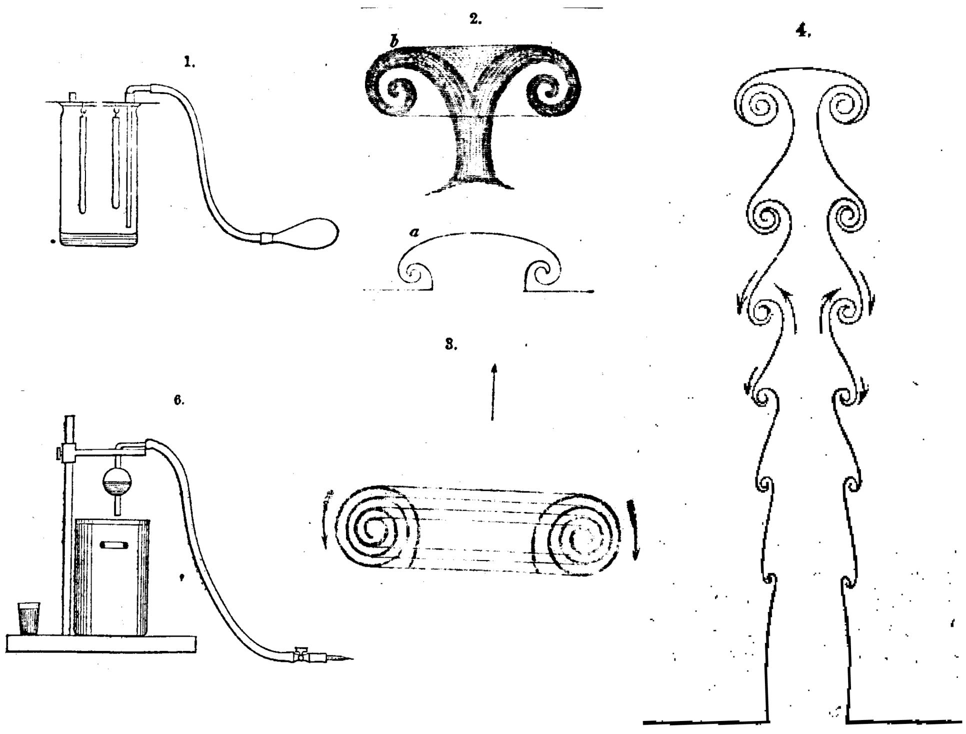


Figure 1: Illustrations from experiments "[on] the formation of rotating rings by air and liquids under certain conditions of discharge" by W. B. Rogers [789], published in 1858, the same year as Helmholtz's seminal paper, see also §2.5.

> problems, after many erroneous attempts, by the gradual generalization of favorable examples and by a series of fortunate guesses. I would compare myself to a mountain climber who, not knowing the way, ascends slowly and painfully and is often compelled to retrace his steps because he can go no farther; who, sometimes by reasoning and sometimes by accident, hits upon signs of a fresh path, which leads him a little farther; and who finally, when he has reached the summit, discovers to his annoyance a royal road on which he might have ridden up if he had been clever enough to find the right starting point at the beginning. In my papers and memoirs I have not, of course, given the reader an account of my wanderings, but have only described the beaten path along which one may reach the summit without trouble.

Until the appearance of Helmholtz's paper the integrals of the hydrodynamical equations had been determined almost exclusively on the assumption that the cartesian components of the velocity of each fluid particle are partial first derivatives – "differential coefficients", in the terminology of the time – with respect to the cartesian coordinates of a certain function, the *velocity potential*. This assumption is valid so long as the motion of the fluid results from the action of forces that have a potential of their own. Helmholtz eliminated this limitation, and took into account the possible friction between different elements of the fluid or between the fluid and a solid boundary. At the time the effect of friction had not been fully understood mathematically. Helmholtz endeavored to identify some of the aspects of the motion that frictional forces can produce in fluids.

It is somewhat rare that a subject in a rather "mature" science such as fluid mechanics has so clear a starting date. Usually when this happens it is due to a seminal paper by a luminary of the field, a paper that is far ahead of anything else produced by the contemporaries of said luminary, and a paper that is immediately embraced by the community and sets the stage for developments for decades to come. The early papers in the new field of vortex dynamics were scattered among many journals in many countries and were written in a multitude of languages, primarily English, French, German, Italian and Russian. This diversity of publication venue and language, unfortunately, often makes the literature rather difficult to identify and access for the modern researcher[5]. About a century after Helmholtz's paper two major journals devoted to fluid mechanics would be started, *Journal of Fluid Mechanics* in 1956 and *Physics of Fluids* in 1957. After their initiation these journals attracted many of the major papers in vortex dynamics. Furthermore, the vast literature on the subject of vortex dynamics since the 1950's may be accessed through serial publications such as *Advances in Applied Mechanics* (published since 1948), *Progress in Aeronautical Sciences* (published since 1961), *Annual Review of Fluid Mechanics* (published since 1969), and several volumes devoted to "Strömungsmechanik" of the extensive *Handbuch der Physik* (published since 1956). Other valuable sources are the proceedings of the International Congresses of Theoretical and Applied Mechanics (held every four years since 1924), along with other topical symposia, conferences and workshops. There also exist many general treatises, surveys, textbooks, Ludwig Prandtl and (since 1957) Lanchester Memorial Lectures [3, 471, 582], etc., a rather complete list of which can be found in the extensive bibliography[6] in Schlichting's book [835].

Hence – and this may be a somewhat optimistic assessment – the period that is most in need of a comprehensive bibliography is the century from 1858 to about 1956. It is such a bibliography that we have tried to provide in this paper. (We have also included some more recent references which either contain historical perspective on the papers of that period or essentially use and further develop classical results in vortex dynamics.) We hope the bibliography will be found useful by researchers working in vortex dynamics – that it may even on occasion improve the scholarship of this field by drawing attention to earlier and sometimes neglected works – and that it will be of interest to the general reader with an affinity for the history of mechanics. We have also included a few quotations and older illustrations (see Figs. 1, 2, 3 and 4) intended to give the flavor of vortex dynamics as an international scientific endeavor with a long history, sometimes simply to amuse the reader, but also to compensate those readers who do not have easy access to a library with an extensive collection of older books and journals.

The bibliography was compiled by the first author during an extended visit in the Department of Theoretical & Applied Mechanics at University of Illinois, Urbana-Champaign, almost a decade ago. Extensive use was made of the remarkable holdings of the library of that institution. Some additional references related to older studies in Russian were collected at the National Vernadskii Library of Ukraine and the Library of the Institute of Hydromechanics, both in Kiev. We have greatly benefitted from the skill and prescience of the librarians in these institutions in assembling and caring for this literature.

We do not claim that the current bibliography is complete, but we do believe it brings to light many important works that ought to be better known and to be more frequently cited than they are today. At the end of the 19th century, Karl Pearson wrote on this issue in the preface to the monumental treatise [951, pp. x-xi]:

> The use of a work of this kind is twofold. It forms on the one hand the history of a peculiar phase of intellectual development, worth studying for the many side lights it throws on general human progress. On the other hand it serves as a guide to the investigator in what has been done, and what ought to be done. In this latter respect the individualism of modern science has not infrequently led to a great waste

[5]Indeed, while the authors can claim fluent command of English and Russian, the multitude of languages has been a challenge even when assembling this bibliography.

[6]Oddly neither the first German edition [834] nor the seventh American edition [835] of this classic text on boundary layer theory, a subject closely related to vortex dynamics, reference Helmholtz's paper [393]. Helmholtz's name is not even among the extensive list of authors mentioned! The ninth German edition [836] does reference Helmholtz but the citation is to a later paper [398].

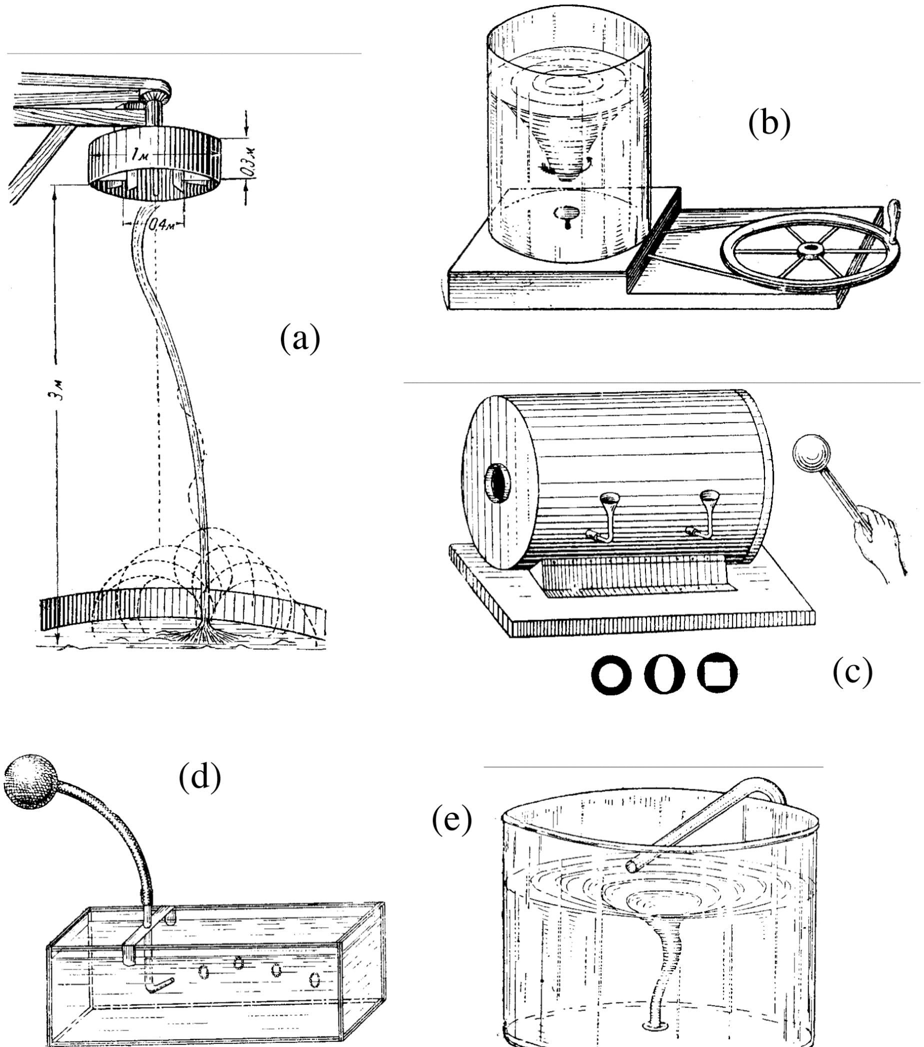


Figure 2: Various examples of vortex generators. From [439].

> of power; the same bit of work has been repeated in different countries at different times, owing to the absence of such histories as Dr. Todhunter set himself to write. It is true that the various *Jahrbücher* and *Fortschritte* now reduce the possibility of this repetition, but besides their frequent insufficiency they are at best but indices to the work of the last few years; an enormous amount of matter is practically stored out of sight in the *Transactions* and *Journals* of the last century and of the first half of the present century. It would be a great aid to science, if, at any rate, the innumerable mathematical journals could be to a great extent specialized, so that we might look to any of them for a special class of memoir. Perhaps this is too great a collectivist reform to expect in the near future from even the cosmopolitan spirit of modern science. As it is, the would-be researcher either wastes much time in learning the history of his subject, or else works away regardless of earlier investigators. The latter course has been singularly prevalent with even some first class British and French mathematicians.

There are, of course, other bibliographies to which the reader might turn. Some older review papers [34, 35, 99, 226, 403, 404, 595, 600–603] contain many references to concrete problems in the dynamics of concentrated vorticity and vortices, with special attention to experimental studies. The 1954 monograph by Truesdell [964] contains a rich and scholarly bibliography covering nominally the same subject and with virtually the same "upper bound" in terms of the period in time. (Truesdell's first reference is Newton's *Principia* so his bibliography covers almost two centuries more than ours.) There is considerable overlap between Truesdell's bibliography and ours. There are also major differences. Truesdell includes a number of papers that, while dealing with basic issues of fluid mechanics, are not particularly – and certainly not solely – addressed to vortex dynamics. Similarly, Truesdell's bibliography is essentially devoid of references to any kind of discrete vortex models, such as point vortices, line vortices, vortex sheets, or so-called "vortex patches" (finite regions of vortical fluid embedded in otherwise irrotational flow), and so on. Thus, the papers [467, 468] by von Kármán – let alone those by Bénard [56–58] – on vortex streets, are not cited by Truesdell. Hill's paper of 1893 [422] on the vortices that today bear his name is not cited. Truesdell's bibliography is also weak on literature from the Eastern block: Russia and the several republics of the former Soviet Union[7]. (We realize that much of this literature may simply not have been accessible to him.) For example, no work of Chaplygin, such as [127, 129], is cited. Even for the literature where Truesdell's bibliography is strong – mainly the countries of Western Europe – the more physical investigations, such as the papers by Lagally [531–533], are not cited although his 1928 lectures on vector calculus make the list. These examples of the difference between the two bibliographies are given not to detract in any way from Truesdell's work but to stress that the outlook and content of the two bibliographies is quite different. Ideally, the reader will find the two collections to complement one another and to both be useful for further research.

We call attention to the unique *Catalogue of Scientific Papers (1800–1900)* compiled and published by the Royal Society of London in 19 volumes during the period 1867–1925 (freely available in electronic form via `http://gallica.bnf.fr`) where a complete list of journal papers of every(!) 19th century author is given in chronological order with complete references. In addition, the subject index to this edition contains short titles of papers (with names of the authors, abbreviated title of the journal, volume, year and first page of the publication) arranged according to the following topics: Volume 2 *Mechanics*, 2450 – Vortex motion. Vortex atoms (130 titles); Volume 3, Part 1 *Physics*, 0500 – Theories of the Constitution of Matter, Vortex theories (20 titles), 0600 – Theories of the Ether (4 titles related to the vortex ether).

We also cite a number of books that have particularly good discussions on aspects of vortex dynamics paying particular attention to several relatively unknown textbooks in French, German, Italian, and Russian. When an English translation exists, we provide a citation to this translation.

In many cases the main work has been reviewed or abstracted elsewhere. Papers written in the 1930's were reviewed in detail in *Zentralblatt für Mechanik* published 1934-1941[8]. Later publications in this area are extensively reviewed in *Applied Mechanics Reviews*, published since 1948 (with S. P. Timoshenko, Th. von Kármán and L. H. Donnell as founding editors). Moreover, some mathematical review journals, such as *Jahrbuch über die Fortschritte der Mathematik* (published in 1875-1942), *Zentralblatt für Mathematik und ihre Grenzgebiete* (published since 1931; both journals are included into the Zentralblatt MATH Database which contains about 2.3 million entries drawn from about 3500 journals and 1100 serials from 1868 to present and is freely available in electronic form via `http://www.zentralblatt-math.org/zmath/en/`), and *Mathematical Reviews* (published since 1940) also contain sections devoted to vortex dynamics.

[7] References to this vast literature may be found to some extent in the bibliographies [16, 200, 607].

[8] It should be noted that the editorial board of the first several volumes was truly international. It included A. Betz (Göttingen), C. B. Biezeno (Delft), J. M. Burgers (Delft), R. Grammel (Stuttgart), E. Hahn (Nancy), Th. von Kármán (Aachen, Pasadena), T. Levi-Civita (Rome), E. L. Nicolai (Leningrad), L. Prandtl (Göttingen), G. I. Taylor (Cambridge), and S. P. Timoshenko (Ann Arbor). For political reasons of the time later issues were edited solely by German editors, W. Flügge and O. Neugebauer, both from Göttingen.

