Fluid•DTU Summer School "Complex Motion in Fluids"
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Wakes behind wings

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European projects

- "C-Wake" (Wake Vortex Characterisation & Control, 2000-2003)
- "FAR-Wake" (Fundamental Research on Aircraft Wake Phenomena, 2005-2008)

Collaborations with Airbus & Eurocopter

Aircraft wake vortices (examples)

Dryden Flight Research Center ECN 3831 Photographed 1974
B-727 vortex study NASA photo

Aircraft wake vortices (examples)

Wake Vortex Study at Wallops Island
NASA Langley Research Center 5/4/1990 Image # EL–1996–00130

Visualisations of aircraft trailing wakes

SEMPRE A BORDO. SEMPRE REFRESCANTE.

Wing tip and flap tip vortices

Photo:
P. Meunier

Wing tip vortex in wind tunnel

Wing tip vortices in catapult facility

Source: ONERA Lille

http://www.onera.fr/cahierdelabo/english/asub8.htm

Visualisations of aircraft trailing wakes

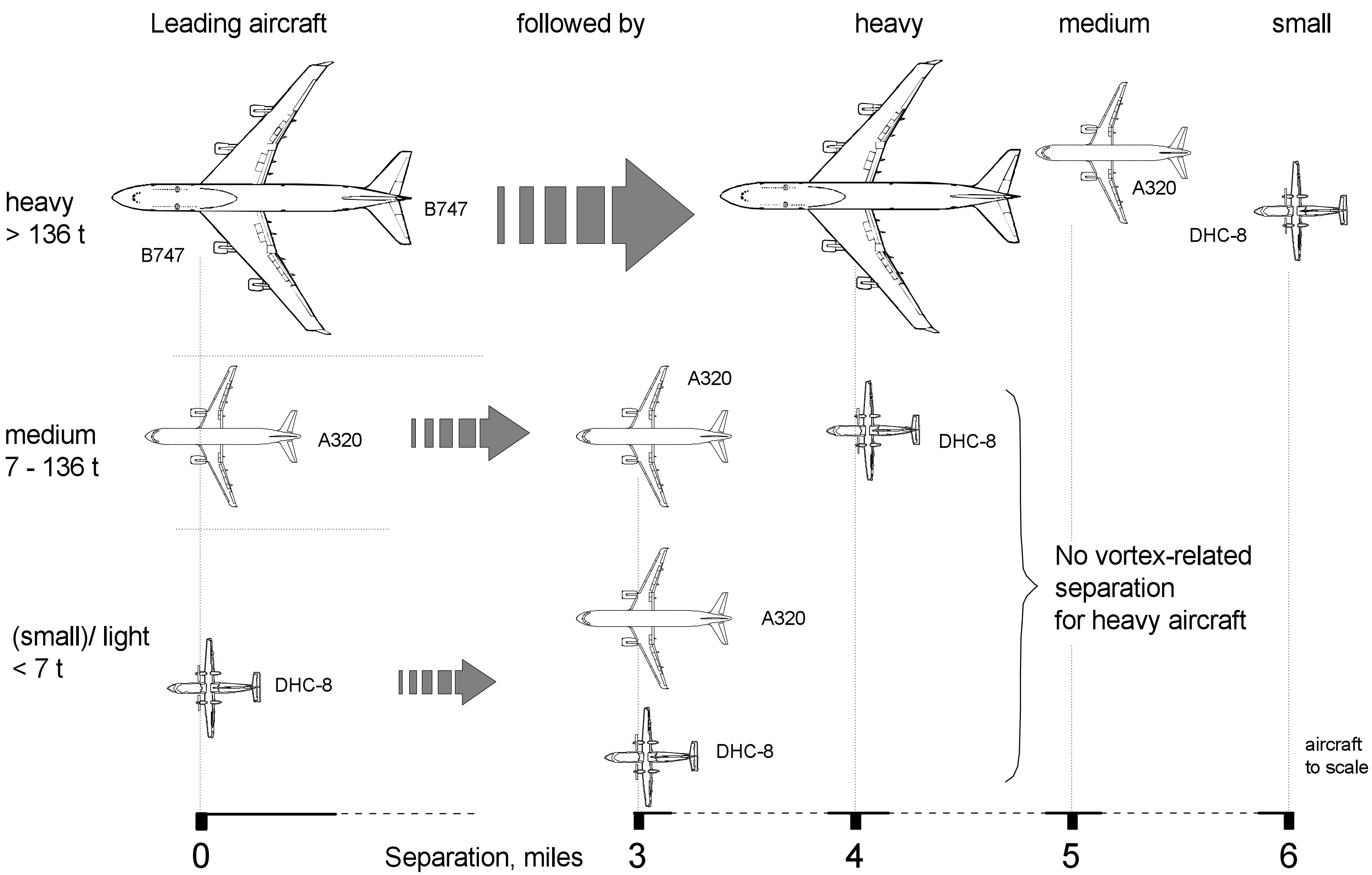
- **danger for following aircraft (downwash, roll)**
- **minimum separation distances → limits airport capacity**

Higuchi (1993)

Photo: Cessna Aircraft Company

Rules for separation distances (before A380)