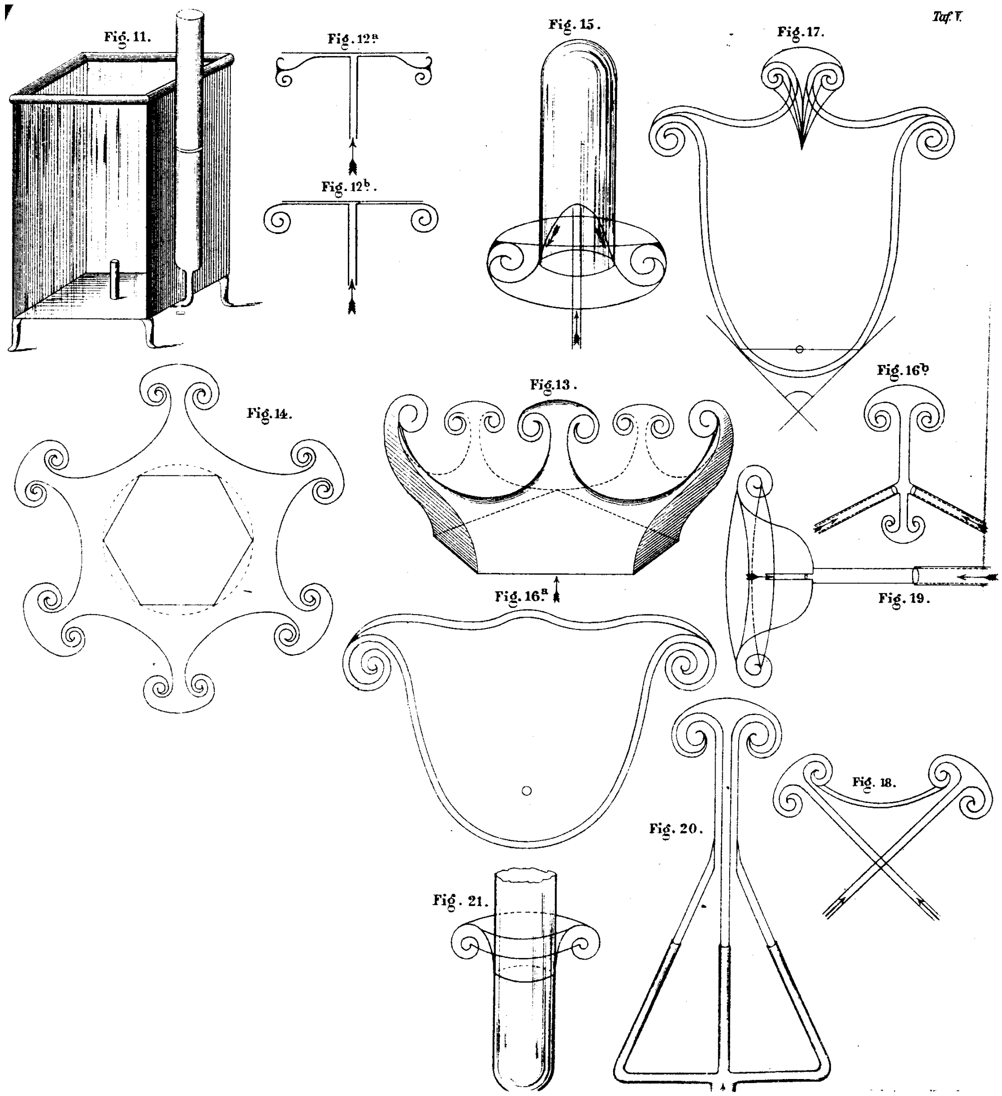


Figure 3: Experiments on interaction of vortex rings with plates of various shapes. After Kötschau [518].

In some cases the abstract or review is in a different language from the original. When we are aware of such a review or abstract, we have included a citation of it along with the citation of the original work.

In the bibliography we have abbreviated the names of most of the journals. Some of these names and abbreviations will be familiar to the reader (although we may have chosen shorter or different abbreviations than are in common use). Others will be to journals and other periodicals that are no longer in circulation. Thus, we give here a list of the full names of the periodicals (as printed on the title page) and other sources where the various papers have appeared, ordered alphabetically by the abbreviations used in the bibliography. The abbreviations are generally formed by abbreviating the principal words in the title in the order in which they occur; the place of publication and other pertinent information is given in parentheses when desirable for clarity.

Abh. Geb. Naturw. Hamburg – Abhandlungen aus dem Gebiet der Naturwissenschaften herausgegeben van Naturwissenschaftlichen Verein in Hamburg.

Adv. Appl. Mech. – Advances in Applied Mechanics.

Aeronaut. J. – Aeronautical Journal.

AIAA. J. – AIAA Journal.

Am. J. Math. – American Journal of Mathematics.

Am. J. Phys. – American Journal of Physics.

Am. J. Sci. – American Journal of Science and Arts.

Ann. Fac. Sci. Univ. Toulouse – Annales de la Faculté des Sciences de l'Université de Toulouse pour les sciences mathématiques et physiques.

Ann. Phys. – Annalen der Physik.

Ann. Phys. Chem. – Annalen der Physik und Chemie.

Ann. Sci. – Annals of Science.

Ann. Sci. Éc. Norm. Super. – Annales scientifiques de lÉcole Normale supérieure.

Ann. Scu. Norm. Sup. Pisa – Annali della Scuola normale superiore di Pisa. Scienze Fisiche e Matematiche.

Annu. Rev. Fluid Mech. – Annual Review of Fluid Mechanics.

Appl. Mech. Rev. – Applied Mechanics Reviews.

Arch. Hist. Exact Sci.. – Archive for the History of Exact Sciences.

Arch. Math. Phys. – Archiv der Mathematik und Physik.

Atti Accad. Naz. Lincei. Mem. Cl. Sci. Fis. Mat. Nat. – Atti dell'Accademia Nazionale dei Lincei. Memorie. Classe di Scienze Fisiche, Matematiche e Naturali.

Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. – Atti dell'Accademia Nazionale dei Lincei. Rendiconti. Classe di Scienze Fisiche, Matematiche e Naturali.

Atti Accad. Pontif. Nuovi Lincei – Atti dell'Accademia Pontificia dei Nuovi Lincei.

Atti Accad. Sci. Lett. Arti Padova – Atti dell'Accademia delle Scienze, Lettere ed Arti di Padova.

Atti Accad. Sci. Torino. Cl. Sci. Fis. Mat. Nat. – Atti dell'Accademia delle Scienze di Torino. Classe di Scienze Fisiche, Matematiche e Naturali.

Atti Ist. Veneto Sci. Lett. Arti. Cl. Mat. Nat. – Atti dell'Istituto Veneto di Scienze, Lettere ed Arti. Classe di Scienze Matematiche e Naturali.

Atti Soc. Ital. Prog. Sci – Atti della Società Italiana per il progresso delle scienze.

Beitr. Geophys. – Beiträge zur Geophysik.

Biogr. Mem. Notes FRS – Biographical Memoirs of Fellows of the Royal Society.

Boll. Un. Mat. Ital. – Bollettino della Unione Matematica Italiana.

Bull. Am. Math. Soc. – Bulletin of the American Mathematical Society.

Bull. Calcutta Math. Soc. – Bulletin of the Calcutta Mathematical Society.

Bull. Inst. Aérodyn. Koutchino – Bulletin de l'Institut aérodynamique de Koutchino.

Byull. Mosk. Obshch. Vozdukhopl. – Byulleten' Moskovskogo obshchestva vozdukhoplavaniya.

Bull. Nat. Res. Council – Bulletin of the National Research Council.

Cambr. Dubl. Math. J. – The Cambridge and Dublin Mathematical Journal

Can. J. Math. – Canadian Journal of Mathematics.

Chem. News J. Ind. Sci. – Chem News and Journal of Industrial Science.

Comment. Math. Helvet. – Commentarii Mathematici Helvetici.

C. R. Acad. Bulg. Sci. – Comptes Rendus de l'Académie Bulgare des Sciences.

C. R. Acad. Sci. Crac. – Comptes Rendus de l'Academie des Sciences Cracovie.

C. R. Acad. Sci. Paris – Comptes rendus hebdomadaires des séances de l'Académie des Sciences.

Dokl. Akad. Nauk SSSR – Doklady Akademii Nauk SSSR.

F. d. M. – Jahrbuch über die Fortschritte der Mathematik

Fluid Dyn. – Fluid Dynamics.

Fluid Dyn. Res. – Fluid Dynamics Research.

Forsch. Geb. IngWes. – Forschung auf dem Gebiet des Ingenieurswesens.

G. Mat. Battaglini – Giornale di Matematiche di Battaglini.

Geophys. Mag. – Geophysical Magazine.

Geofys. Publ. – Geofysiske publikationer.

God. Sof. Univ. – Godishnik na Sofiiskiya Universitet. Fiz-Mat Fakultet.

Helv. Phys. Acta – Helvetica Physica Acta.

Ill. Aeronaut. Mitt. – Illustrierte aeronautische Mitteilungen.

Ing.-Arch. – Ingenieur-Archiv.

Ing. Grav. – Ingenieur. 's Gravenhage.

Izv. Akad. Nauk SSSR. Mekh. Zhid. Gaza – Izvestiya Akademii Nauk SSSR. Mekhanika zhidkosti i gaza.

Izv. Akad. Nauk SSSR. Otd Mat. Estest. Nauk – Izvestiya Akademii Nauk SSSR. Otdelenie matematicheskikh i estestvennykh nauk.

Izv. Akad. Nauk SSSR. Otd. Tekh. Nauk – Izvestiya Akademii Nauk SSSR. Otdelenie tekhnicheskikh tauk.

Izv. Imp. Obshch. Lyub. Estest. Antrop. Etnogr. Imp. Mosk. Univ. – Izvestiya Imperatorskogo obshchestva lyubitelei estestvoznaniya, antropologii i etnografii pri Imperatorskom Moskovskom universitete.

Izv. RAN. Mekh. Zhid. Gaza – Izvestiya Rossiiskoi Akademii Nauk. Mekhanika zhidkosti i gaza.

Jahrb. Wiss. Ges. Luftf. – Jahrbuch der Wissenschaftlichen Gesellschaft fúr Luftfahrt.

Jahresber. Dt. Mat. Verein. – Jahresbericht der Deutschen Matematiker-vereinigung.

J. Aeronaut. Sci. – Journal of the Aeronautical Sciences.

J. Appl. Math. Mech. – Journal of Applied Mathematics and Mechanics.

J. Appl. Phys. – Journal of Applied Physics.

J. Chem. Soc. – Journal of the Chemical Society.

J. Coll. Sci. Imp. Univ. Tokyo – Journal of the College of Science, Imperial University of Tokyo.

J. Fluid Mech. – Journal of Fluid Mechanics.

J. Franklin Inst. – Journal of the Franklin Institute.

J. Japan Soc. Mech. Engrs – Journal of the Japan Society of Mechanical Engineers.

J. Lond. Math. Soc. – Journal of the London Mathematical Society.

J. Math. Phys. – Journal of Mathematics and Physics.

J. Math. Pures Appl – Journal de mathématiques pures et appliquées.

J. Phys. Soc. Japan – Journal of the Physical Society of Japan.

J. R. Aeronaut. Soc. – Journal of the Royal Aeronautical Society.

J. Reine Angew. Math. – Journal für die reine und angewandte Mathematik.

J. Sci. Instrum. – Journal of Scientific Instruments.

J. Soc. Appl. Mech. Japan – Journal of the Society of Applied Mechanics of Japan.

Mat. Sbor. – Matematicheskii Sbornik.

Math. Ann. – Mathematische Annalen.

Math. Naturwiss. Unterr. – Mathematische und naturwissenschaftliche Unterricht.

Math. Rev. – Mathematical Review

Math. Z. – Mathematische Zeitschrift.

Mem. Accad. Pontif. Nuovi Lincei – Memorie dell'Accademia Pontificia dei Nuovi Lincei.

Mem. R. Accad. Sci. Ist. Bologna – Memorie della R. Accademia delle scienze dell'Istituto di Bologna.

Mess. Math. – Messenger of Mathematics.

Met. Z. – Meteorologische Zeitschrift.

Mitt. math.-naturwiss. Ver. Württemberg – Mitteilungen des mathematisch-naturwissenschaftlichen Vereins ins Württemberg.

Monatsber. Akad. Wiss. Berlin – Monatsberichte der Akademie der Wissenschaften in Berlin.

Nachr. Ges. Wiss. Göttingen Math.-Phys. Kl. – Nachrichten von der (Königliche) Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-physikalische Klasse.

NACA Tech. Memo. – National Advisory Committee for Aeronautics. Technical Memoranda.

NACA Tech. Notes – National Advisory Committee for Aeronautics. Technical Notes.

Note Esercit. Mat. Catania – Note ed esercitazioni matematiche. Circolo matematico di Catania.

Phil. Mag. – Philosophical Magazine.

Phil. Trans. R. Soc. London – Philosophical Transactions of the Royal Society of London.

Phys. Fluids – Physics of Fluids.

Phys. Z. – Physikalische Zeitschrift.

Popul. Sci. Rev. – Popular Science Reviews.

Prikl. Mat. Mekh. – Prikladnaya Matematika i Mekhanika.

Pod Znam. Marxisma – Pod Znamenem Marxisma.

Proc. Am. Acad. Arts Sci. – Proceedings of the American Academy of Arts and Sciences.

Proc. Benares Math. Soc. – Proceedings of the Benares Mathematical Society.

Proc. Camb. Phil. Soc. – Proceedings of the Cambridge Philosophical Society.

Proc. Dublin Sci. Soc. – Proceedings of the Dublin Scientific Society.

Proc. Edinb. Math. Soc. – Proceedings of the Edinburgh Mathematical Society.

Proc. Glasgow Phil. Soc. – Proceedings of the Glasgow Philosophical Society.

Proc. Ind. Acad. Sci. – Proceedings of the Indian Academy of Sciences.

Proc. Inst. Automob. Eng. – Proceedings of the Institution of Automobil Engineers.

Proc. Inst. Mech. Engrs – Proceedings of the Institution of Mechanical Engineers.

Proc. Lit. Phil. Soc. Manch. – Proceedings of the Literary and Philosophical Society of Manchester.

Proc. Lond. Math. Soc. – Proceedings of the London Mathematical Society.

Proc. Nat. Acad. Sci. USA – Proceedings of the National Academy of Sciences of the United States of America.

Proc. Phys.-Math. Soc. Japan – Proceedings of the Physico-Mathematical Society of Japan.

Proc. R. Dublin Soc. – Proceedings of the Royal Dublin Society.

Proc. R. Instn Gt Brit. – Proceedings of the Royal Institution of Great Britain.

Proc. R. Irish Acad. – Proceedings of the Royal Irish Academy.

Proc. R. Soc. Edinb. – Proceedings of the Royal Society of Edinburgh.

Proc. R. Soc. London – Proceedings of the Royal Society of London.

Proc. Sect. Sci. K. Ned. Akad. Wet – Proceedings of the Section of Sciences Koninklijke Nederlandse Akademie van Wetenschappen.

Publ. Scient. Tech. Minist. Air – Publications scientifiques et techniques du Ministère de l'Air. Paris.

Quart. J. Math. – Quarterly Journal of Mathematics. Oxford ser.

Quart. J. Pure Appl. Math. – Quarterly Journal of Pure and Applied Mathematics.

Rend. Circ. Mat. Palermo – Rendiconti del Circolo matematico di Palermo.

Rend. Ist. Lomb. Sci. Lett. – Rendiconti dell'Istituto Lombardo di scienze e lettere. Milano.

Rend. Semin. Mat. Fis. Milano – Rendiconti del Seminario matematico e fisico di Milano.

Rend. Semin. Mat. Univ. Padova – Rendiconti del Seminario matematico della Università Padova.

Rep. Aeronaut. Res. Inst. Tokyo – Report of the Aeronautical Research Institute, Tokyo Imperial University.

Rep. Brit. Ass. Advmt Sci. – Report of the British Association for the Advancement of Science.

Rep. David Taylor Model Basin – Reports David Taylor Model Basin. United States Navy Department.

Rep. Memo. Advis. Comm. Aeronaut. – Reports and Memoranda. Advisory Committee for Aeronautics. London.

Sb. Inst. Inzh. Put. Soobshch. – Sbornik Instituta inzhenerov putei soobshcheniya. St.Petersburg (Leningrad).

SchReihe ForschInst Math. – Schriftenreihe des Forschungsinstituts für Mathematik.

Scient. Am. Suppl. – Scientific American Supplement.

Sber. Akad. Wiss. Wien – Sitzungsberichte der Akademie der Wissenschaften in Wien. Abh. IIa. Mathematik, Physik, Astronomie.

Sber. Bayer. Akad. Wiss. – Sitzungsberichte der Bayerischen Akademie der Wissenschaften zu München. Mathematische-physikalische Klasse.

Sber. Preuss. Akad. Wiss. – Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin.

Sci. Abs. – Scientific Abstracts.

Spis. Bulg. Akad. Nauk. – Spisanie na Bulgarskato akademiya na naukite.

Tech. Rep. Advis. Comm. Aeronaut. – Technical Report of the Advisory Committee for Aeronautics.

Trans. Cambr. Phil. Soc. – Transactions of the Cambridge Philosophical Society.

Trans. R. Irish Acad. – Transactions of the Royal Irish Academy.

Trans. R. Soc. Canada – Transactions of the Royal Society of Canada.

Trans. R. Soc. Edinb. – Transactions of the Royal Society of Edinburgh.

Trudy Avia. Rasch.-Ispyt. Byuro – Trudy Aviacionnogo raschetno-ispytatel'nogo byuro. Moskva.

Trudy Glav. Geofiz. Obs. – Trudy Glavnoi geofizicheskoi observatorii imeni A.I. Voeikova, Leningrad.

Trudy Inst. Istor. Estestv. Tekhn. – Trudy Instituta istorii estestvoznaniya i tekhniki. Moskva.

Trudy Otd. Fiz. Nauk. Mosk. Obshch. Lyub. Estest. Antr. Etn. – Trudy otdela fizicheskikh nauk Moskovskogo obshchestva lyubitelei estestvoznaniya, antropologii i etnografii.

Trudy Sev.-Kavkaz. Assoc. – Trudy Severo-Kavkazskoi Associacii. Rostov-na-Donu.

Trudy Tsentr. Aero-Gidrodin. Inst. – Trudy Tsentral'nogo Aero-Gidrodinamicheskogo Instituta. Moskva.

Uchen. Zap. Imp. Kazan. Univ. – Uchenye zapiski Imperatorskogo Kazanskogo universiteta.

Uchen. Zap. Imp. Moskov. Univ. – Uchenye zapiski Imperatorskogo Moskovskogo universiteta.

Uchen. Zap. Moskov. Gos. Univ. – Uchenye zapiski Moskovskogo gosudarstvennogo universiteta imeni M.V. Lomonosova.

Uchen. Zap. Saratov. Gos. Univ. – Uchenye zapiski Saratovskogo gosudarstvennogo universiteta imeni N.G. Chernyshevskogo.

Unterrbl. Math. Naturw. – Unterrichtsblätter für Mathematik und Naturwissenschaften.

Usp. Mat. Nauk – Uspekhi matematicheskikh nauk. Moskva.

Verh. Naturh.-med. Ver. Heidelb. – Verhandlungen des Naturhistorisch-medizinischen Vereins zu Heidelberg.

Vest. AN SSSR – Vestnik Akademii Nauk SSSR. Moskva.

Vest. Moskov. Univ. – Vestnik Moskovskogo universiteta.

Vierteljschr. Naturf. Ges. Zürich – Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich.

Zap. Imp. Novoross. Univ. – Zapiski Imperatorskogo Novorossiiskogo universiteta. Odessa. Fiz-mat fakul'tet.

Zap. Mat. Otd. Novoross. Obshch. Estest. – Zapiski matematicheskogo otdeleniya Novorossiiskogo obshchestva estestvoispytatelei. Odessa.

Z. Angew. Math. Mech. – Zeitschrift für angewandte Mathematik und Mechanik.

Z. Flugtech. Motorluftschiff. – Zeitschrift für Flugtechnik und Motorluftschiffahrt.

Z. Flugwiss. – Zeitschrift für Flugwissenschaften.

Z. Ges. Naturw. – Zeitschrift für die gesamte Naturwissenschaft.

Z. Math. Phys. – Zeitschrift für Mathematik und Physik.

Z. Phys. – Zeitschrift für Physik.

Z. Tech. Phys. – Zeitschrift für technische Physik.

Z. VDI – Zeitschrift des Vereins deutscher Ingenieure.

Zentr. Math. – Zentralblatt für Mathematik.

Zentr. Mech. – Zentralblatt für Mechanik.

ZhETF – Zhurnal eksperimental'noi i teoreticheskoi fisiki. Moskva.

Zh. Nauchno-issled. Kafedr Odessa – Zhurnal nauchno-issledovatel'skikh kafedr v Odesse.

Zh. Russk. Fiz.-Khim. Obshch – Zhurnal Russkogo fiziko-khimicheskogo obshchestva pri Imperatorskom St-Peterburgskom universitete. Chast' fizicheskaya.

2. **Case studies**

Some of the older papers collected in this bibliography have maintained themselves into modern research while others have been long forgotten. For example, the thesis of Gröbli [365, 366] and the later paper by Synge [892] on the solution of the three-vortex problem were revived about 30 years ago through the independent rediscoveries by Novikov [697] and Aref [26]. For a review of the history of solution, neglect and re-discovery see [30]. While the three-vortex problem is very interesting of its own accord, the discovery of chaos in the four-vortex problem (cf. [27]) immediately propelled this kind of problem to the front lines of "modern science". See also §2.2 below.

Another example of this kind may be found in the extensive series of works by Da Rios, [171–186], on vortex filament motion under the so-called *localized induction approximation*. In spite of having been done as a thesis under T. Levi-Civita, one the most illustrious mathematicians of his day, this work, somehow, never "took". It was not until the 1960's when Arms & Hama [31] and Betchov [73] re-introduced this idea – and Batchelor included it in his well known text [46] – that it finally became a standard part of the subject. The beautiful transformation of Hasimoto [385], and the idea that vortex filaments can support soliton waves, also played a role in this "assimilation" into modern research. The history of Da Rios' work has been reviewed by Ricca [784, 785].

2.1 *Helmholtz's paper* Helmholtz must rank as the discoverer of a series of fundamental propositions in hydrodynamics that had entirely escaped his predecessors. He pointed out that already Euler had mentioned cases of fluid motion in which no velocity-potential exists, for example, the rotation of a fluid about an axis where every element has the same angular velocity. A minute

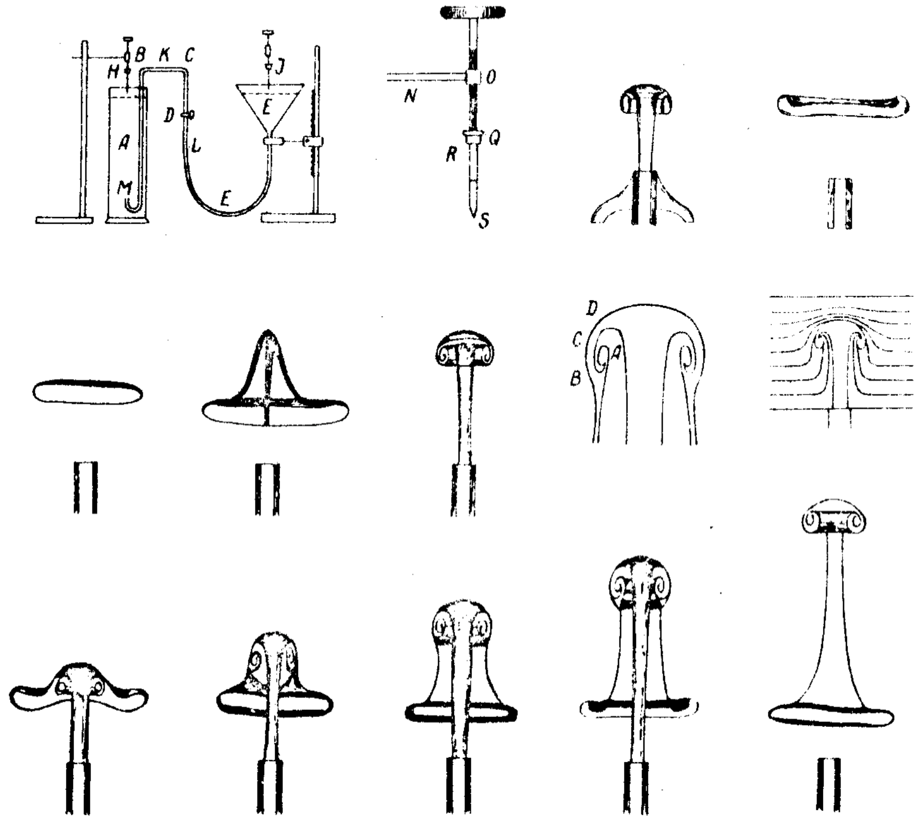


Figure 4: Experiments on vortex ring formation by injecting one fluid into another through an orifice. After Mack [609].

sphere of fluid may move as a whole in a definite direction, and change its shape, all while rotating about an axis. This last motion is the distinguishing characteristic of vorticity. Helmholtz was the first to elucidate key properties of those portions of a fluid in which vorticity occurs. His investigation was restricted to a frictionless, incompressible fluid. He proved that in such an ideal substance vortex motion could neither be produced from irrotational flow nor be destroyed entirely by any natural forces that have a potential. If vorticity exists within a group of fluid particles, they are incapable of transmitting it to particles that have none. They cannot be entirely deprived of their vorticity themselves (although the vorticity of any individual particle may change in three-dimensional flow; in two-dimensional flow the vorticity of each particle is a constant of the motion). For an ideal fluid the laws of vortex motion establish a curious and indissoluble fellowship between fluid particles and their state of rotation.

In the Introduction to his paper Helmholtz[9] states:

> Hence it appeared to me to be of importance to investigate the species of motion for which there is no velocity-potential.
>
> The following investigation shows that when there is a velocity-potential the elements of the fluid have no rotation, but that there is at least a portion of the fluid elements in rotation when there is no velocity-potential.

[9] According to Tait's translation [394, p.486] of his paper.

By *vortex-lines* (*Wirbellinien*) I denote lines drawn through the fluid so as at every point to coincide with the instantaneous axis of rotation of the corresponding fluid element.

By *vortex-filaments* (*Wirbelfäden*) I denote portions of the fluid bounded by vortex-lines drawn through every point of the boundary of an infinitely small closed curve.

The investigation shows that, if all the forces which act on the fluid have a potential, —

1. No element of the fluid which was not originally in rotation is made to rotate.

2. The elements which at any time belong to one vortex-line, however they may be translated, remain on one vortex-line.

3. The product of the section and the angular velocity of an infinitely thin vortex-filament is constant throughout its whole length, and retains the same value during all displacements of the filament. Hence vortex-filaments must either be closed curves, or must have their ends in the bounding surface of the fluid.

According to Truesdell [964, p.58] the name *vorticity* was introduced by Lamb [542, §30] for the vector, $\boldsymbol{\omega}$, whose Cartesian components, (ξ, η, ζ), are given in terms of the components (u, v, w) of the (Eulerian) velocity vector $\boldsymbol{u}$ by

$$\xi = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \quad \eta = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \quad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \tag{1}$$

In modern vector notation

$$\boldsymbol{\omega} = \nabla \times \boldsymbol{u}. \tag{2}$$

Helmholtz [393] and Stokes [883] used the symbols ξ, η, ζ and ω', ω'', ω''', respectively, to denote the quantities:

$$\frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right), \quad \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right), \quad \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right). \tag{3}$$

Helmholtz called these *Rotationsgeschwindigkeiten*, Stokes *angular velocity*; and W. Thomson [937] called them *component rotations*. Basset [44] called the entities (3) *molecular rotations*. For the general case of motion in which $\boldsymbol{\omega}$ does not vanish Helmholtz [393] used the term *Rotations-bewegung*, which was translated by Tait [394] as *vortex motion*.

The notion of vorticity had already appeared in earlier works by d'Alembert (1749), Euler (1752-1755), Lagrange (1760), and Cauchy (1815) – in Truesdell's view [964, p.59], "all these early works are purely formal and somewhat mystifying" – where the vorticity vector is described either as a mean value of the rates of rotation about all or several directions by Cauchy [126] or as a local angular velocity by Stokes [883]. Subsequently, additional interpretations as a circulation per unit area were given by Hankel [377] and W. Thomson [937].

Helmholtz's result in §1 of his paper that an arbitrary instantaneous state of continuous motion of a deformable medium is at each point the superposition of a uniform velocity of translation, a motion of extension, a shearing motion, and a rigid rotation, is called by Truesdell [964, p.66] the "Cauchy-Stokes decomposition theorem" (referring to [126] and [883]). Saint-Venant in a letter of 22 January 1862 to Stokes, cited in [966], insisted on the clear priority of Cauchy in this regard.

In 1867 the French academician Bertrand published [69] various criticisms regarding the universality of Helmholtz's use of the word *rotation* and the correctness of Helmholtz's result. Bertrand contended that in some cases *oblique*, infinitesimal parallelepipeds could be chosen that would transform into other parallelepipeds, whose edges would remain parallel with those of the former, i.e., the continuous motion would consist of the superposition of a translation and an extension (or contraction) along three *nonorthogonal* axes, without rigid rotation of any element of the fluid. He also pointed out that in a simple shearing motion, $u = y$, $v = 0$, $w = 0$, fluid particles move along straight lines, while according to Helmholtz's notation the motion is rotational, $\zeta = -\frac{1}{2}$. These examples produced an animated discussion [70–72, 395–397], "an acrimonious public controversy" as Truesdell [964, p.58] calls it. Helmholtz proved that any extension (or contraction) along three *nonorthogonal* axes is equivalent to the superposition of an extension (or contraction) along three *orthogonal* axes and a rotation. Concerning Bertrand's example, while the fluid particles, which are *points*, do indeed not rotate in orbits like planets about the sun, any *volume*, however small, suffers rotation with respect to its initial configuration, and to an (Eulerian) observer a small rigid object convected by this motion would appear to rotate.

In §2 of his paper [393] Helmholtz gives proofs of his three laws of vortex motion based upon kinematical considerations and ingenious transformations of the dynamical equations for incompressible, homogeneous, inviscid fluid into his now famous vorticity equations.

The first law of Helmholtz is closely related to the celebrated Lagrange-Cauchy velocity-potential theorem. Lagrange [537, §§17-19] and [538, Part II, Section 11, §§16-17] stated that if a velocity potential exists at one time in a motion of an inviscid incompressible fluid, subject to conservative extraneous forces, it will exist at all future times, i.e., a motion once irrotational is always irrotational. Some objections to Lagrange's proof were put forward by Power [727] and especially by Stokes [883, Section II], (see Truesdell [964, §§104-107] for a thorough discussion). Cauchy [126] gave a clear statement and correct proof of the proposition that in a continuous motion of such a fluid, a particle once in irrotational motion is at all times in irrotational motion. The argument was later repeated by Stokes [884]. Later, Stokes [886] added to his paper [884] a note "that two of Helmholtz's fundamental propositions respecting vortex motion follow immediately from Cauchy's integrals" and gave the proof.

The third law contains two statements, *viz* that "vortex-filaments must either be closed curves", or that they "must have their ends in the bounding surface of the fluid". The first statement excludes the possibility of vortex lines that wander aperiodically and never close, as one finds, for example, in a chaotic, three-dimensional flow[10]. The second is, in principle, correct only for vortex lines, although an example of a thin vortex filament that ends at a point in the interior of the fluid has, so far as we are aware, never been given. The vorticity distribution in such a structure would be near-singular.

In §3 of his paper [393] Helmholtz addresses the inverse problem of finding the components of the velocity u, v, w from the components of vorticity ξ, η, ζ (up to a potential flow that covers the boundary conditions). He independently obtains the representations of Stokes [885] for the classical problem of vector analysis of determining a vector field of known divergence ("hydrodynamic integrals of the first class" in his terminology) and curl ("hydrodynamic integrals of the second class"). Determination of the velocity field for incompressible fluid leads to the *Biot-Savart law* of electromagnetism, which in the present case reads that each rotating element of fluid induces in every other element a velocity with direction perpendicular to the plane through the second element that contains the axis of the first element. The magnitude of this induced velocity is directly proportional to the volume of the first element, its angular velocity, and the sine of the angle between the line that joins the two elements and the axis of rotation, and is inversely proportional to the square of the distance between the two elements.

Helmholtz also establishes analogies between the induced velocity and the forces on magnetized particles. Most of these relations would today come under the heading of potential theory.

In §4 of his paper [393] Helmholtz derives an elegant expression for the conserved kinetic

[10] The best known examples may be the *ABC flows* studied by several authors ever since their introduction in 1965-66 by Arnold and Hénon; see [28] for a brief description in the context of "chaotic advection". There are many other instances where vortex lines do not close. Indeed, closed vortex lines are the exception.

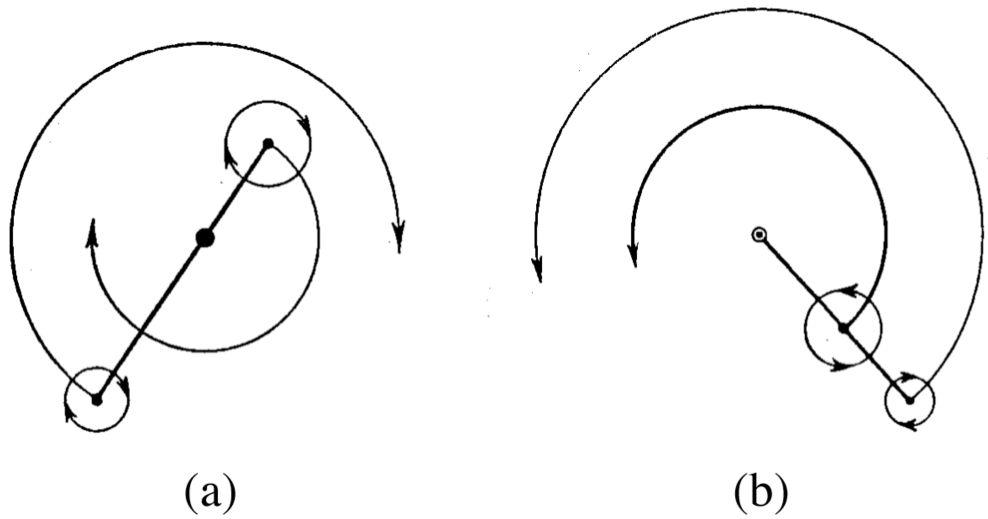


Figure 5: Motion of two parallel rectilinear vortices (or point vortices) (a) circulations of the same sign; (b) circulations of opposite sign. From [439].

energy – *"vis viva"* in his terminology – of infinite fluid with a compact distribution of vorticity within it.

In §5, entitled "Straight parallel vortex-filaments", Helmholtz studies certain simple cases in which the rotation of the elements occurs only in a set of parallel rectilinear vortex-filaments. In particular, he considers several infinitely thin, parallel vortex-filaments each of which carries a finite, limiting value, m, of the product of the cross-sectional area and the angular velocity. This is the now celebrated concept of a *point vortex*. Helmholtz considers simple cases of the dynamics of such vortices. He establishes the law of conservation of the *center of vorticity* of an assembly of point vortices. The discussion is phrased in terms of the "center of gravity" of the vortices (considering their values of m as the analog of "masses"): "The centre of gravity of the vortex-filaments remains stationary during their motions about one another, unless the sum of the masses be zero, in which case there is no centre of gravity." Without further explanation Helmholtz notes the following two consequences:

> 1. If there be a single rectilinear vortex-filament of indefinitely small section in a fluid indefinite in all directions perpendicular to it, the motion of an element of the fluid at finite distance from it depends only on the product ($\zeta da db = m$) of the velocity of rotation and the section, not on the form of that section. The elements of the fluid revolve about it with tangential velocity $= \frac{m}{\pi r}$, where r is the distance from the centre of gravity of the filament. The position of the centre of gravity, the angular velocity, the area of the section, and therefore, of course, the magnitude m remain unaltered, even if the form of the indefinitely small section may alter.
>
> 2. If there be two rectilinear vortex-filaments of indefinitely small section in an unlimited fluid, each will cause the other to move in a direction perpendicular to the line joining them. Thus the length of this joining line will not be altered. They will thus turn about their common centre of gravity at constant distances from it. If the rotation be in the same direction for both (that is, of the same sign) their centre of gravity lies between them. If in opposite directions (that is, of different signs), the centre of gravity lies in the line joining them produced. And if, in addition, the product of the velocity and the section be the same for both, so that the centre of gravity is at an infinite distance, they travel forwards with equal velocity, and in parallel directions perpendicular to the line joining them.