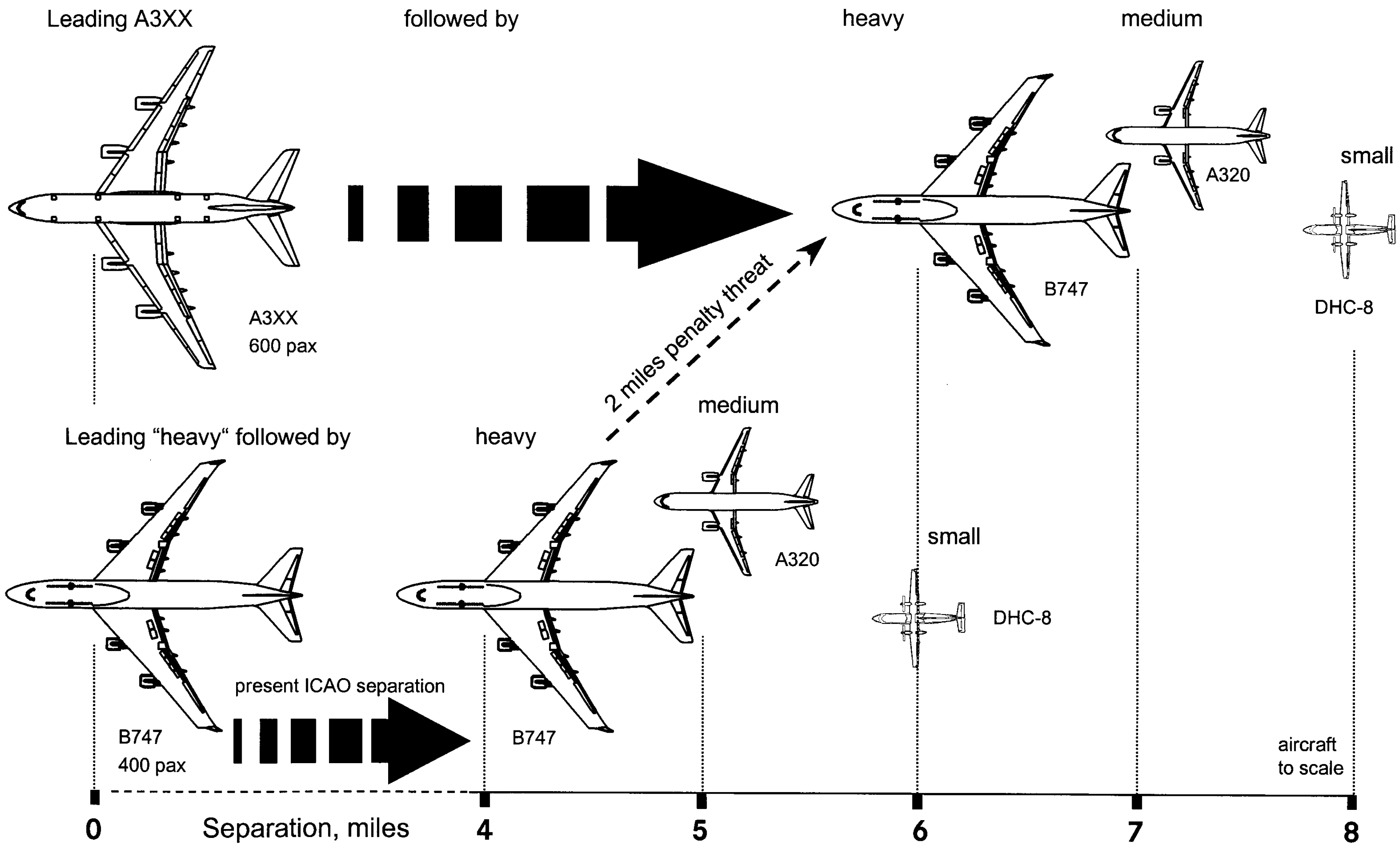
(source: International Civil Aviation Organization ICAO)



Airbus A380

	A380	B747
wing span	79.8 m	64.4 m
MTOW	560 t	400 t

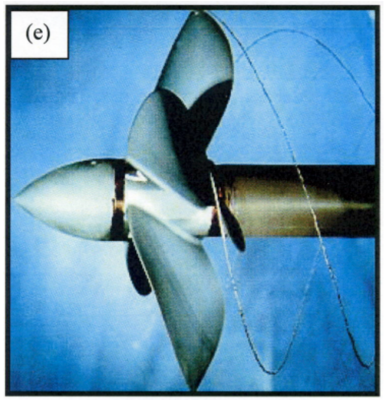
Current rules for separation distances



Wakes behind rotating wings

- helicopters
- propellers
- wind turbines

Hand *et al*. (2001)

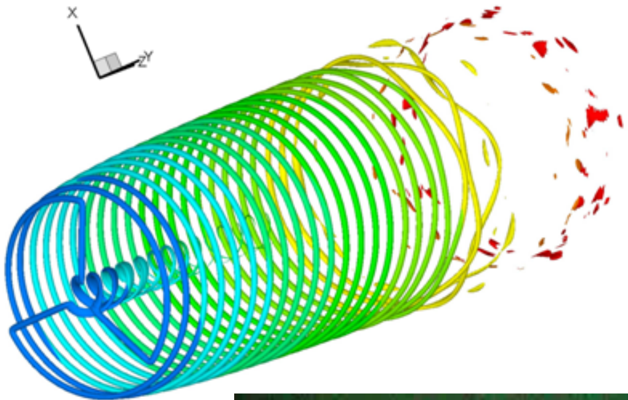


Senocak *et al*. (2002)

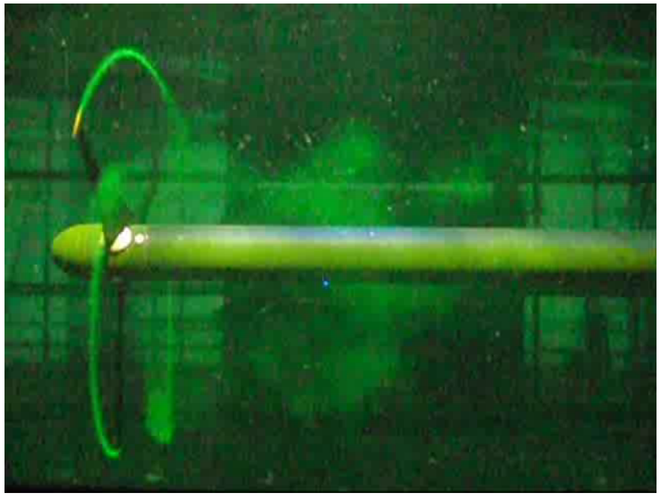
Wakes behind rotating wings

wind turbines

- destabilisation/decay of the helical vortex wake



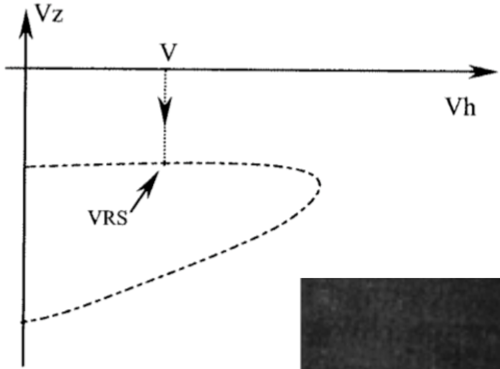
Ivanell *et al.* (2010)

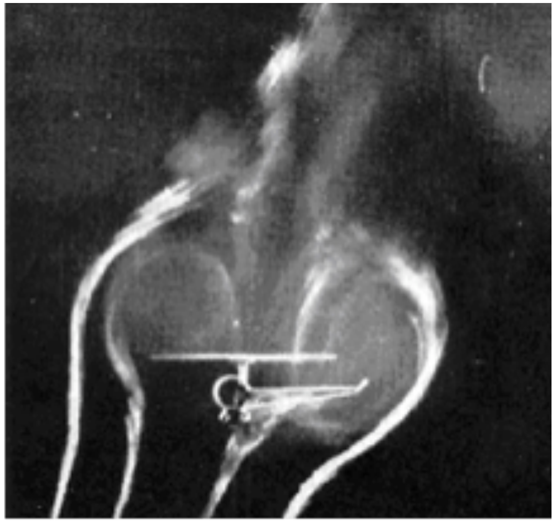


Mikkelsen (2010)

helicopters

- transition from helical wake to Vortex Ring State (VRS) in steep descent





Drees & Hendal (1950)

Overview

- **Basic elements of vortex dynamics and wing wakes**
  - Vorticity/circulation, vortices, lifting surface, wake vortex systems
- **Merging of co-rotating vortices**
- **Three-dimensional instabilities**
  - Long wavelength (Crow instability)
  - Medium wavelength
  - Short wavelength (elliptic instability)
- **Vortex reconnection**
- **Meandering**
- **Pairing instability of helical vortices**

Nomenclature and definitions

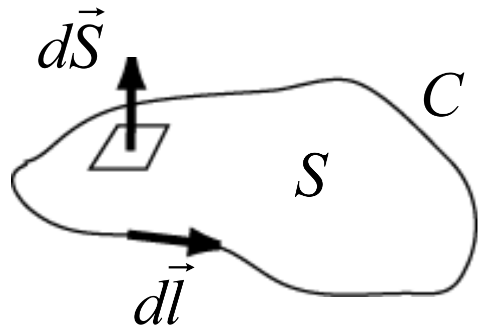
- **velocity** $\vec{u} = [u(x,y,z,t),\ v(x,y,z,t),\ w(x,y,z,t)]$

- **vorticity** $\vec{\omega} = \vec{\nabla} \times \vec{u} = \left(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y},\ \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z},\ \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right)$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \quad \Rightarrow \quad \vec{\nabla} \cdot \vec{\omega} = 0$$

- **circulation** $\Gamma = \oint_C \vec{u} \cdot d\vec{l}$

$$= \int_S \vec{\omega} \cdot d\vec{S}$$



Common hypotheses

- **Newtonian fluid**
  - ↳ **stresses ∝ velocity gradients**

- **constant-density fluid,** $\rho\,(x,y,z,t) = \text{const.}$
  - ↳ **incompressible**
  - ↳ **barotropic**

- **conservative volume forces**
  - ↳ $\vec{F} = -\vec{\nabla}\Phi$
  - ↳ **example: gravity**

Balance and evolution equations

- Conservation of mass ("continuity")

$$\vec{\nabla}\vec{u} = 0$$

- Navier-Stokes equation

$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho}\vec{\nabla}p' + \nu\,\Delta\vec{u}$$

- Vorticity equation

$$\frac{D\vec{\omega}}{Dt} = (\vec{\omega}\cdot\vec{\nabla})\vec{u} + \nu\,\Delta\vec{\omega}$$

$\Delta = \nabla^2$
Laplacian

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\vec{u}\cdot\vec{\nabla})$$

material derivative

$p' = p + \rho\,\Phi$
p: pressure

ν: kinematic viscosity

Laws and theorems

- **Biot-Savart relation**

$$\vec{u}(\vec{r},t) = -\frac{1}{4\pi}\int_V \frac{(\vec{r}-\vec{r}\,')\times\vec{\omega}(\vec{r}\,',t)}{|\vec{r}-\vec{r}\,'|^3}\,d^3r'$$

- **Kelvin's Theorem for an ideal fluid (ν = 0)**

"The circulation of any closed material line is conserved during its motion"

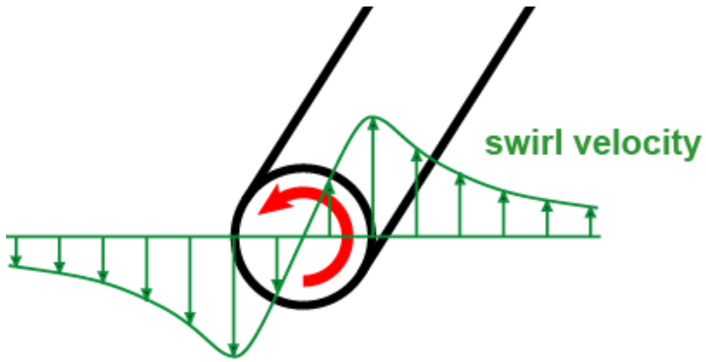
$$\frac{D\Gamma}{Dt} = 0$$

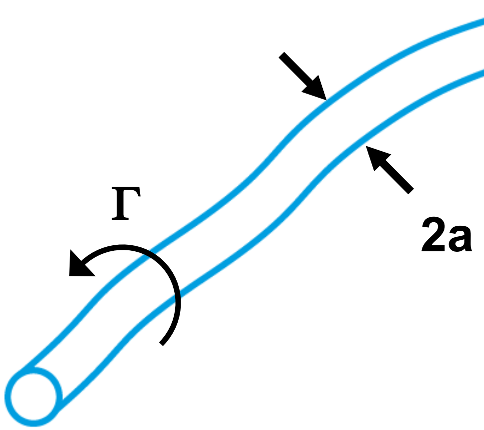
↳ **Theorems and laws of Lagrange and Helmholtz**

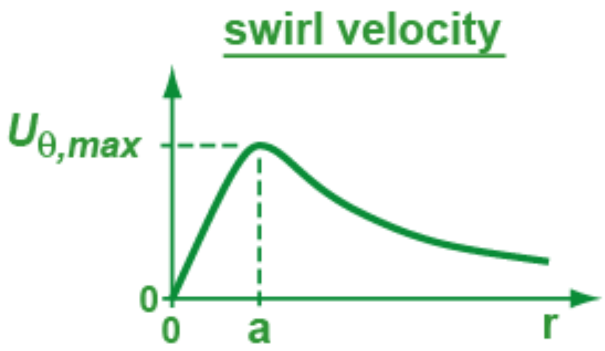
↳ ***summary:*** **In an ideal fluid, the circulation of each fluid element is constant in time and advected by the velocity field**

Vortices

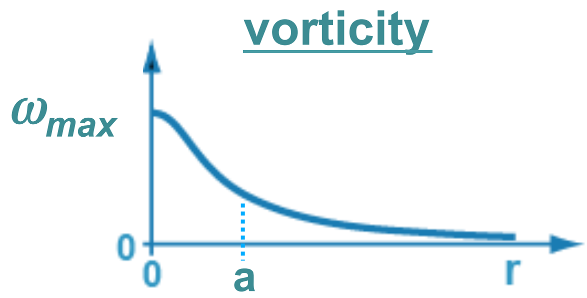
- local concentration of vorticity
- (fairly) axisymmetric
- tube-like structure