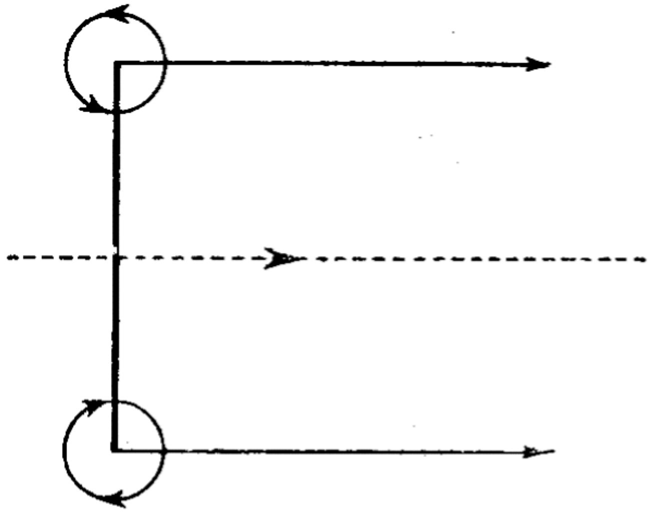


Figure 6: Motion of a vortex pair. From [439].

See Figs. 5 and 6 for later illustrations of these motions.

In addition to introducing this notion of a "vortex pair" Helmholtz describes the motion of a single vortex-filament near an infinite plane to which it is parallel. He states that the boundary condition will be fulfilled if instead of the plane there is an infinite mass of fluid with another vortex-filament as the image (with respect to the plane) of the first, and concludes: "From this it follows that the vortex-filament moves parallel to the plane in the direction in which the elements of the fluid between it and the plane move, and with one-fourth of the velocity which the elements at the foot of a perpendicular from the filament on the plane have."

In §6, entitled "Circular vortex-filaments", Helmholtz studies the axisymmetric motion of several circular vortex-filaments whose planes are parallel to the xy-plane, and whose centers are on the z-axis. Here he considers the problem in full detail (with some minor errors in the formulae corrected in later translations – see [394] and, especially, [401] – without changing the final results) and arrives at the conclusion that "in a circular vortex-filament of very small section in an indefinitely extended fluid, the centre of gravity of the section has, from the commencement, an approximately constant and very great velocity parallel to the axis of the vortex-ring, and this is directed towards the side to which the fluid flows through the ring." (See Fig.7 for a later illustration.)

When two such rings of infinitesimal cross-section have a common axis and the same direction of rotation, they travel in the same direction. As they approach, the first ring widens and travels more slowly, the second contracts and travels faster. Finally, if their velocities are not too different, the second ring overtakes the first and travels through it. (For a later illustration see Fig.10. The corresponding and simpler case for vortex pairs is shown in Fig.8. This same process of "leapfrogging" is then repeated indefinitely (in principle – in reality the finite cores of the rings and the effects of viscosity will only allow one or two cycles of this motion). If two vortex rings have equal radii and opposite angular velocities, they will approach each other and widen one another; and when they are very near to one another, their velocity of approach becomes smaller and smaller, and their rate of widening faster and faster. Just as in the case of the straight vortex filament near the plane wall, this motion is similar to the motion of a single vortex ring running

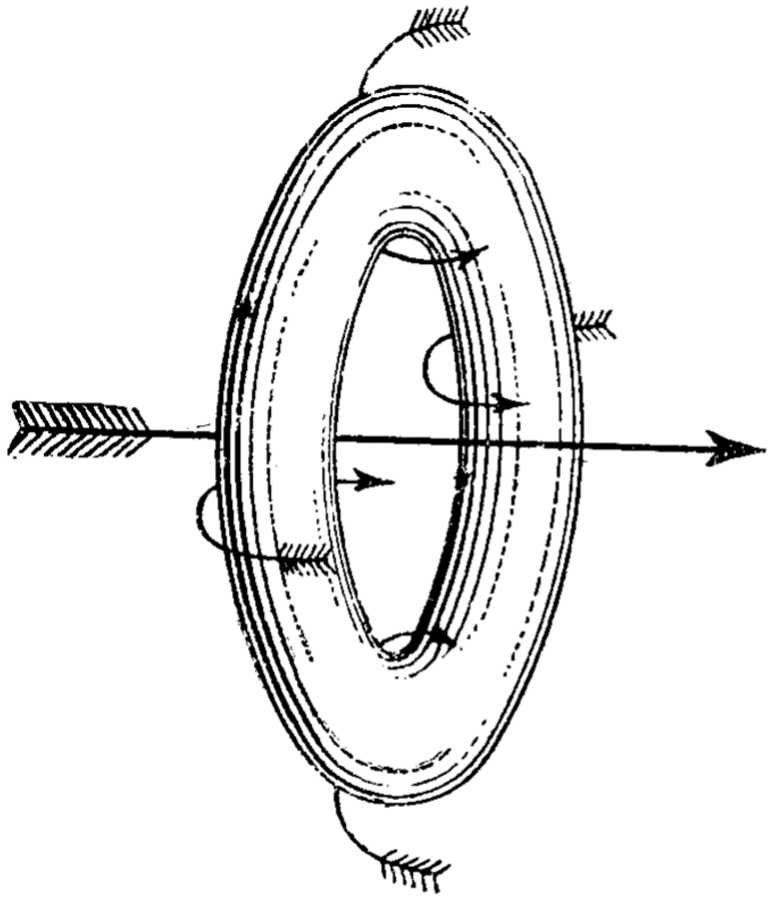


Figure 7: Self-induced forward motion of a vortex ring. From [439].

up against a plane wall. The image of the ring in the wall is another similar ring with the opposite sense of circulation.

Lanchester saw this type of motion involving several vortices to be relevant to vortex formation behind a wing of finite span (cf. Fig.9 later). He wrote [545, p. 122]:

> Groups of filaments or rings behave in a similar manner to pairs: thus a group of rings may play "leap-frog" collectively so long as the total number of rings does not exceed a certain maximum; congregations of vortex filaments likewise by their mutual interaction move as a part of a concentrated system, like waltzers in a ball-room; when the number exceeds a certain maximum the whole system consists of a number of lesser groups.

This quote seems to embody the mechanism of leap-frogging along with the ultimate merging of vortices with cores of finite size.

Helmholtz concludes the paper with the following interesting thought experiment:

> In addition it may be noticed that it is easy in nature to study these motions of circular vortex-rings, by drawing rapidly for a short space along the surface of a fluid a half-immersed circular disk, or the nearly semicircular point of a spoon, and quickly withdrawing it. There remain in the fluid half vortex-rings whose axis is in the free surface. The free surface forms a boundary plane of the fluid through the axis, and thus there is no essential change in the motion. These vortex-rings travel on, widen when they come to a wall, and are widened or contracted by other vortex-rings, exactly as we have deduced from theory.

This experiment was also described qualitatively by Klein [495] and this led to discussions [2, 79, 82] on the possibility of vortex generation in an inviscid fluid. Quantitative calculations were performed by Taylor [909]; see also his excellent review paper [910][11].

English translations of Helmholtz's paper [393] have been published at least twice [394, 402]. The paper was included in the well known *Ostwalds Klassiker der exakten Wissenschaften* series [400] (edited by A. Wangerin). In turn, this book was translated into Russian [401] (edited by S. A. Chaplygin, with extensive comments). In view of the widespread use of vector notation today it is interesting to note that when P. G. Tait, Professor of Natural Philosophy at the University of Edinburgh, an intimate friend of Sir W. Thomson and J. C. Maxwell, and a champion of using quaternions (which were flatly rejected by Thomson), wanted to translate Helmholtz's work for his own use and wrote to Helmholtz on the subject, Helmholtz replied [508, p. 170]:

> If you find quaternions useful in this connection, it would be highly desirable to draw up a brief introductory explanation of them, so far as is necessary in order to make their application to vortex-motion intelligible. Up to the present time I have found no mathematician, in Germany at any rate, who was able to state what quaternions are, and personally I must confess that I have always been too lazy to form a connected idea of them from Hamilton's innumerable little notes on this subject.

Only in rare cases does a single paper put forward so many profound ideas and open so many avenues for further investigation. Almost fifty years later, in 1906, Lord Kelvin, who had himself conducted many great studies developing vortex dynamics further, wrote in the preface to a book about Helmholtz [508] that "his admirable theory of vortex rings is one of the most beautiful of all the beautiful pieces of mathematical work hitherto done in the dynamics of incompressible fluids." Surprisingly Helmholtz never continued his investigations of the topic established in his groundbreaking paper [393]. Instead he wrote another remarkable paper [398] on discontinuous motion of an inviscid fluid in which he used the notion of a vortex sheet from [393] (see [1025] for details).

In the following century Helmholtz's theory of vortices was described in great detail in articles in leading general and specialized encyclopedias in various countries [13, 19, 34–36, 96, 261, 264, 339, 341, 348, 361, 592, 600–603, 642, 730]. Note the appearance of the great Maxwell among the authors. Also note that the articles [339, 341, 592] carefully avoided referencing Helmholtz by name because this was considered bad form by the ruling Marxist-Leninist philosophy of the time in the Soviet Union – politics sometimes has unusual influences on science! The theory was elaborated in major German courses of theoretical physics by (in chronological order) Kirchhoff [491], Planck [717], Haas [372], Schaefer [831] and Sommerfeld [866] resulting in now classic texts.

The Helmholtz theory of vortex motion was described, mainly from a qualitative point of view, in several popular courses of general physics [243, 364, 489, 720], and found its place in important treatises on the history of mathematics[12] [496] and physics [428, 791]. In 1866 Maxwell set Helmholtz's theorems as a question on the Cambridge Mathematical Tripos [637]! The vortex theorems were also treated in important lectures presented on various occasions in various places [8, 18, 76, 77, 95, 106, 108, 265, 392, 410, 439, 453, 571, 593, 604, 605, 639, 728, 736, 738, 759, 771, 775, 782, 787, 807], including the famous 1904 lecture by Prandtl. Many of these were accompanied by illustrative experiments. Here is a partial list of studies where the results of Helmholtz's paper [393] were used in one way or another: Classical textbooks and monographs on fluid mechan-

[11] The remarkable life, career and many seminal contributions to fluid dynamics by G. I. Taylor are chronicled with great scholarship and affection by Batchelor [47, 48].

[12] Klein [496] points out that the same vortex theorems were independently discovered by Dirichlet [201] at approximately the same time.

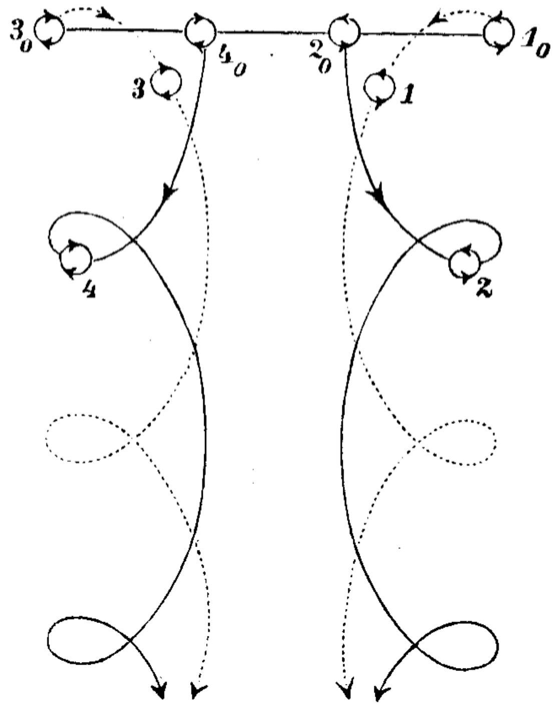


Figure 8: Illustration of “leapfrogging” by two vortex pairs. The induced velocities are indicated by arrows. From [439] based upon Gröbli’s calculations [365, 366].

ics[13] and aerodynamics [23, 44, 298, 306, 448, 455, 504, 542, 543, 545, 655–657, 740, 742, 743, 809]; general textbooks and monographs on fluid mechanics and aerodynamics [33, 37, 45, 83, 86, 148, 244, 280, 283, 287, 288, 376, 478, 580, 594, 662, 678, 716, 729, 739, 745, 753, 1021, 1024]; review papers [81,99,100,147,150,187,300,307,349,403,404,472,535,595,684,685,724,734,735,876,903]; chapters or sections of popular books [275, 712, 896, 930, 931, 993–995]; early specialized books on vortex dynamics [51, 278, 721, 1001]; dissertations [67, 75, 352, 365, 367, 436, 585, 714, 752, 814, 815, 976, 978, 1012]. We also mention the following books in Russian that are less well known in the West[14] [14, 32, 253, 320, 335, 437, 460–463, 487, 505, 506, 530, 591, 658–660, 700, 708, 723, 746, 820, 826, 830, 985, 986].

2.2 *Point vortices* A vast area of research started by Helmholtz’s paper is the study of the motion of straight, parallel, infinitely thin vortex filaments (rectilinear vortices) in incompressible inviscid fluid or, equivalently, the two-dimensional problem of point vortices on a plane. Through pioneering work of Rosenhead [799] and Westwater [1018] in the 1930’s the discretization of two-dimensional hydrodynamics provided by such vortex elements became the foundation for an entire family of numerical methods for flow simulation today collectively known as *vortex methods*.

[13] Apparently, Lamb [540] (see also the review essay by Reynolds [773] and Lamb’s reply [541]) was the first author to treat vortex dynamics in a separate chapter in a major textbook on fluid mechanics.

[14] Many of the authors are also less well known, not to say unknown, in the West. For example, A.A. Satkevitch (1869-1938) was a professor at many St. Petersburg high schools, a corresponding member of the USSR Academy of Sciences (since 1933), and the author of several excellent (text)books on hydromechanics, hydraulics, and gas dynamics, published between 1903 and 1934. He was also a Lieutenant-General in the army of the Russian Empire. In February 1938 he was arrested in Leningrad and accused in the “white-guard officers’ conspiracy” and was executed in July 1938.

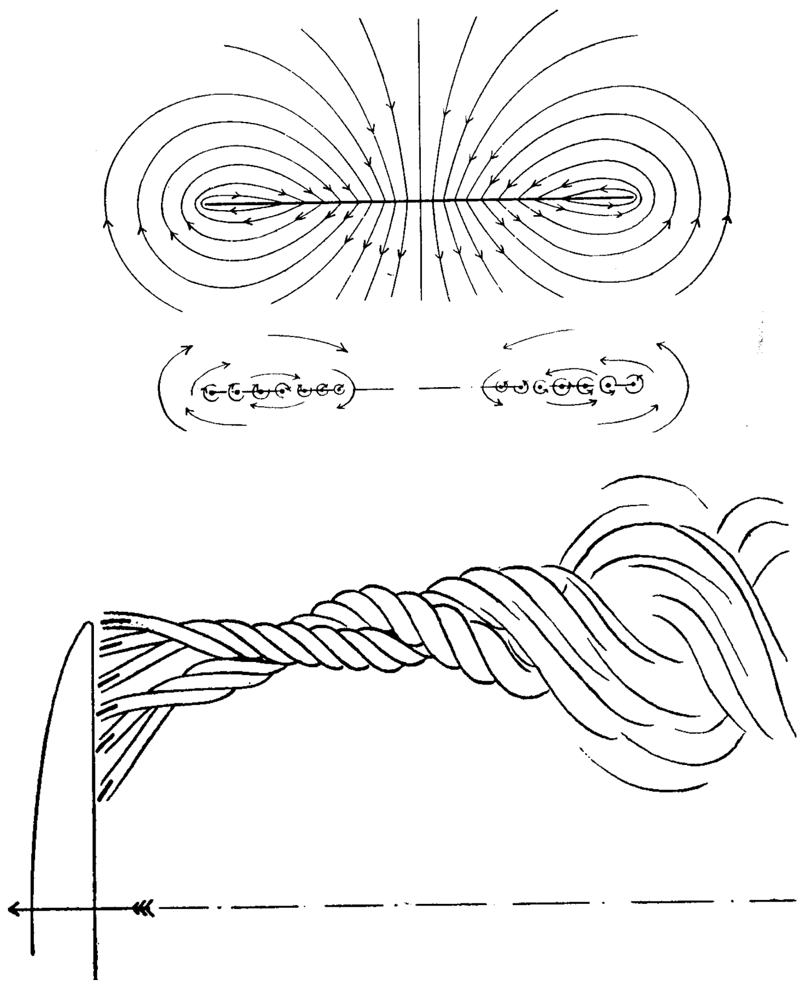


Figure 9: One of the early major applications of vortex dynamics was to the vortex system of a wing and the generation of lift. T hese illustrations of the streamlines and vortex formation behind a wing of finite span are from Lanchester's work, see [545].