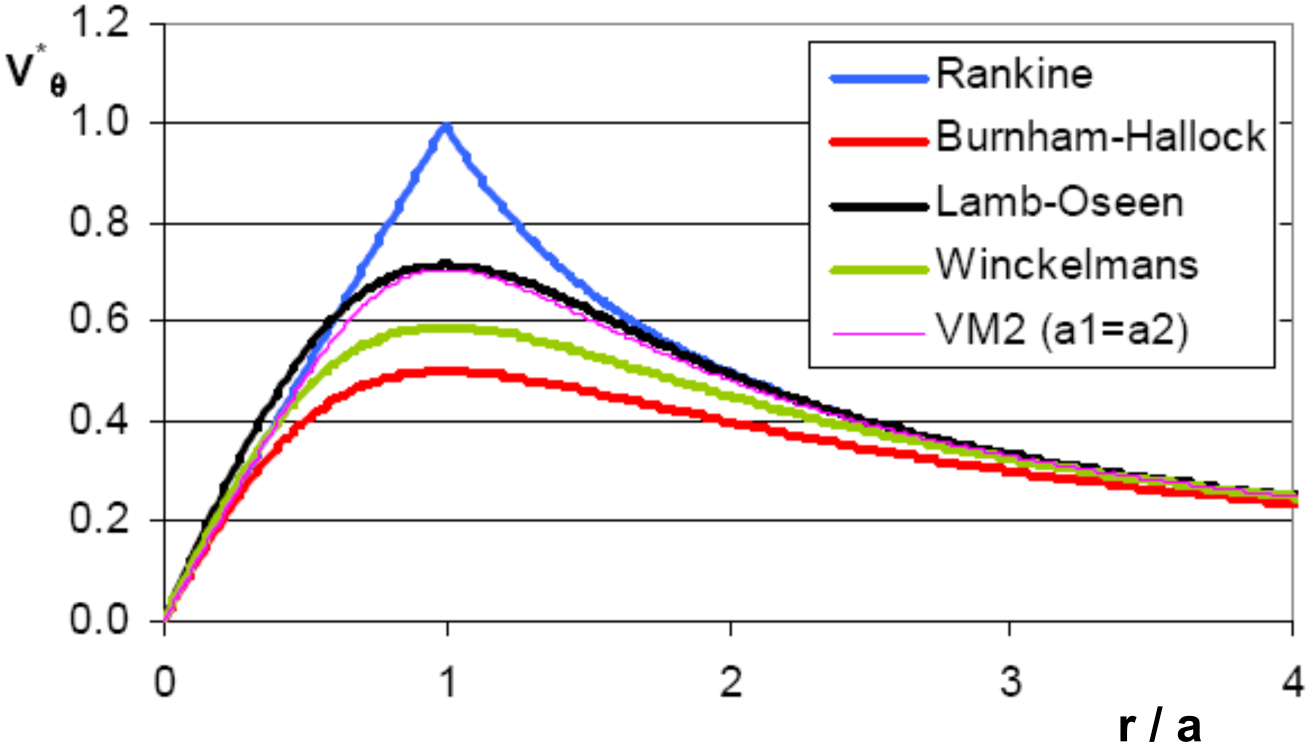






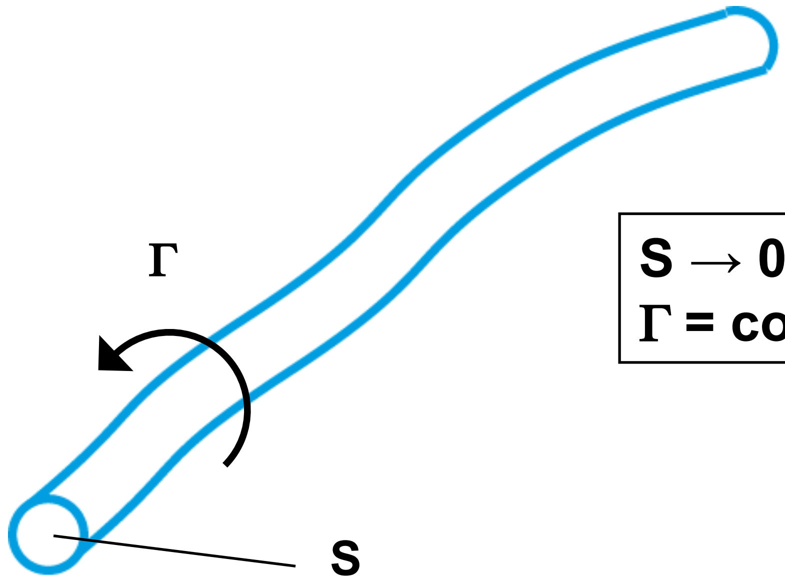
- circulation Γ
- core radius a
- Reynolds number $Re = \Gamma / \nu$



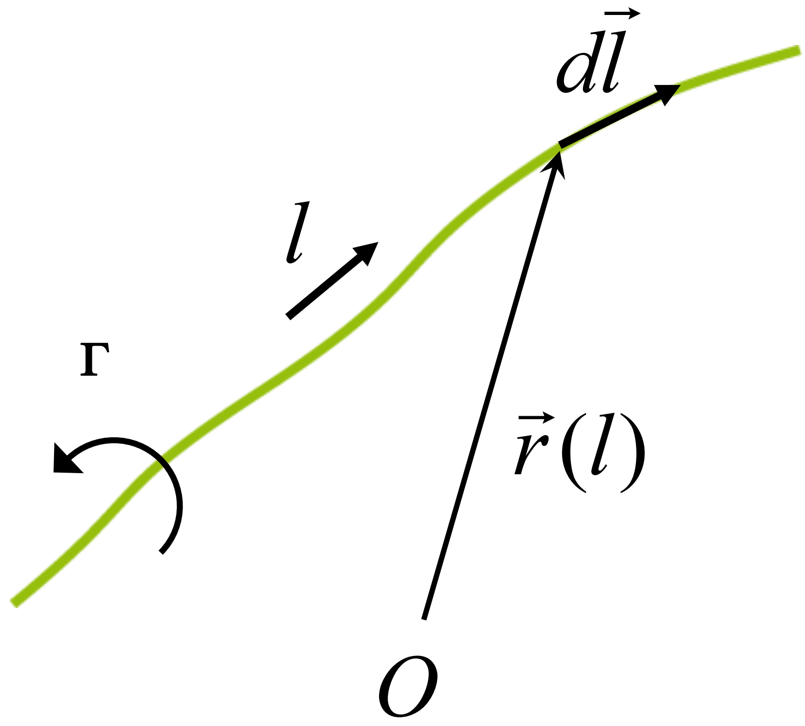


Velocity profiles, different analytical models.

Vortex filaments



S → 0
Γ = const.



Evolution of filament shape:

- **calculate** $\vec{u}(\vec{r}(l)) = \vec{u}_{ext} + \vec{u}_{ind}$
- **using Biot-Savart**

$$\vec{\omega} d^3 r' \rightarrow \Gamma d\vec{l}$$

$$\vec{u}_{ind}(\vec{r},t) = -\frac{\Gamma}{4\pi}\int_L \frac{(\vec{r}-\vec{r}\,') \times d\vec{l}}{|\vec{r}-\vec{r}\,'|^3}$$

Vortex filaments

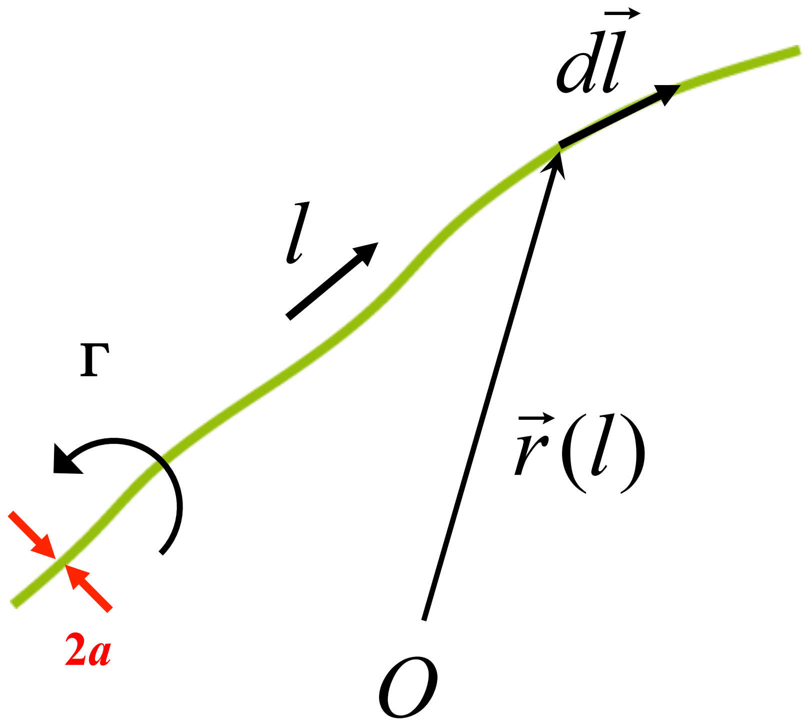
Problem:

- singularity for $\vec{r} = \vec{r}\,'$

Solution:

- reconsider finite core size a

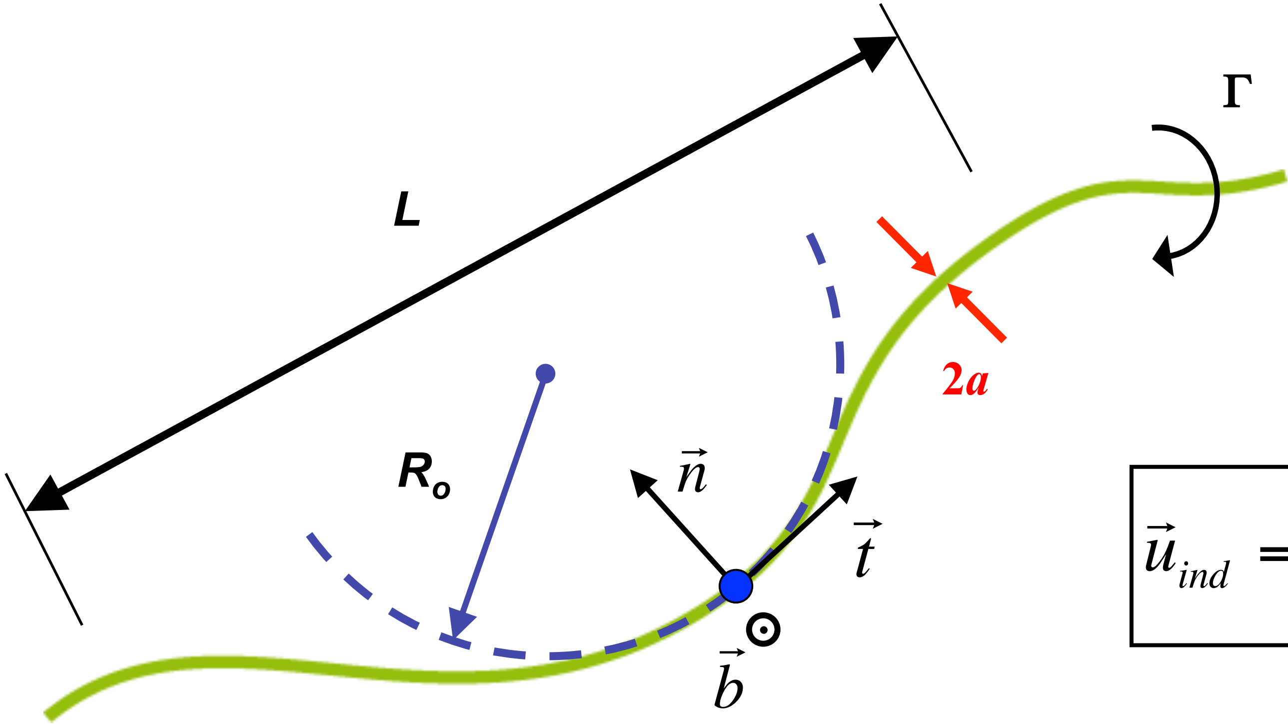
- $a \ll R_o$, $a \ll L$



$$\vec{u}_{ind}(\vec{r},t) = -\frac{\Gamma}{4\pi}\int_L \frac{(\vec{r}-\vec{r}\,')\times d\vec{l}}{|\vec{r}-\vec{r}\,'|^3}$$

Vortex filament evolution

Local Induction Approximation



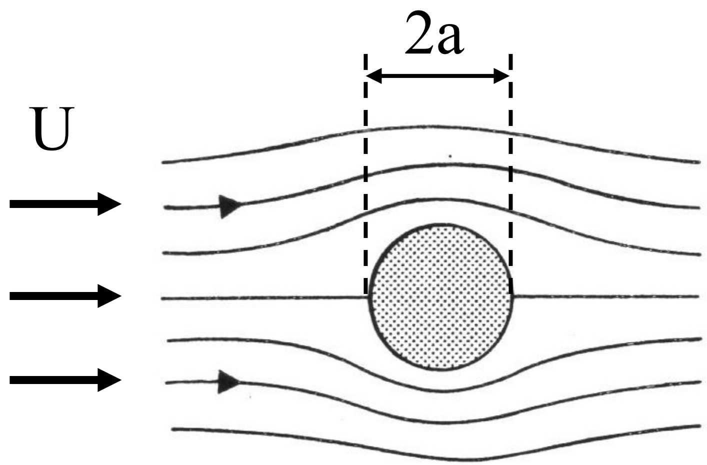
$$\vec{u}_{ind} = \frac{\Gamma}{4\pi R_o} \ln\frac{L}{a}\ \vec{b}$$

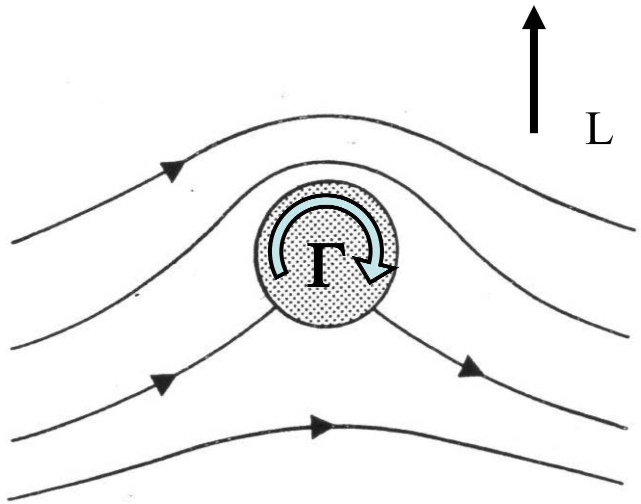
error of $O\left(\frac{\Gamma}{4\pi R_o}\right)$

Flow around a wing (1)

Circular cylinder in 2D (degenerated wing)