The problem of N interacting point vortices on the unbounded xy-plane, with vortex $\alpha = 1, \dots, N$ having strength Γ_α (which is constant according to Helmholtz's theorems) and position (x_α, y_α), consists in solving the following system of $2N$ first-order, nonlinear, ordinary differential equations

$$\begin{aligned} \frac{dx_\alpha}{dt} &= -\frac{1}{2\pi}\sum_{\beta=1}^{N}{}' \, \Gamma_\beta \, \frac{y_\alpha - y_\beta}{l_{\alpha\beta}^2}, \\ \frac{dy_\alpha}{dt} &= \frac{1}{2\pi}\sum_{\beta=1}^{N}{}' \, \Gamma_\beta \, \frac{x_\alpha - x_\beta}{l_{\alpha\beta}^2}, \end{aligned} \tag{4}$$

where $\alpha = 1, 2, \dots, N$, $l_{\alpha\beta} = \sqrt{(x_\alpha - x_\beta)^2 + (y_\alpha - y_\beta)^2}$ is the distance between vortices α and β, and the prime on the summation indicates omission of the singular term $\beta = \alpha$. Typically, an initial value problem is addressed with the initial positions of the vortices and their strengths given so as to capture or model some flow situation of interest.

The system (4) can also be written as N ODEs for N complex coordinates $z_\alpha = x_\alpha + \mathrm{i}y_\alpha$

$$\frac{dz_\alpha^*}{dt} = \frac{1}{2\pi\mathrm{i}} \sum_{\beta=1}^{N}{}' \frac{\Gamma_\beta}{z_\alpha - z_\beta}\,, \qquad \alpha = 1, 2, \dots, N, \tag{5}$$

where the asterisk denotes complex conjugation.

In his lectures [491, Lecture 20] Kirchhoff demonstrated that the system (4) can be cast in Hamilton's canonical form[15]:

$$\Gamma_\alpha \frac{dx_\alpha}{dt} = \frac{\partial H}{\partial y_\alpha}\,, \qquad \Gamma_\alpha \frac{dy_\alpha}{dt} = -\frac{\partial H}{\partial x_\alpha}\,, \qquad \alpha = 1, 2, \dots, N, \tag{6}$$

where the Hamiltonian,

$$H = -\frac{1}{4\pi} \sum_{\alpha,\beta=1}^{N}{}' \, \Gamma_\alpha \Gamma_\beta \log l_{\alpha\beta}, \tag{7}$$

is conserved during the motion of the point vortices. (Here and in what follows $\log$ denotes the natural logarithm.)

In addition to H the Hamiltonian system (6) has three independent first integrals:

$$Q = \sum_{\alpha=1}^{N} \Gamma_\alpha x_\alpha\,, \quad P = \sum_{\alpha=1}^{N} \Gamma_\alpha y_\alpha\,, \quad I = \sum_{\alpha=1}^{N} \Gamma_\alpha (x_\alpha^2 + y_\alpha^2)\,. \tag{8}$$

Regardless of the values of the vortex strengths, the integrals H, I, and $P^2 + Q^2$ are *in involution*, that is, the Poisson bracket between any two of them is zero; see the review paper [27] or the monograph [690][16]. According to Liouville's theorem in analytical dynamics the Hamiltonian system (6) for $N = 3$ is then integrable regardless of the values of the vortex strengths. A terse general statement to this effect was included by Poincaré in his lectures [721, §77] during the second semester of 1891-1892 at the Sorbonne.

An extensive analytical study of integrability and of several special cases of three-vortex motion had already been performed by Gröbli[17] in his noteworthy 1877 Göttingen dissertation [365] (later also published as an extensive paper [366]) that must rightly be considered a classic of the vortex dynamics literature. The solution of the three-vortex problem and the dissertation itself were mentioned in footnotes by Kirchhoff in the third (1883) edition of his lectures [491, Lecture 20, §3] and in the fundamental treatise by Lamb [543, §155] (although in a way that does not fully reveal the comprehensive nature of Gröbli's investigations). Based on these cursory citations it is not difficult to understand that almost a century later Batchelor would write in his important text [46] that the details of motion of three point vortices "are not evident". A lengthy excerpt (in English translation) from Gröbli's dissertation is given in [30].

The Hamiltonian (7) depends only on the mutual distances $l_{\alpha\beta}$ between the vortices which suggests that one can write equations of motion that involve only these distances. Such equations

[15] A complete correspondence follows by setting the "generalized coordinates" $q_\alpha = x_\alpha$ and the "generalized momenta" $p_\alpha = \Gamma_\alpha y_\alpha$. This results in the remarkable insight that the "phase space" – in the sense of Hamiltonian dynamics – for a point vortex system is, in essence, its configuration space, a fact later exploited by Onsager in a seminal paper [701] on the statistical mechanics of a system of point vortices.

[16] This recent monograph also contains a very useful bibliography connecting vortex dynamics and dynamical systems theory.

[17] An account of the life, scientific achievements and tragic death of the Swiss scientist and mathematician Walter Gröbli (1852-1903) may be found in [30].

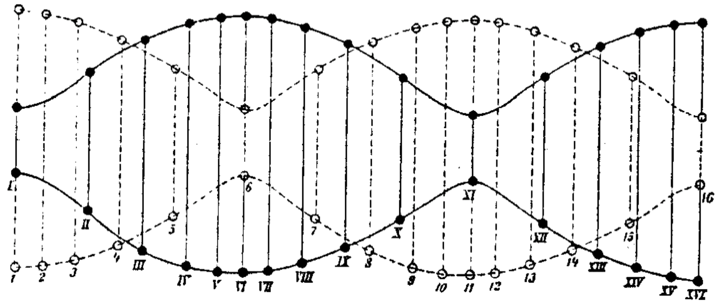


Figure 10: Illustration of “leapfrogging” by two vortex rings. The trajectories of the two rings have been calculated in detail. This is the analog of the vortex pair motion illustrated in Fig.8. From [34].

were obtained by Gröbli and later by Laura [553] who also expounded on the canonical formalism. They are

$$\frac{dl_{\alpha\beta}^2}{dt} = \frac{2}{\pi}\sum_{\lambda=1}^{N}{}''\Gamma_\lambda \epsilon_{\alpha\beta\lambda} A_{\alpha\beta\lambda}\left(\frac{1}{l_{\beta\lambda}^2} - \frac{1}{l_{\lambda\alpha}^2}\right), \qquad \alpha,\beta = 1,2,\dots,N, \tag{9}$$

where the two primes on the summation sign now mean that $\lambda \neq \alpha, \beta$. The quantity $\epsilon_{\alpha\beta\lambda} = +1$ if vortices α, β and λ appear counterclockwise in the plane, and $\epsilon_{\alpha\beta\lambda} = -1$ if they appear clockwise. Finally, $A_{\alpha\beta\lambda}$ is the area of the vortex triangle $\alpha\beta\lambda$ which can, in turn, be expressed in terms of the three vortex separations (the sides of the vortex triangle) by Hero's formula. Interestingly, Eqs.(9) were re-discovered independently at least twice: by Synge [892] in 1949 and by Novikov [697] in 1975. For N vortices one has $\frac{1}{2}N(N-1)$ quantities $l_{\alpha\beta}$ and, thus, $\frac{1}{2}N(N-1)$ equations of the form (9). However, only $2N-3$ of these are independent.

It can be shown that

$$\frac{1}{2}\sum_{\alpha,\beta=1}^{N}\Gamma_\alpha\Gamma_\beta l_{\alpha\beta}^2 = \left(\sum_{\alpha=1}^{N}\Gamma_\alpha\right) I - P^2 - Q^2. \tag{10}$$

The equations (9), then, have two general first integrals, *viz* the Hamiltonian (7) and the quantity on the left hand side of (10). Using these two integrals the three ODEs for l_{12}, l_{23} and l_{31} may be reduced to a single ODE that can be solved by quadrature, and this was, in essence, the solution method outlined by Gröbli in his dissertation [365, 366]. The case $N = 3$ thus appears as a critical one since for more vortices additional “scales of motion” appear without any obvious integrals to constrain them. One may, therefore, expect the problem to become non-integrable. Indeed, this is what happens and the connection to the recent interest in the emergence of chaos in nonlinear dynamics is established. The appearance of chaos in point vortex dynamics as one goes from three to four vortices is analogous to the appearance of chaos in the gravitational N-body problem of celestial mechanics as one goes from two to three bodies. For the case of point masses the appearance of chaos or the absence of integrability became part of the legacy of Poincaré. For inexplicable reasons the analogous discussion for point vortices had to wait for more than a century after the solution of the three-vortex problem. Both Gröbli [365, 366] and later Laura [552, 553] outlined how to determine the “absolute motion” of the vortices provided the solution for the “relative motion” as given by equations (9) was already known.

Over the years the equations for point vortex motion on the unbounded plane have been extensively analyzed in most textbooks and monographs on fluid dynamics, e.g., [23, 45, 148, 306, 455, 459, 504, 542, 543, 655–657, 742, 808] just to mention a few. Several dissertations, e.g., [67, 352, 365, 752, 976, 1012], also addressed various problems of integrable motion of a small number of point vortices.

The nature of the motion of two vortices had already been outlined by Helmholtz [393]. The motion of three vortices – both the relative and the absolute motion – with various intensities and initial conditions was extensively analyzed by Gröbli [365, 366]. The relative motion of three arbitrary vortices, based upon Eqs.(9), was studied and classified by Synge [892] by introducing triangular coordinates in a "phase space" of the three distances between the vortices. Gröbli had actually found such a representation for the case of three identical vortices, and this construction was found independently a century later by Novikov [697]. Synge's comprehensive analysis was re-discovered independently in [26]. Thus, today the three-vortex problem may be considered to have a rather complete solution. Gröbli [365, 366] also discovered an unusual case where the three vortices converge on a point in a *finite time*. Except for Synge's study [892], which was itself overlooked, this intriguing case of *vortex collapse* also went unnoticed for a century. It is admittedly a somewhat special case requiring both that the harmonic mean of the three vortex strengths be zero and that the integral of motion (10) vanish.

The problem of stability of uniformly rotating configurations of three vortices with general values of the strengths was addressed by Morton [672] and Sona [867–869]. In our opinion, the papers of the Italian school from the 1920's and 1930's on point vortices by Agostinelli [4, 6, 7], Caldonazzo [111–118], Laura [554–556], and Masotti [615–628] deserve to be better known. A "revival" of this large and poorly cited body of work was started in the German dissertation by Wehner [1012], possibly with political motivations considering the time at which it was written.

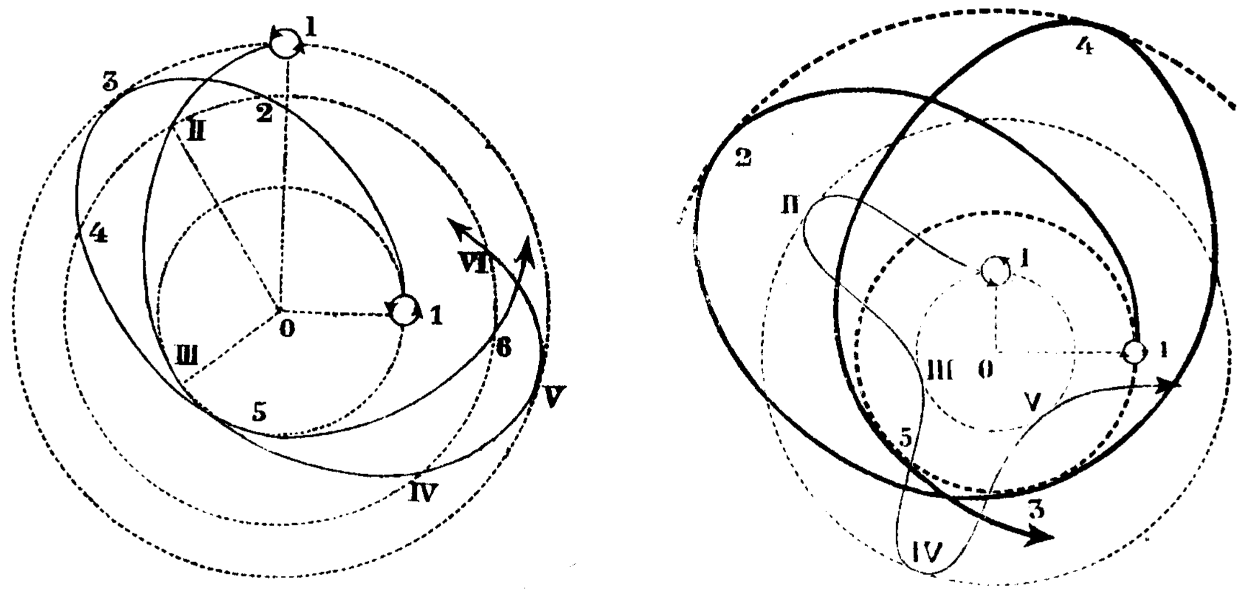
The integrable problem of four vortices arranged as two coaxial pairs has been addressed in many papers. Gröbli [365, 366] investigated the case of "leapfrogging" when all vortices have the same absolute strength, and obtained an analytical representation for the vortex trajectories, cf. Fig.8. His analysis was repeated independently by Love [598] and Hicks [416].

Centrally symmetric configurations of vortices either of equal strengths or of two vortex pairs were studied in detail by Goriachev [351, 352], cf. Fig.11. The stability problem for a certain configuration of four point vortices that rotates like a rigid body was addressed in [253][18].

The case of uniform rotation of a regular polygon of N vortices was addressed in the Adams Prize essay of J. J. Thomson [923]. He proved that the regular N-gon is stable to infinitesimal perturbations for $N = 2, 3, 4, 5, 6$ but becomes unstable for $N > 7$. (For $N = 7$ the polygon is marginally stable to linear order and one must go to the next order to decide the stability issue.) This study was extended by Havelock [388], Laura [554] and Morton [673, 674] and the problem continues to be addressed in the literature in various forms.

Helmholtz was also the first to address problems of point vortices interacting with rigid boundaries [393]. As we have seen, he considered the case of a point vortex in the space bounded by a plane wall. Using an "image" vortex of opposite strength situated at the reflection of the original vortex in the plane boundary he reduced the problem to that of the motion of a vortex pair on the unbounded plane. This use of the "method of images" has since been widely employed in various problems of the motion of a single point vortex in various bounded domains: in a wedge with an opening angle less than $180°$ [359, 365, 366, 440], in a parallel channel [498, 679], in a triangle [596], in a rectangle [17, 359, 679], within and outside a circular cylinder [359, 501]; see also [404, 595] for review. Many of these problems appeared either as examples or exercises in textbooks by Basset [44], Lamb [540, 542, 543], Milne-Thomson [655, 656], Kochin and co-

[18] Note the review of this paper in *Appl. Mech. Rev.* written by a then very young M. J. Lighthill.



Черт. 8. Черт. 11.

Figure 11: Results on 4- and 5-vortex motion from Goriachev's 1898 doctoral thesis [352] at Moscow University done under Joukovskii's supervision. The figures show motion of vortices with center of symmetry. Thus, only two trajectories need to be shown. For five vortices, one vortex is at the origin and there are two pairs with a center of symmetry.

authors [504–506], Villat [1001, 1002], and others. Symmetric motion of a vortex pair (two point vortices of opposite strengths) inside and outside a circular cylinder was first studied in [359]. A particular case of an equilibrium of a vortex pair behind a cylinder in a uniform potential flow is known as the "Föppl problem" after the seminal paper [279]. This problem is also frequently discussed in textbooks [504–506, 543, 655, 656, 1001, 1002].

The general case of the motion of point vortices in an arbitrary domain was studied by Routh [810] using the theory of conformal mappings. The velocity of a point vortex in the transformed plane is not equal to the velocity obtained by simple substitution of the conformal mapping into the expression for the velocity in the original plane – one requires also the influence of the images which is captured by the so-called "Routh correction". Later this approach was extended by Joukovskii [440], Lagally [533] and Caldonazzo [111, 116]. A complete mathematical theory was developed by Lin [583–585] who showed that the problem is always Hamiltonian with a Hamiltonian function that is a hybrid of Kirchhoff's Hamiltonian (7) for the unbounded plane and the Hamiltonian that Routh found for motion of a single vortex in a bounded domain [810]. This technique has since been applied by several authors [43, 375, 440, 663–666, 709, 975] to study the problem of motion of a point vortex outside and near a sharp wedge. In order to prevent infinite velocity at the apex of the wedge (which occurs for inviscid potential flow) the authors introduce certain conditions on the flow velocity at infinity, some of which seem quite artificial and are, in some sense, equivalent to introducing a time-dependent circulation for the vortex. An improved theory based upon vortex shedding from the sharp edge was developed in [101].