- **potential flow ($\vec{\omega} = 0$ everywhere)**





without circulation

↳ **no force on cylinder**

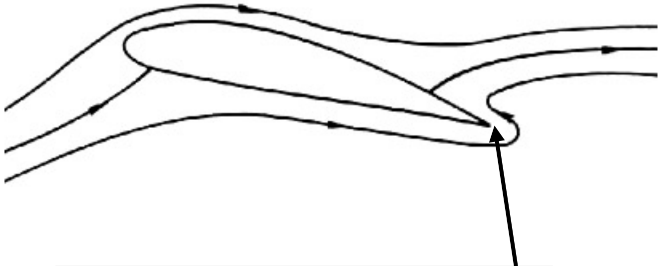
with circulation, $|\Gamma| < 4\pi U a$

↳ **lift force**

$L = \rho\, U\, \Gamma$

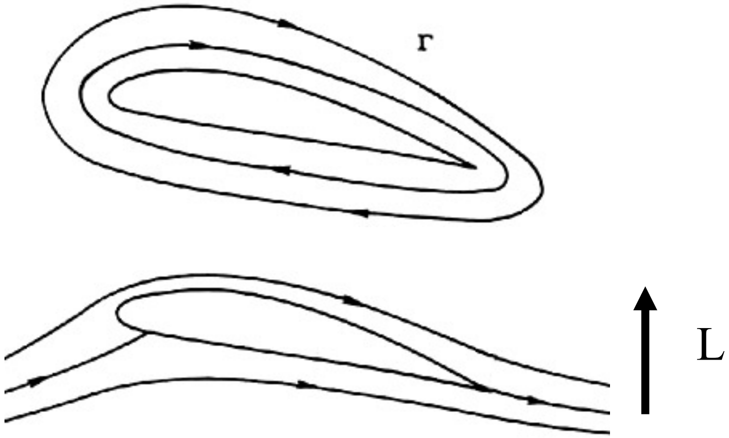
Flow around a wing (2)

Airfoil at incidence in 2D



infinite velocity at trailing edge

+



finite velocity everywhere
(Kutta-Joukowsky condition)

without circulation

↳ **no force on wing**

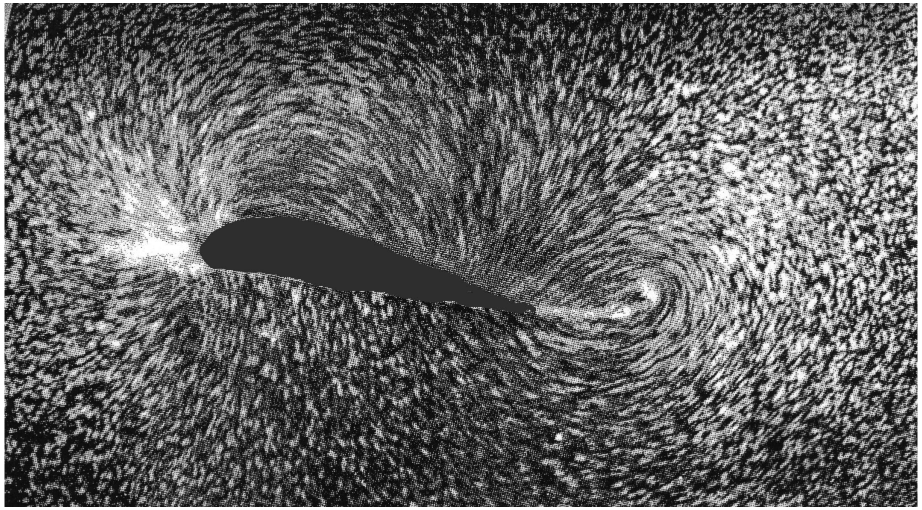
with circulation

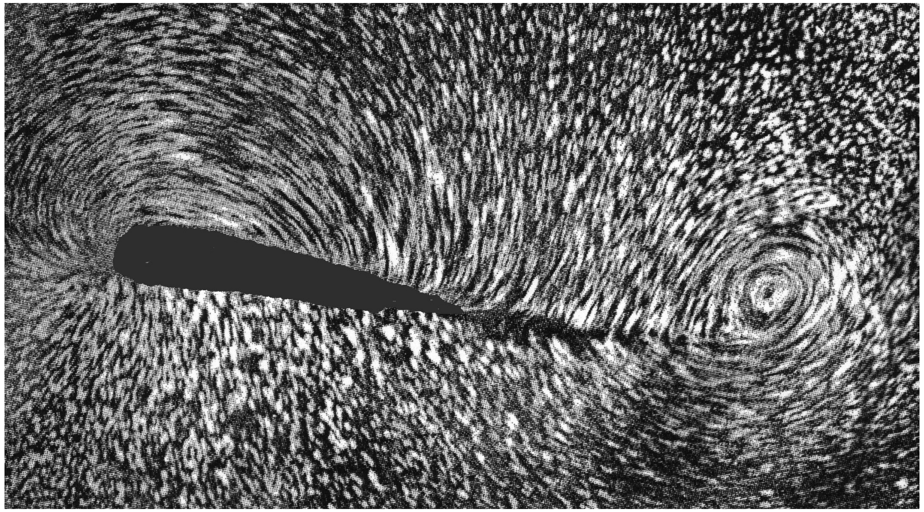
↳ **lift force**

$$L = \rho \, U \, \Gamma$$

Flow around a wing (3)

Airfoil at incidence in 2D

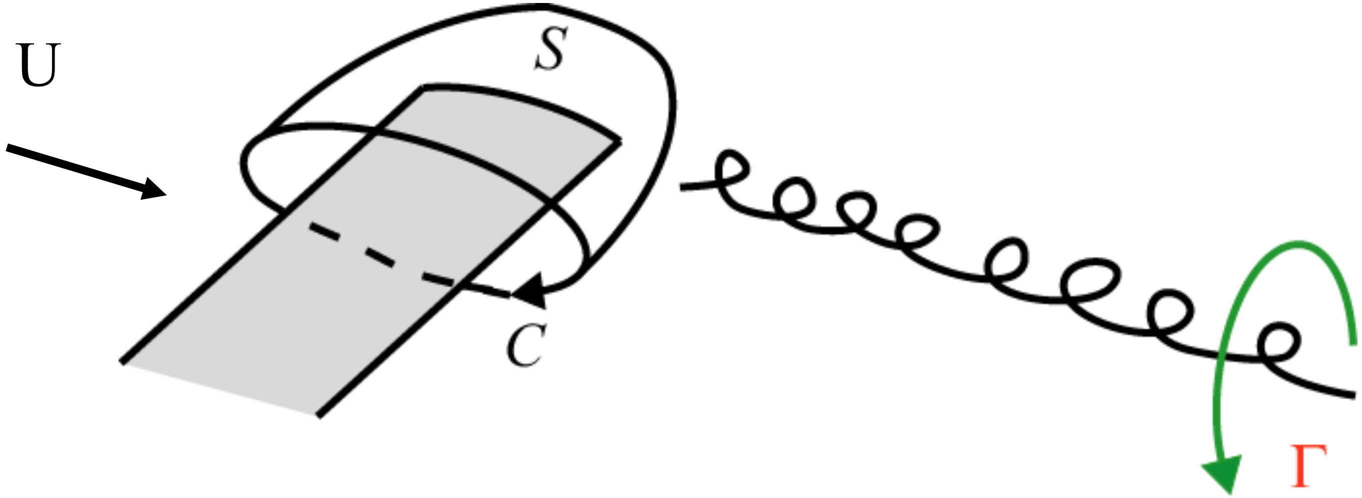




starting vortex behind an impulsively translated airfoil (Prandtl & Tietjens 1934)