The paper [440] by Joukovskii is considered [877–881] to be one origin of the *Kutta-Joukovskii* *condition* on flow at the trailing edge of a wing[19]. The development of the Kutta [527–529]-Joukovskii [441, 442, 444–447]-Lanchester [545–547] theory for lift on a wing and what is today usually called the Kutta-Joukovskii condition at the trailing edge of an airfoil has a long and colorful history with many contributors. An important early contribution was made by B. Robins in 1771, well before the period covered by this bibliography. The term *Magnus force* for the side force experienced by a spinning body refers to work by G. Magnus [610] published in 1853; see also [10, 49, 737] for details. Lord Rayleigh [758] codified much of this early work and provided a derivation of a formula for the lift force, later corrected slightly by Greenhill [360]. Important contributions were also made by Blasius [93] and Chaplygin [130]. This pre-history is extensively reviewed in [294, 863] and also by Khristianovich [488] in a paper that was later criticized by Stepanov [878] who took exception to Khristianovich's proposal that the condition of smooth flow at the trailing edge of an airfoil not be named for Joukovskii!

A classical paper by Joukovskii [442] concerns the important problem of equilibrium of a symmetrical vortex pair behind a finite thin plate placed normally to a flow that is uniform at infinity. It is clearly shown that obtaining stationary positions of the two vortices and at the same time achieving finite velocities of the flow at the sharp edges of the plate (i.e., satisfying the Kutta-Joukovskii condition) is impossible. Interestingly, this key problem has since been reconsidered many times: some authors [102, 781] claimed the existence of solutions (and even stated finding these as an exercise [50, p.254]), others [381, 998, 1001] flatly ruled out the possibility of a solution. The discussion was re-visited in the 1970's [164, 812, 865].

Various practical problems of point vortex interaction with rigid obstacles and airfoils were studied in the main aviation centers of the USSR (TsAGI and VVA imeni Zhukovskogo) by the famous Russian scientists Sergei Alekseevich Chaplygin (1869-1941)[20] [130–132, 134–137] and Vladimir Vasil'evich Golubev (1884-1954)[21] [308–310, 313–318, 320–323, 344, 345], and in a few of their joint publications [138–140]. Being written mainly in Russian (sometimes with a short English summary and/or a German abstract) these papers remain almost unknown in the West.

W. Thomson [935] was the first to show that a vortex pair in steady motion on the unbounded plane is accompanied by an "atmosphere", i.e., a fixed, closed volume (area) of fluid particles that move forward with the vortex pair. The bounding curve of this "atmosphere" is today sometimes called the "Kelvin oval". Figure 12 reproduces the original drawing from [935] where we find this description:

> The diagram represents precisely the convex outline referred to, and the lines of motion of the interior fluid carried along by the vortex, for the case of a double vortex consisting of two infinitely long, parallel, straight vortices of equal rotations in opposite directions. The curves have been drawn by Mr. D. McFarlane, from calculations which he has performed by means of the equation of the system of curves, which is
>
> $$\frac{y^2}{a^2} = \frac{2x}{a}\,\frac{N+1}{N-1} - \left(1 + \frac{x^2}{a^2}\right), \text{where } \log N = \frac{x+b}{a}.$$
>
> The motion of the surrounding fluid must be precisely the same as it would be if the space within this surface were occupied by a smooth solid.

[19]Besides these papers a detailed overview of the voluminous contributions made by Nikolai Egorovich Joukovskii (1847-1921) – alternatively spelled Zhukovskii, Joukowski, Joukovsky, or Žukovskij – in the fields of hydromechanics, aerodynamics, general mechanics, etc., may be found in the books [221, 324, 516, 561] and in the review articles [294, 329, 330, 332, 334, 342, 363, 464, 513, 989]. A detailed bibliography of his works with short comments is collected in [845].

[20]A detailed overview of the life and work of Chaplygin (Tschaplygin, Tchapliguine) in the fields of hydromechanics and aerodynamics may be found in the book [331] (in Russian) and in the review articles [294, 325, 336, 340, 347, 362, 653, 873, 877–879, 990, 991]. A detailed bibliography of his work with short comments is collected in [846].

[21]A detailed overview of the life and work of Golubev in the field of aerodynamics may be found in the books [749, 974] (in Russian), in an interesting autobiography [747], a lecture [346], and in review articles [53, 514, 515, 517, 748].

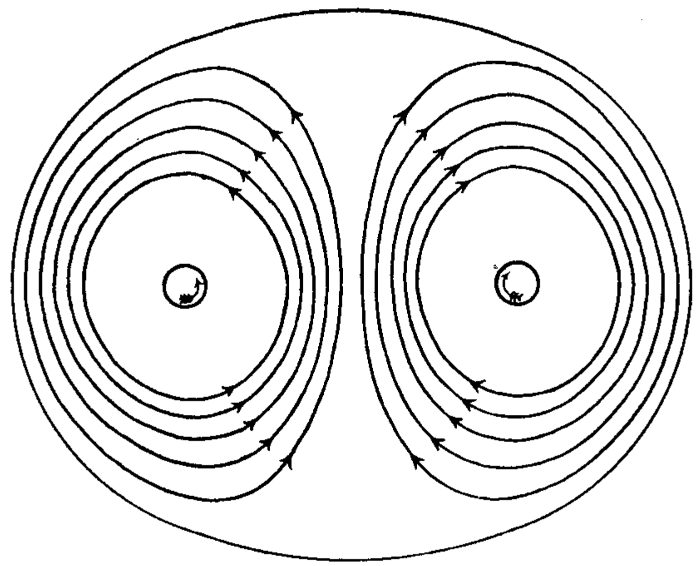


Figure 12: The "atmosphere" traveling with a vortex pair. From [935].

Hicks [415] generalized the result and proved that when a configuration of point vortices moves steadily through otherwise irrotational fluid, it is possible to distinguish three definite regions of fluid motion: (i) a region of rotational motion (the point vortices), which conserves its identity; (ii) a region of irrotational cyclic motion surrounding the first, which also retains its identity and volume and travels uniformly through the fluid with an undeformed boundary; and (iii) a region of irrotational acyclic motion, outside the second region. The fluid in this region remains at rest at infinity and is never displaced over more than a small distance.

Each passive fluid particle may be considered "a point vortex of zero strength", and the equations of motion for all particles advected by the translating vortex system are integrable. The deformation of a line of fluid connecting two vortices within the moving body was studied analytically by Riecke [788]; see [654] for additional illustrations.

2.3 *Vortex atoms* In the 1860's William Thomson, later Lord Kelvin[22] became very interested in vortex dynamics since he was convinced that atoms were to be modeled as vortex configurations in the aether. According to [864, p.417] no evidence exists that Thomson knew of Helmholtz's paper [393] prior to early 1862 when Tait mentioned it to him[23]. Tait made a complete English translation [394] of Helmholtz's paper [393] for his own use. He also devised some extremely clever experiments to illustrate the vortex theory using smoke vortex rings in air. Following completion of their famous *Treatise on Natural Philosophy*, referred to simply as "Thomson and Tait", and the successful laying of the Atlantic cable in 1866 (for which Thomson was knighted and became Sir William Thomson), Thomson visited Tait in Edinburgh in mid-January 1867 and saw the

[22]For a popular survey of Kelvin's long and adventuresome life (1824-1907) we may direct the reader to several books [273, 356, 586, 852, 864, 920, 1028] written at various times, to the obituary notice [919], to the century jubilee notes [245, 702] (the second in a rather unusual journal), and to one among many encyclopedia articles [104]. Kelvin was one of the greatest scientists of the 19th century – his grave is in Westminster Abbey, next to Newton's – and is credited with having foreseen in his lecture at the Royal Institution on April 27, 1890, [486] two major discoveries in theoretical physics of the 20th century, *viz* the special theory of relativity and quantum mechanics.

[23]Helmholtz and Thomson had met for the first time in 1855. In a letter of August 30, 1859, Helmholtz reported to Thomson that he had been working on hydrodynamic equations including friction, but did not mention his *Wirbelbewegungen* study.

Figure 13: Illustrations to accompany Kelvin's letter to Helmholtz of 22nd January, 1867 [920].

smoke rings with his own eyes. On 22nd January he wrote to Helmholtz [920, pp. 513-515]:

> ... Just now, however, *Wirbelbewegungen* have displaced everything else, since a few days ago Tait showed me in Edinburgh a magnificent way of producing them. Take one side (or the lid) off a box (any old packing-box will serve) and cut a large hole in the opposite side. Stop the open side AB loosely with a piece of cloth, and strike the middle of the cloth with your hand. If you leave anything smoking in the box, you will see a magnificent ring shot out by every blow. A piece of burning phosphorus gives very good smoke for the purpose; but I think nitric acid with pieces of zinc thrown into it, in the bottom of the box, and cloth wet with ammonia, or a large open dish of ammonia beside it, will answer better. The nitrite of ammonia makes fine white clouds in the air, which, I think, will be less pungent and disagreeable than the smoke from the phosphorus. We sometimes can make one ring shoot through another, illustrating perfectly your description; when one ring passes near another, each is much disturbed, and is seen to be in a state of violent vibration for a few seconds, till it settles again into its circular form. The accuracy of the circular form of the whole ring, and the fineness and roundness of the section, are beautifully seen. If you try it, you will easily make rings of a foot in diameter and an inch or so in section, and be able to follow them and see the constituent rotary motion. The vibrations make a beautiful subject for mathematical work. The solution for the longitudinal vibration of a straight vortex column comes out easily enough. The absolute permanence of the rotation, and the unchangeable relation you have proved between it and the portion of the fluid once acquiring such motion in a perfect fluid, shows that if there is a perfect fluid all through space, constituting the substance of all matter, a vortex-ring would be as permanent as the solid hard atoms assumed by Lucretius and his followers (and predecessors) to account for the permanent properties of bodies (as gold, lead, etc.) and the differences of their characters. Thus, if two vortex-rings were once created in a perfect fluid, passing through one another like links of a chain, they never could come into collision, or break one another, they would form an indestructible atom; every variety of combinations might exist.
>
> Thus a long chain of vortex-rings, or three rings, each running through each of the other, would give each very characteristic reactions upon other such kinetic atoms. I am, as yet, a good deal puzzled as to what two vortex-rings through one another would do (how each would move, and how its shape would be influenced by the other). By experiment I find that a single vortex-ring is immediately broken up and destroyed in air by enclosing it in a ring made by one's fingers and cutting it through. But a single finger held before it as it approaches very often does not cut it and break it up, but merely causes an indentation as it passes the obstacle, and a few vibrations after it is clear.

Tait's translation of Helmholtz's paper was published that same year in *Philosophical Magazine* [394]. One must imagine that Kelvin encouraged his friend and colleague to prepare this translation for publication.

Thomson's prodigious talent produced several first rate studies of vortex dynamics which, although ultimately wrong-headed in terms of atomic physics, have had a lasting influence on fluid dynamics. The idea of *circulation*, for example, is from this period. The circulation is defined as the contour integral of the projection of the flow velocity on the tangent to the contour,

$$\Gamma = \oint_C \mathbf{V} \cdot d\mathbf{s}. \tag{11}$$

He showed that for any material contour moving according to Euler's equation for incompressible flow, the circulation is an integral of the motion, a result known today as *Kelvin's circulation theorem*[24]. This profound insight has continued to exert an influence on the entire field of fluid mechanics, including in such areas as the assessment of the accuracy of numerical methods and in turbulence modeling. Circulation is a distinctly topological entity, independent of the shape of the vortex and measurable by integration along any circuit that loops around the vortex. In this sense, the notion of circulation may be taken as one of the earliest introductions of topological considerations into fluid mechanics[25]. For a broader discussion see [248]. Today the intersection of fluid mechanics and topology, in its multiple forms, has matured into a subfield often referred to as topological fluid dynamics. The permanence of circulation in an ideal fluid was one of the cornerstones of vortex atom theory. Like atoms, vortices in the aether could neither be created nor destroyed.

Thomson's fascination with the floating magnet experiments by Mayer [643–648], and his role in the re-publication of these works in journals such as *Nature* and *Philosophical Magazine*, were also outgrowths of his conviction that vortices and atoms are intimately related. See Snelders' article [861] for a comprehensive historical review of this topic.

It is interesting to trace Thomson's presentations at the meetings of the *Royal Society of Edinburgh* and their subsequent publication:

1. "On vortex atoms" (read 18th February 1867 – with an abstract in the newspaper the *Scotsman* on February 19, 1867!); published as [935].
2. "On vortex motion" (read 29th April 1867); published as [937] (abstract in *Proc. R. Soc. Edinb.* **7**, 576–577).
3. "On vortex motion" (read 18th December 1871).
4. "On vortex motion" (read 3rd March 1873); title in *Proc. R. Soc. Edinb.* **8**, 80.
5. "On the oscillation of a system of bodies with rotating portions. Part II. Vibrations of a stretched string of gyrostats (dynamics of Faraday's magneto-optic discovery) with experimental illustrations" (read 19th April 1875); title in *Proc. R. Soc. Edinb.* **8**, 521.
6. "Vortex statics" (read 20th December 1875); published as [938].
7. "On two-dimensional motion of mutually influencing vortex-columns, and on two-dimensional approximately circular motion" (read 3rd January 1876); title in *Proc. R. Soc. Edinb.* **9**, 98.
8. "On the vortex theory of gases, of the condensation of gases, and of the continuity between the gaseous and liquid state of matter" (read 3rd April 1876); title in *Proc. R. Soc. Edinb.* **9**, 144.
9. "On vortex vibrations, and on instability of vortex motion" (read 15th April 1878); title in *Proc. R. Soc. Edinb.* **9**, 613.
10. "A mechanical illustration of the vibrations of a triad of columnar vortices" (read 20th May 1878); title in *Proc. R. Soc. Edinb.* **9**, 660.

[24] This theorem was considered by Einstein [245] among the most important scientific results of W. Thomson (Lord Kelvin).

[25] Tait's seminal work on the classification of knots on closed curves is a spin-off of his interest in vortex atoms. It has stood the test of time and is today recognized as an important contribution to topology, knot theory and graph theory. Maxwell was an important catalyst for Tait's work on knots, since he had also become interested in topological ideas. There is a letter from Maxwell to Tait of 13 November 1867 [638] in which he points out the importance of linking of vortices, see [502].

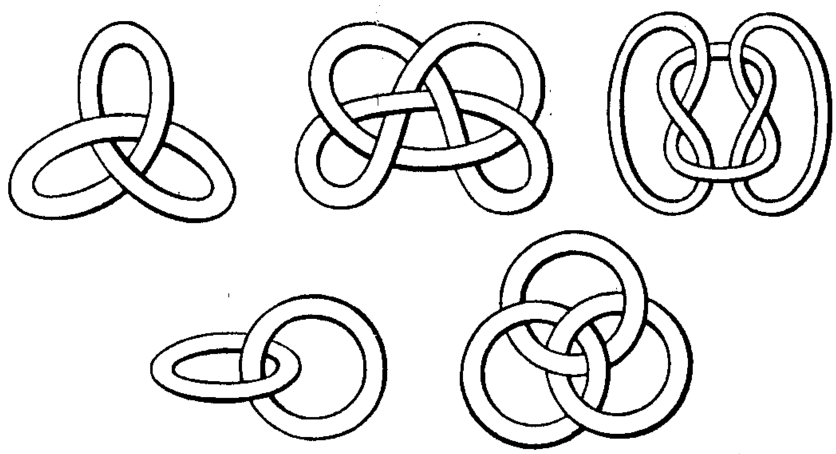


Figure 14: Tait's drawings, reproduced in [937], that capture Thomson's ideas on how atoms arise as vortex structures in the aether.

11. "On vortex motion: Gravitational oscillations in rotating water" (read 17th March 1879).
12. "Vibrations of a columnar vortex" (read 1st March 1880); published as [940].
13. "On vortex sponge" (read 21st February 1881); title in *Proc. R. Soc. Edinb.* **11**, 135.
14. "On the average pressure due to impulse of vortex rings on a solid" (read 18th April 1881[26]); title in *Proc. R. Soc. Edinb.* **11**, 204; published as [944].
15. "On energy in vortex motion" (read 16th February 1885); title in *Proc. R. Soc. Edinb.* **13**, 114.
16. "On the instability in fluid motion" (read 18th April 1887); title in *Proc. R. Soc. Edinb.* **14**, 194.
17. "On a mechanism for the constitution of Ether; illustrated by a model" (read 17th March 1890).

The famous quote from Thomson that "Helmholtz's [vortex] rings are the only true atoms" summarizes the theme of this research thrust. Figure 14 depicts the kind of things he envisioned.

Although it ultimately faded, the vortex atom idea maintained itself for many years and through Kelvin's boundless energy and great influence spread widely in the scientific community. Schuster [840, p.34] recounts that Kirchhoff, who otherwise was not easily excited about physical theories, once told him: "I like it... because it excludes everything else," to which he added with a sigh:"If only it could explain gravitation." The work by J. J. Thomson, discoverer of the electron, on vortex dynamics in [922–932] and [934] was stimulated by vortex atom theory. Even in his great paper of 1897 entitled "Cathode Rays", in which the discovery of the electron is announced, we find these remarks: "If we regard the system of magnets as a model of an atom, the number of magnets being proportional to the atomic weight, ... we should have something quite analogous to the periodic law...", where by "periodic law" he means the periodic table of the elements. The reference to the floating magnets is to Mayer's experiments mentioned above. We see what a profound role these demonstration experiments played in the thinking of these great scientists. We should not forget that at the time analog experiments were one of the only ways of exploring solutions to nonlinear

[26]Helmholtz was the second speaker at this meeting according to [248].

equations that did not easily yield to analytical methods. Computers and numerical solutions were still many years in the future.