Flow around a wing (4)

3D (rectangular) airfoil (finite wing span)



At wing tip:

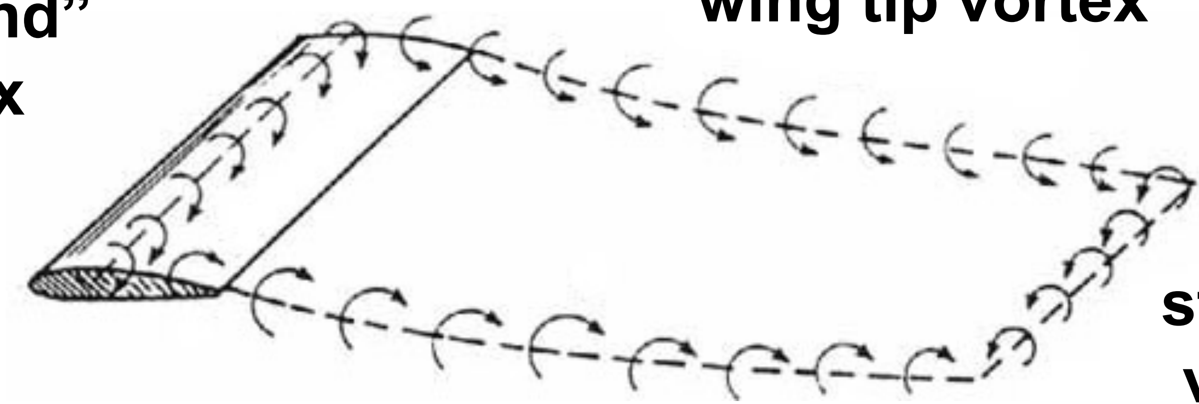
Vorticity flux across S :

$$\int_S \vec{\omega} \cdot d\vec{S} = \oint_C \vec{u} \cdot d\vec{l} = \Gamma$$

↳ **longitudinal wing tip vortex**

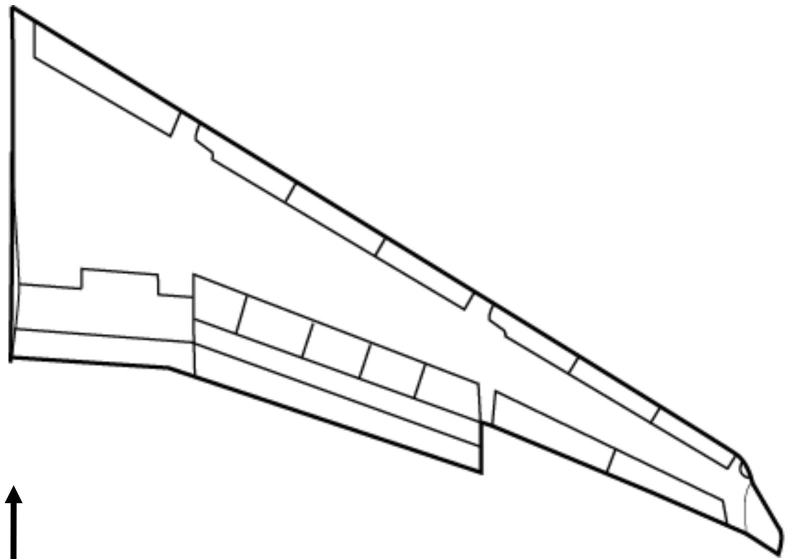
Flow around a wing (5)

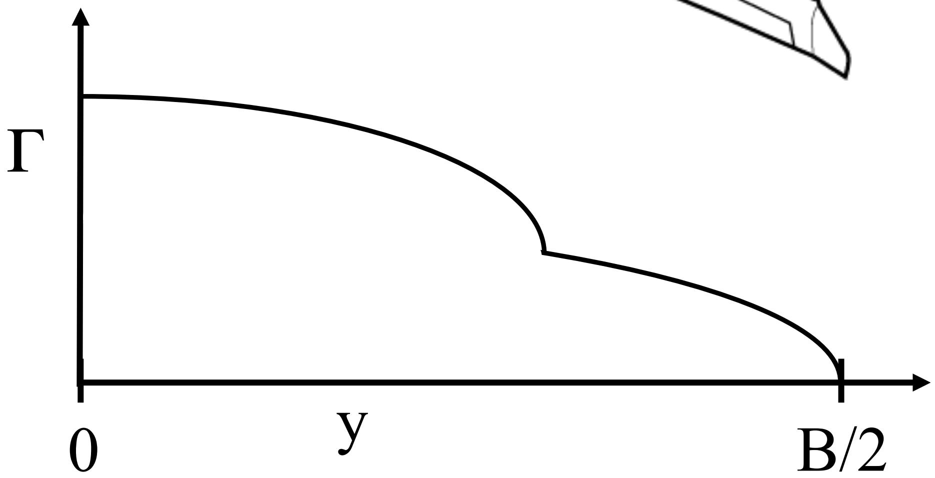
Global vortex system of a finite-length airfoil in motion



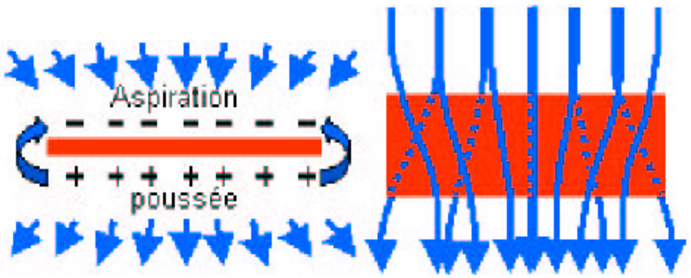
Flow around a wing (6)