The vortex atom theory received considerable attention from Maxwell [642] in his article on the atom written for the 1878 edition of *Encyclopedia Britannica*. He provided a detailed description of properties of vortices in ideal fluid and strongly supported the idea of vortex atoms. Apparently, he was reminded of his own earlier articles [633–636] in which his celebrated electromagnetic theory was initially formulated based upon a mechanical model that also made reference to Helmholtz's paper [393]. Here the energy of a magnetic system was represented by the rotation of an inviscid fluid about the lines of magnetic force, each unit tube of force being depicted as a circular Rankine vortex [755]. Maxwell concluded:

> His [Thomson's] primitive fluid has no other properties than inertia, invariable density, and perfect mobility, and the method by which the motion of this fluid is to be traced is pure mathematical analysis. The difficulties of this method are enormous, but the glory of surmounting them would be unique.

As one might expect, Boltzmann thought deeply about molecular vortices. In the translations of Maxwell's papers into German, that Boltzmann produced and annotated, one finds extensive, penetrating comments on two-dimensional vortices elucidated in terms of pressure, velocity, energy, and so on, all very enlightening of the physical properties of this model. These translations and comments are mentioned in the bibliography in conjunction with [633–636].

Accounts of the vortex atom theory may be found in the books [220, pp. 28-33], [229, pp.169-176], [896, pp.294-301], [1022, pp.290-295], and in the papers [855, 861, 1027]. See also [519] for a recent account.

Much recent work has transpired on the problem of *Vortex Statics*, as Kelvin called it (see item 5 above and [938]). The reader may be interested in a recent review in this series [29]. The term "vortex crystals" appears to be gaining favor in current research. The terminology of celestial mechanics would suggest the term "relative equilibria" for the states being investigated.

We conclude this brief description of vortex atoms by citing the following poem which appeared in *Nature* **26** (1882) 297:

THE LAY OF THE LAST VORTEX-ATOM
(*Vide* THE UNSEEN UNIVERSE)
Melody — *Lorelei*

The Vortex-Atom was dying
The last of his shivering race —
With lessening energy flying
Through the vanishing realms of Space.

No more could he measure his fleeting —
No milestones to mark out his way;
But he knew by his evident heating
His motion was prone to decay.

So he stayed in his drift rectilinear
For Time had nigh ceased to exist,
And his motion grew ever less spinnier
Till he scattered in infinite mist.

But as his last knot was dissolving
Into the absolute nought —
“No more,” so sighed he resolving,
“Shall I as atom be caught.

“I’ve capered and whirled for ages,
“I’ve danced to the music of spheres,
“I’ve puzzled the brains of the sages —
“Whose lives were but reckoned by years.

“They thought that my days were unending,
“But sadly mistaken were they;
“For, alas! my ‘life-force’ is expending
“In asymptotic decay!”

Edinburgh University K.

2.5 *Vortex rings* In spite of the great popularity of Tait’s [896, pp. 291-294] smoke box for generating vortex rings in air, the first observation of vortex rings probably corresponds with the introduction of smoking tobacco! Northrup[27] [695, p.211] writes:

> It is not improbable that the first observer of vortex motions was Sir Walter Raleigh; if popular tradition may be credited regarding his use of tobacco, and probably few smokers since his day have failed to observe the curiously persistent forms of white rings of tobacco smoke which they delight to make. But some two hundred eighty years went by, after the romantic days of Raleigh and Sir Francis Drake, who made tobacco popular in England, before a scientific explanation of smoke rings was attempted.

By curious coincidence the first experimental observations of the generation of vortex rings in air were performed by Rogers[28] [789] in the same year (1858) that Helmholtz published his seminal paper [393], see Fig.1.

Later Ball [38–40] provided a detailed description of the experimental set-up and gave some data concerning the uniform velocity of propagation of single vortex rings of various sizes. Using the smoke-box apparatus Dolbear carefully repeated all Tait’s experiments and produced [220, pp.28-31] a long list of characteristics of smoke vortex rings:

1. It retains its ring form and the same material rotating as it starts with.
2. It can travel through the air easily twenty or thirty feet in a second without disruption.
3. Its line of motion when free is always at right angles to the plane of the ring.
4. It will not stand still unless compelled by some object. If stopped in the air, it will start up itself to travel on without external help.
5. It possesses momentum and energy like a solid body.

[27] Edwin Fitch Northrup (1866-1940) was a professor of physics at Princeton and author of a science fiction book entitled “Zero to Eighty: Being my Lifetime Doings, Reflections, and Inventions; also my Journey around the Moon.” The book was published in Princeton in 1937 under the pseudonym Akkad Pseudoman. It gives a fictional account, supported by valid scientific data, of a Morris County resident’s trip around the moon. It appears to have a sustained following in the world of science fiction.

[28] William Barton Rogers (1804-1882) will be better known today as the founder of MIT. Indeed, he was heavily engaged in this enterprise at about the time his paper on vortex rings was written.

6. It is capable of vibrating like an elastic body, making a definite number of such vibrations per second, – the degree of elasticity depending upon the rate of vibration. The swifter the rotation, the more rigid and elastic it is.

7. It is capable of spinning on its own axis, and thus having rotary energy as well as translatory and vibratory.

8. It repels light bodies in front of it, and attracts into itself light bodies in its rear.

9. If projected along parallel with the top of a long table, it will fall upon it every time, as a stone thrown horizontally will fall to the ground.

10. If two rings of the same size be traveling in the same line, and the rear one overtakes the other, the front one will enlarge its diameter, while the rear one will contract its own till it can go through the forward one, when each will recover its original diameter, and continue on in the same direction, but vibrating, expanding, and contracting its diameter with regularity.

11. If two rings be moving in the same line, but in opposite directions, they will repel each other when near, and thus retard their speed. If one goes through the other, as in the former case, it may quite lose its velocity, and come to a standstill in the air till the other has moved on to a distance, when it will start up its former direction.

12. If two rings be formed side by side, they will instantly collide at their edges, showing strong attraction.

13. If the collision does not destroy them, they may either break apart at the junction of the collision, and then weld together into a single ring, with twice the diameter, and then move on as if a single ring had been formed, or they may simply bound away from each other; in which case they always rebound *in a plane at right angles* to the plane of collision. That is, if they collided on their sides they would rebound so that one went up and the other down.

14. Three may in like manner collide, and fuse into a single ring.

Experiments in water on the generation and interaction of vortex rings were started independently by Rogers [789] and Reusch [768] and then reported in a long series of papers including [41, 42, 52, 84, 97, 170, 196–198, 384, 433, 518, 521, 544, 608, 609, 698, 699, 704, 750, 751, 769–772, 776, 790, 888, 901, 904, 934, 953, 954, 962, 963, 992, 1011, 1019, 1020, 1032]. The extensive study by Northrup [695,696] should also be mentioned here. It contains a very detailed description of a "vortex gun", including all the parameters, together with beautiful photos of interacting vortex rings and vortex rings interacting with rigid obstacles, e.g., with a small watch chain. The modern reader may be intrigued to see in these near-century old papers an essentially contemporary elucidation of the interaction of two circular vortex rings tilted towards one another so as to interact after having propagated for some distance, cf. Fig.15.

Theoretical studies of the motion of a circular vortex ring of closed toroidal shape with core radius a and radius R of the center line of the torus, where $a \ll R$, in an ideal fluid led to a formula for the self-induced translational velocity V_{ring}, directed normally to the plane of the ring:

$$V_{ring} = \frac{\Gamma}{4\pi R}\left(\log\frac{8R}{a} - C\right) + O(a/R)$$

Here Γ is the (constant) intensity of the vortex ring, equal to the circulation along any closed path around the vortex core, and C is a constant. There was some disagreement in the literature concerning the value of C. The value $C = \frac{1}{4}$ was given (without proof) by W. Thomson [936] and later by Hicks [407], Basset [44], Dyson [238] and Gray [358]. This corresponds to the case where the vorticity inside the core varies directly as the distance from the centerline of the ring. The value $C = 1$ was given by Lewis [577], J. J. Thomson [923], Chree [142], Joukovskii [443], and Lichtenstein [579] for a uniform distribution of vorticity within the core. Detailed discussion

FIG. 14.

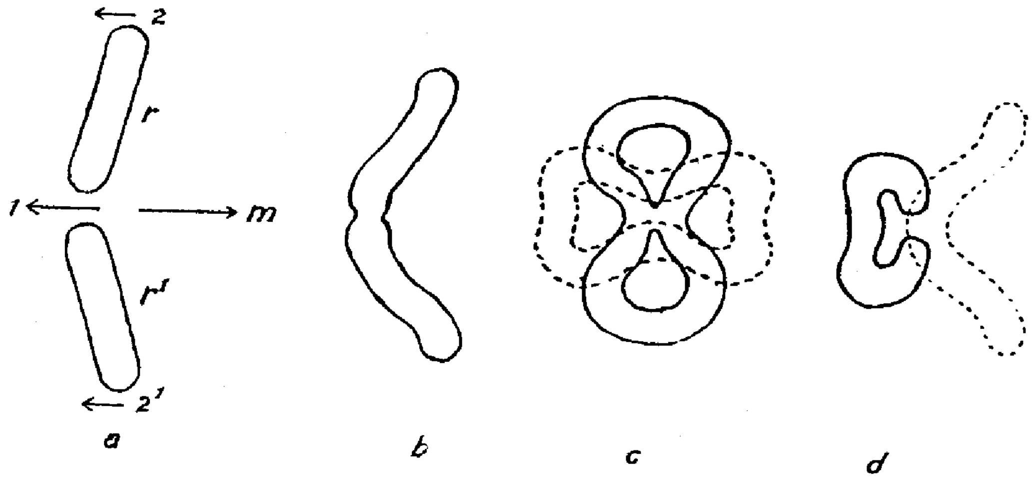


Figure 15: Sketch of interaction of two identical vortex rings launched on a collision course. From Northrup [696].

of both these hypotheses and also the circular form of the core is given in [543, 579, 847] and later repeated by Fraenkel [281]. For a hollow vortex core, or if one assumes the fluid inside the core is stagnant, the value $C = \frac{1}{2}$ results [719].

The reader may wish to consult the more recent review by Shariff and Leonard [851] on vortex ring dynamics for the further evolution of this intriguing subject.

2.5 *Vortex streets* Most students of fluid mechanics know that the common staggered array of vortices that forms in the wake of a cylinder (or any bluff body) is called the *Kármán vortex street.* The concept of the vortex street is among the best known in all of fluid mechanics, in the same "league" as *Reynolds number*, *Bernoulli's equation* and the concept of the *boundary layer*. The formation and structure of vortex wakes downstream of bluff bodies had been studied extensively in experiments (see Fig.16 for an example) going back to Leonardo da Vinci [292, 382, 383, 562] but von Kármán's theory was the first real analysis of the phenomenon. In his charming book [470] he explains that his interest was aroused by an early picture of such vortices in a fresco in one of the churches in Bologna, Italy, where St. Christopher is shown carrying the child Jesus across a flowing stream. Alternating vortices are seen behind the saint's foot; see [667] for a beautiful color picture of this fresco at the Church of St Dominic, entitled *Madonna con bambino tra I Santi Domenico, Pietro Martire e Critoforo*, painted by an unknown artist of the fourteenth century.

Alternating vortices in air were observed and imaged by the English scientist Mallock [612, 613] (cf. Fig.16) while impressive photos of such vortices in water were obtained by the German scientist Ahlborn [8]. The French scientist Bénard [56] also observed the alternating formation of detached vortices on the two sides of a bluff obstacle in water and later in many viscous fluids and in colloidal solutions:

> Pour une vitesse suffisante, au-dessous de laquelle il n'y a pas de tourbillons (cette vitesse limite croît avec la viscosité et decroît quand l'épaisseur transversale des obstacles augmente), *les tourbillons produits périodiquement se détachent alternativement à droite et à gauche du remous d'arrière qui suit le solide; ils gagnent presque immédiatement leur emplacement définitif, de sorte qu'à l'arrière de l'obstacle se forme une double rangeée alternée d'entonnoirs stationnaires, ceux de droite dextrogyres, ceux de gauche*

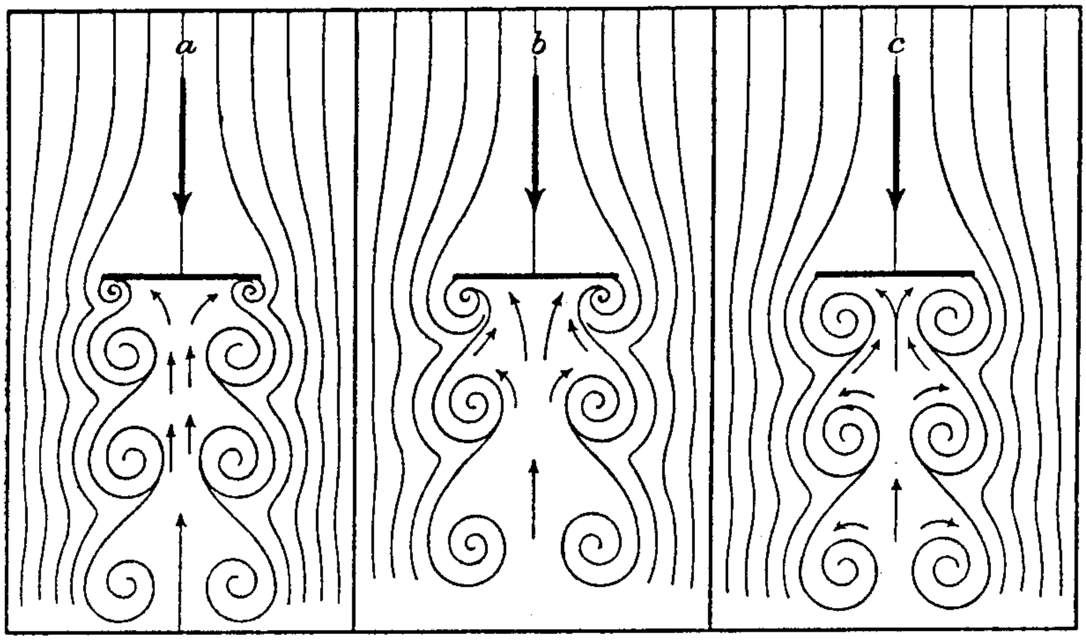


Figure 16: Sketch of vortex wake formation downstream of a flat plate. After Mallock [612].

lévogyres, séparés par des intervalles égaux.

Analysis shows that only two such configurations will propagate in the streamwise direction: The vortices must either be arranged in a symmetric or in a staggered configuration. Numerically the intensities of the vortices, Γ, are all equal, but the vortices on the two horizontal rows have opposite signs. In remarkable theoretical investigations [467,468] von Kármán examined the question of stability of such processions in unbounded, incompressible, inviscid, two-dimensional flow with embedded point vortices[29]. He became interested in this problem when he was appointed as a graduate assistant in Göttingen in Prandtl's laboratory in 1911. Prandtl had a doctoral candidate, K. Hiemenz, to whom he had given the task of constructing a water channel in order to observe the separation of the flow behind a cylinder. Much to his surprise, Hiemenz found that the flow in his channel oscillated violently, and he failed to achieve symmetrical flow about a circular cylinder.

Von Kármán wrote [470, p. 70]

> When he [Hiemenz] reported this to Prandtl, the latter told him: "Obviously your cylinder is not circular."
>
> However, even after very careful machining of the cylinder, the flow continued to oscillate. Then Hiemenz was told that possibly the channel was not symmetric, and he started to adjust it.
>
> I was not concerned with this problem, but every morning when I came into the laboratory I asked him, "Herr Hiemenz, is the flow steady now?"
>
> He answered very sadly, "It always oscillates."

Von Kármán addressed the model problem of two infinite rows of point vortices and derived a criterion for when such a configuration is not unstable to *linearized* perturbations. He showed that the symmetric configuration, cf. Fig.17, is always unstable and that the staggered configuration is

[29]Contrary to von Kármán's reminiscences (see later) the first paper was submitted to the *Königlichen Gesselschaft der Wissenschaften zu Göttingen* by Felix Klein on September 14, 1911 and the second paper by Carl Runge on December 23, 1911.

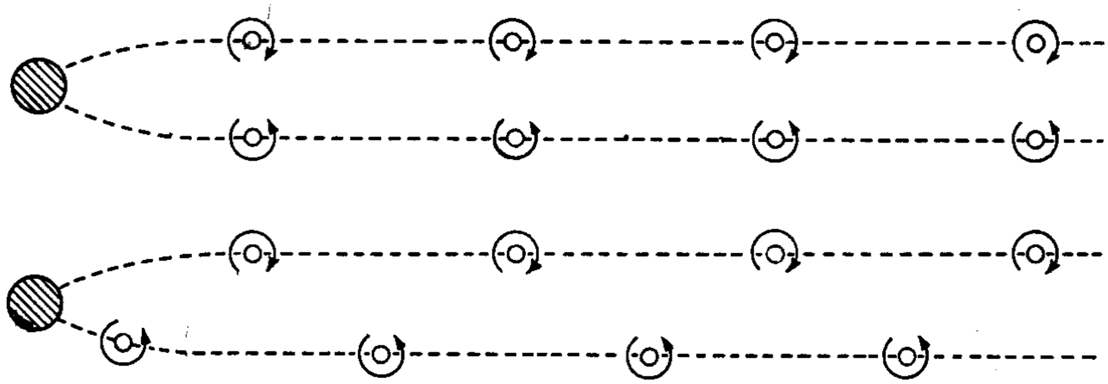


Figure 17: Schematic of a symmetric and a staggered vortex street downstream of a bluff body. From [470].

also unstable unless the spacing between successive vortices in either row and the distance between the rows has a definite ratio.

Later he reminisced [470, p. 70]:

> One weekend I tried to calculate the stability of the system of vortices, and I did it in a very primitive way. I assumed that only one vortex was free to move, while all other vortices were fixed, and calculated what would happen if this vortex were displaced slightly. The result I got was that, provided a symmetric arrangement was assumed, the vortex always went off from its original position. I obtained the same result for asymmetric arrangements but found that, for a definite ratio of the distances between the rows and between two consecutive vortices, the vortex remained in the immediate neighborhood of its original position, describing a kind of small closed circular path around it.
>
> I finished my work over the weekend and asked Prandtl on Monday, “What do you think about this?” “You have something,” he answered. “Write it up and I will present your paper in the Academy.”
>
> This was my first paper ([467]) on the subject. Then, because I thought my assumption was somewhat too arbitrary, I considered a system in which all vortices were movable. This required a little more complicated mathematical calculation, but after a few weeks I finished the calculation and wrote a second paper ([468]).

If the spacing between successive vortices in the same row is called l, and if the distance between the two parallel rows is called h, von Kármán’s criterion [468] is

$$\cosh\frac{\pi h}{l}=\sqrt{2},\quad \text{or}\quad \frac{h}{l}=0.283. \tag{12}$$

The velocity, U, of horizontal translation of the infinite rows is found to be

$$U=\frac{\Gamma}{2l}\tanh\frac{\pi h}{l}. \tag{13}$$

This is today very well known. What is probably less well known is that in the original paper [467] von Kármán found the criterion (12) with $\sqrt{3}$ (or $h/l=0.365$) on the right hand side rather than (the correct) $\sqrt{2}$, which was confirmed subsequently in [474, 814] (with reference to the then newly created theory of an infinite system of linear differential equations due to O. Toeplitz in 1907)[30] and by Lord Rayleigh [760]. The original drawings [474] of the streamlines in a coordinate system moving steadily with the vortices are reproduced in Fig.18. (When the ratio h/l is given by Eq.(12), the propagation speed of the street, U, in Eq.(13) is $\Gamma/l\sqrt{8}$.)

[30] It may appear strange but in his book [470] von Kármán did not mention Rubach’s name at all.

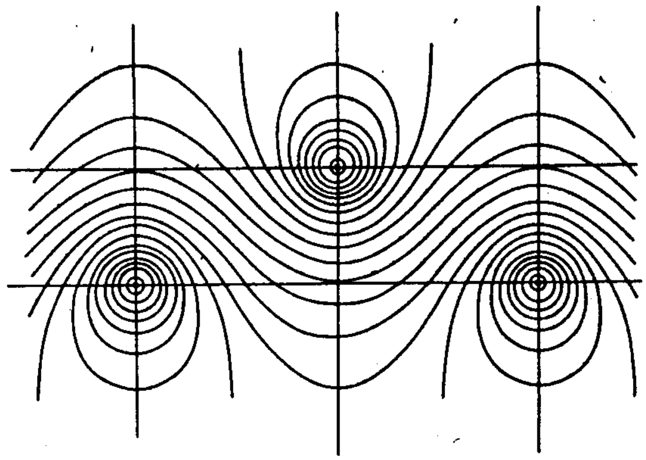


Figure 18: Streamlines of a vortex street in the co-moving frame based on the point vortex model [474].

Von Kármán commented [470, p. 71]: "Some people asked, 'Why did you publish two papers in three weeks? One of them must be wrong.' Not exactly wrong, but I first gave a crude approximation and afterwards refined it. The result was essentially the same; only the numerical value of the critical ratio was different."

The erroneous value was, for example, later used by Synge [891] in his re-derivation of the Kármán drag formula, although the analysis is easily corrected[31].

Measurements by H. Rubach [474, 814] gave the values $h = 1.8$ cm, $l = 6.4$ cm ($h/l = 0.282$) for a circular cylinder of diameter 1.5 cm, and $h = 3.0$ cm, $l = 9.8$ cm ($h/l = 0.305$) for a thin plate of width 1.75 cm.

On the other hand, Joukovskii in a talk in November 1913 [451] and in lectures [459, §29] read in 1911-1912 at Moscow High Technical School, later translated into French [455], repeated the concise derivation of von Kármán's first paper [467] in full detail and naturally arrived at the condition $\cosh(\frac{\pi h}{l}) = \sqrt{3}$ and $h/l = 0.365$ for the stability condition of a vortex street. (All publications contain a photo of Prandtl in front of the small water channel in Göttingen where the discovery of vortex streets was made.) His measurements at the laboratory of Moscow High Technical School [459, p.185] provided the following values: $h/l = 0.320; 0.337; 0.366; 0.363$ for thin plates of various widths placed perpendicular to the flow. Later this work led to an intensive discussion in Russia [1,309,311,312]. The problems considered by von Kármán [467] – Joukovskii [451, 455, 459] and von Kármán [468] are essentially different: in the latter case *all* vortices are "free" and may be perturbed, while in the former case only one vortex is "free" while the others are all fixed at their positions in the street configuration. These two problems are not equivalent and so the stability conditions are different.

Von Kármán's analysis precipitated huge amounts of work, both experimental, analytical – and much later – numerical; see the review papers [523, 801]. On July 18, 1922, a young W. Heisenberg, then a student of A. Sommerfeld at the Institute for Theoretical Physics at University of München, submitted an article [389] in which he tried to define an absolute size of the Kármán vortex street behind a flat plate of width d placed perpendicularly to the oncoming flow of velocity

[31] The authors have a version of the paper [891] with Prof. Synge's annotations effecting the necessary changes.

U_∞ far upstream[32]. Based on physical arguments he arrived at the numerical values $l/d = 5.45$ and $h/d = 1.54$. These values fit von Kármán's second value for the ratio of width to intra-row spacing, $h/l = 0.283$ and the ratio of the speed of propagation of the row relative to the flow speed at infinity, $U/U_\infty = 0.229$. In a short Appendix (written on July 29, 1922) to this paper Prandtl put forward three objections to the analysis of Heisenberg and concluded:

> In my opinion, Mr. Heisenberg's computation, though very instructive, is only adapted to yield definite conclusions when used in connection with experimental data concerning the correction factors referred to above.

Heisenberg's thoughts at this time were already turning to other topics, most notably the creation of a new"matrix" version of quantum mechanics for which he was to receive the Nobel prize in 1933 together with E. Schrödinger and P. A. M. Dirac. Nevertheless, his doctoral dissertation, completed in July 1923, was on hydrodynamics, in particular stability theory and turbulence, and he would return briefly to the topic of fully developed turbulence in the period following World War II.

The necessary condition for absence of linear instability was generalized to vortex streets moving obliquely to the direction of the "free stream" by Dolaptschiew and Maue. While the paper by Maue [630] will probably be familiar, in part because this work was highlighted in the well-known lectures of Sommerfeld [866], the extensive work of Dolaptschiew [202–219] is less well known than it ought to be. Insofar as assimilation into the literature in the West is concerned, the situation was not helped by several of Dolaptschiew's papers being published in Bulgarian and Russian, albeit usually with an abstract or summary in German.

2.6 *Coherent vortex structures* The theoretical analysis of solutions of the Euler equations that represent isolated regions of distributed vorticity in a two- or three-dimensional flow domain have attracted attention of several investigators since the mid-19th century. A detailed survey of the results obtained up to the beginning of the 20th century may be found in the books [44, 542] and the review articles [34, 595, 601]. However, the number of analytically known solutions to the nonlinear Euler or Helmholtz equations is quite limited. For two-dimensional patches of uniform vorticity in an ambient unbounded potential flow, the circular Rankine vortex [755, §633][33] and the Kirchhoff elliptical vortex [491, Lecture XX] are examples of such exact solutions. Rankine's model of the so-called "molecular vortices" [754,756] was used by Maxwell [633–636] in his early attempts to understand the nature of electrical and magnetic phenomena. In contrast to the circular Rankine vortex, which represents a steady flow about a stationary vortex, the flow associated with the Kirchhoff elliptical vortex is unsteady since the elliptical patch performs a steady rotation about its center while preserving its shape. The elliptical vortex in a shear flow, a generalization of the Kirchhoff vortex, was first considered by Chaplygin [127]:

> In this paper we consider the following case of motion in an unbounded mass of fluid: all motion is parallel to the coordinate plane OXY; the velocity components u and v are continuous in the entire flow domain; all the fluid contains vortices; vortex lines are parallel to the OZ axis; the angular velocity Ω of vortex rotation inside an elliptical cylinder with OZ axis is constant and equals $A + \omega$, and in the rest of the fluid $\Omega = A$; the velocity infinitely far from OZ is parallel to OX and $u = -2Ay$. We will show that the inner vortex cylinder will change its form according to a certain law, rotating with a variable angular velocity around the OZ-axis.

[32] The editors of Heisenberg's collected works call this "the apprentice work of [a] second semester student".

[33] The year of publication is 1858! Rankine worked on vortices in conjunction with his use of "molecular vortices" in the then emerging theory of thermodynamics.

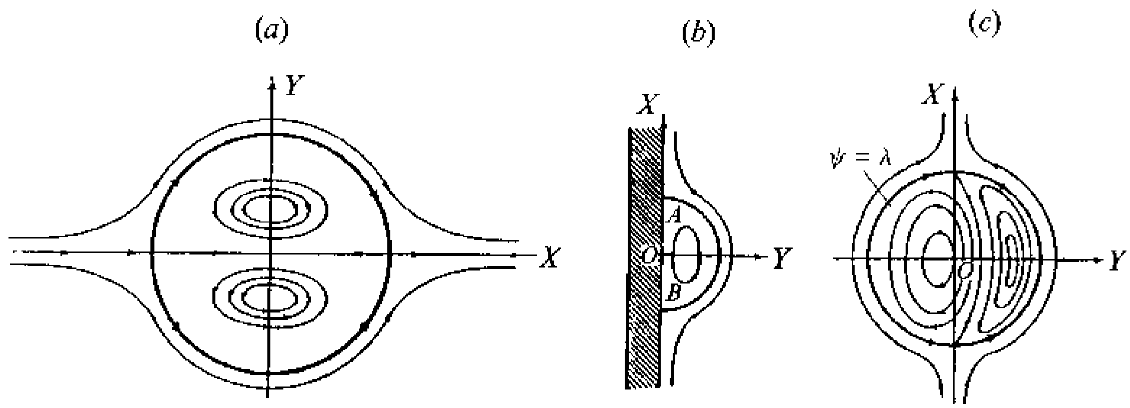


Figure 19: Streamlines of various coherent vortex structures determined analytically by Chaplygin.

Chaplygin provided a complete analytical solution to this problem, see [653] for details. Although Chaplygin's paper [127] was mentioned in review papers by Love [601, p.123] and Auerbach[34] [34, p.1061], it seems to have escaped the attention of later fluid dynamicists. Thus, many years later this solution was rediscovered and generalized by Moore and Saffman [668] and Kida [490] for an elliptical patch of uniform vorticity in both irrotational strain and in a simple external shear flow. For an arbitrary two-dimensional compact vorticity distribution a general approach based on Clebsch variables was introduced by Hill [417–421]. Even after more than a century this method has not received any attention and seems to deserve further extension and application.

A considerable simplification in the analysis of two-dimensional vortex flows is obtained by considering the case of steady motion. In this case Stokes [882] proved that any flow field represented by a stream function, $\psi(x, y)$, is a solution of the two-dimensional Euler equations provided the stream function satisfies an equation of the form

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = f(\psi), \tag{14}$$

where $f(\psi)$ is an arbitrary function of ψ. An important question of stability of any steady solution was formulated by the young Maxwell in a draft [631] and in a letter to William Thomson [632] (we use modern notation for partial derivatives):

> I have been investigating fluid motion with reference to stability and I have got results when the motion is confined to the plane xy. I do not know whether the method is new. It only applies to an incompressible fluid moving in a plane. Put $\chi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}$. Hence, (A) $\frac{d\chi}{dt} = 0$ or $\chi = f(\psi)$ is the condition of steady motion as is otherwise known. (B) $f'(\psi)$ or $\frac{d\chi}{d\psi}$ must be negative for stability.
> When $f'(\psi)$ is positive the motion is unstable.
> When $f'(\psi) = 0$, χ is constant or 0.
> When χ is constant I think equilibrium is neutral.
> When $\chi = 0$ the whole motion is determined by the motion at a limiting curve so that there can be no finite displacement.

In 1895 and later in 1906 Lamb [542] was the first to try to solve (14) for a non-uniform vorticity distribution for the case of a linear relationship $f(\psi)$. Chaplygin [129] addressed the problem of steady motion of a compact vorticity distribution in two-dimensional unbounded inviscid flow:

[34] Felix Auerbach (1856-1933) was Jewish and committed suicide when Hitler came to power. There is a novel [651] based upon his life.

> Consider an unbounded mass of incompressible fluid in which the motion is parallel to the OXY plane; let the motion outside some circular cylinder be irrotational, the velocity being equal to zero at infinity. The question is to find a distribution of vortex lines inside the cylinder that gives rise to a uniformly translating vortex column with a continuous velocity distribution and with a positive pressure all around.

Chaplygin presented an extensive analysis of the characteristics of this solution, see [653] for details. Since no cross-references exist, it is assumed that Chaplygin and Lamb arrived at the same vortex dipole solution independently. This solution is now generally referred to as the *Lamb dipole* (or sometimes the *Batchelor dipole* because of the description of this vortex structure in the influential text [46, §7.3]). According to the chronological order of publication, the name *Chaplygin-Lamb dipole* would seem to be more appropriate[35]. Figure 19 gives examples of the type of coherent vortex solutions Chaplygin was able to determine both in unbounded flow and in flow with a solid, plane wall.

Three-dimensional compact vortex structures are even more complicated. There exists a single exact solution, known as *Hill's spherical vortex* [422, 423], with a special non-uniform distribution of vorticity inside a sphere that moves rectilinearly at constant speed. This solution has been used by Synge and Lin [893] in their model of three-dimensional turbulence. So far attempts [379, 380] to generalize such a solution to an ellipsoidal vortex have failed.

3. **Conclusion**

History, to paraphrase Leibnitz, is a useful thing, for its study not only gives to the researchers of the past their just due but also provides those of the present with a guide for the orientation of their own endeavors. While Helmholtz's 1858 paper on vortex dynamics and vorticity is of great importance and spawned the new subfield of vortex dynamics, one must admit that in the greater scheme of things Helmholtz is today primarily remembered for other contributions to science. The bibliography below contains several names that would not today be immediately associated with the field of vortex dynamics since the individual did work in other fields – often well outside of fluid mechanics – that became of even greater importance. In this category we may list Dirichlet, Friedmann, Hankel, Heisenberg, Klein, Lin, Love, J.J. Thomson, Zermelo and probably even Lord Kelvin. Considering the caliber of these scientists and mathematicians, it may be appropriate to recall Abel's statement, quoted in the remarkable lecture by Truesdell [965, p.39], that he had reached the front rank quickly "by studying the masters, not their pupils."

In the "case studies" in Section II we have focused on what one may call the classical applications of Helmholtz's vortex theory. It is the test of any significant advance that it elicits interest far beyond the boundaries anticipated by its creator. Thus, the importance of vortex dynamics was realized in meteorology and oceanography by such towering figures as Vilhelm Bjerknes [247, 266, 890] whose seminal work [92] bears the title "On the dynamics of the circular vortex : with applications to the atmosphere and atmospheric vortex and wave motions." At the other end of the size-scale spectrum we may cite the application of classical vortex dynamics to superfluid Helium [263], where the famous footnote in [701] announced the quantization of circulation in this case. As one surveys the now vast literature in vortex dynamics some 150 years after Helmholtz's paper one is struck by the richness of the subject matter, and by how the various aspects enter different applications in almost infinitely varied ways. Küchemann's figurative characterization of vortices as "the sinews and muscles of fluid motions" [525] is no less apt today than it was when it was written 40 years ago.

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[35] One of us (H.A.) recalls a meeting where the issue of a name for these solutions came up. George Batchelor was present and immediately stated that whatever the resolution might be, attributing them to him was certainly not appropriate. The attribution of names to physical phenomena, equations and the like often has deeper roots in sociology than in the history of science!

The period we are covering, 1858-1956, begins with Helmholtz's seminal paper and concludes in the year of publication of the intriguing paper by Domm [224]. In his paper Domm considered the dynamics of four point vortices, two of either sign, all with the same absolute circulation, in a strip with periodic boundary conditions. This work showed *inter alia* that even when the Kármán criterion is satisfied, the vortex street is unstable to higher order perturbations. (It is only marginally stable to linear perturbations.) Von Kármán's collected works were also published in 1956, in celebration of his 75th birthday, and so – along with the reasons stated previously – this year forms a fitting closing for our bibliography.

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