Non-rectangular wing



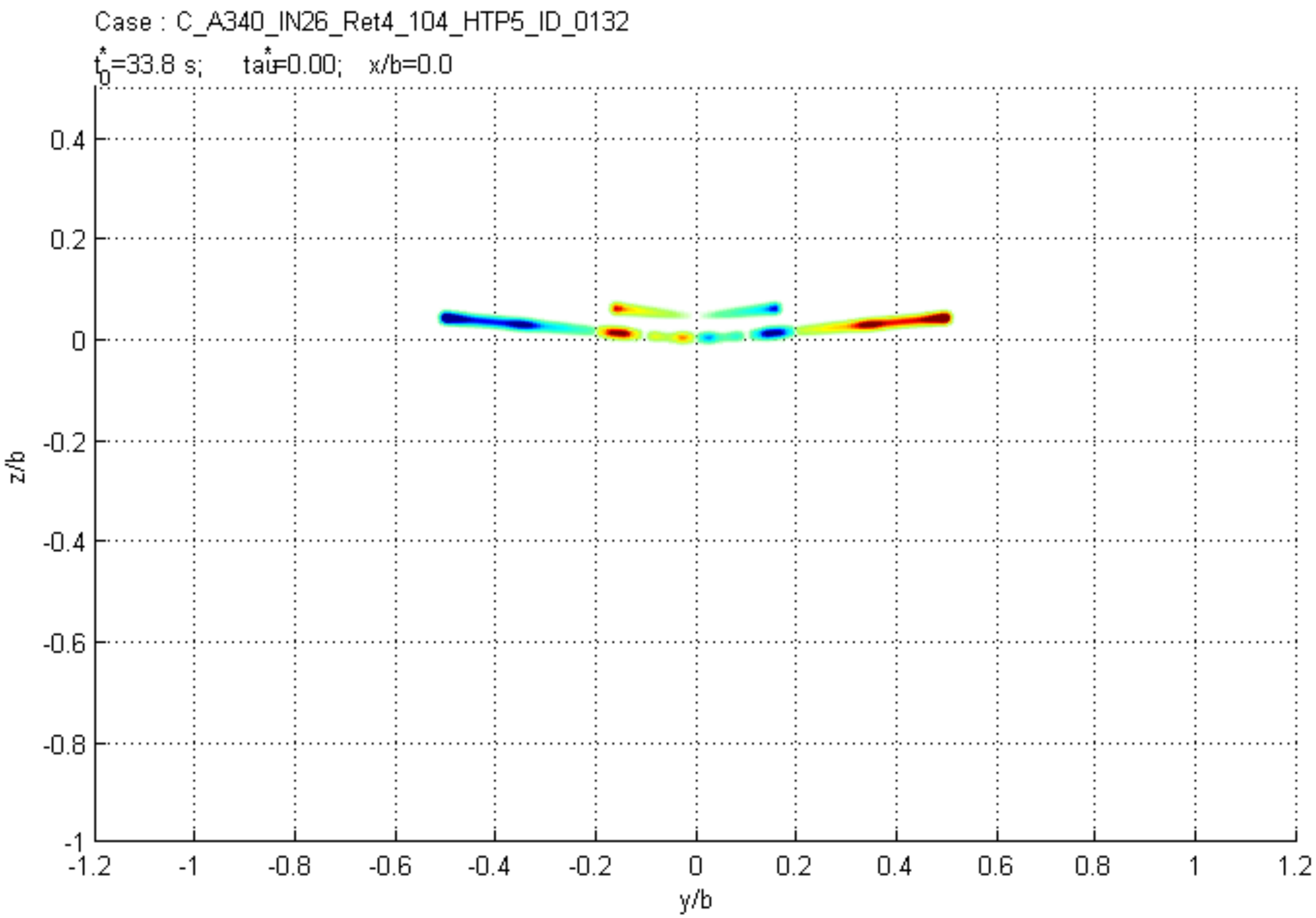


- **distribution** $\Gamma(y)$ **not constant**
- **shedding of a vorticity sheet between** $y = 0$ **and** $y = B/2$
- **circulation density** $\gamma(y) = d\Gamma/dy$



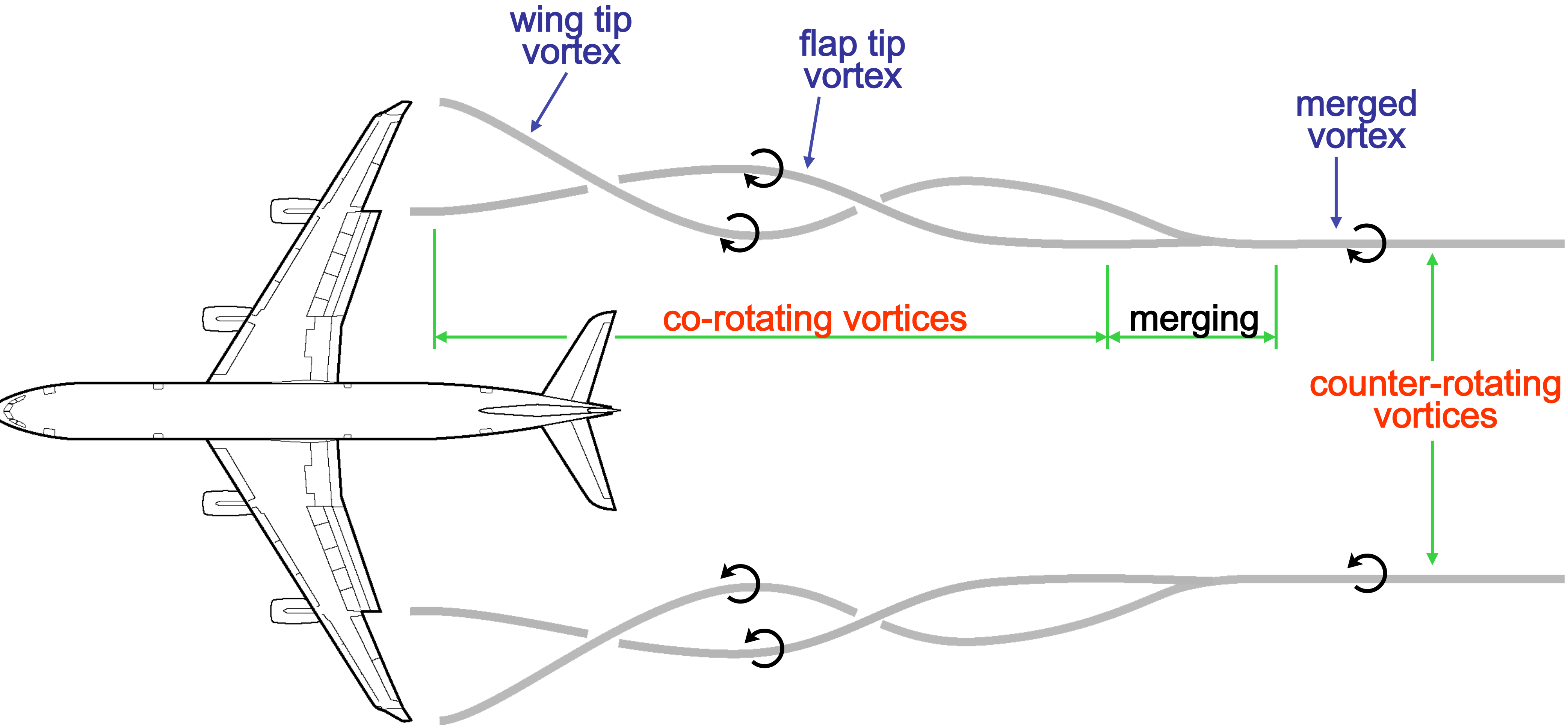
view from behind **view from above**

Vortex system in the wake of a civil aircraft
– typical take-off/landing configuration –
(including horizontal tail plane)



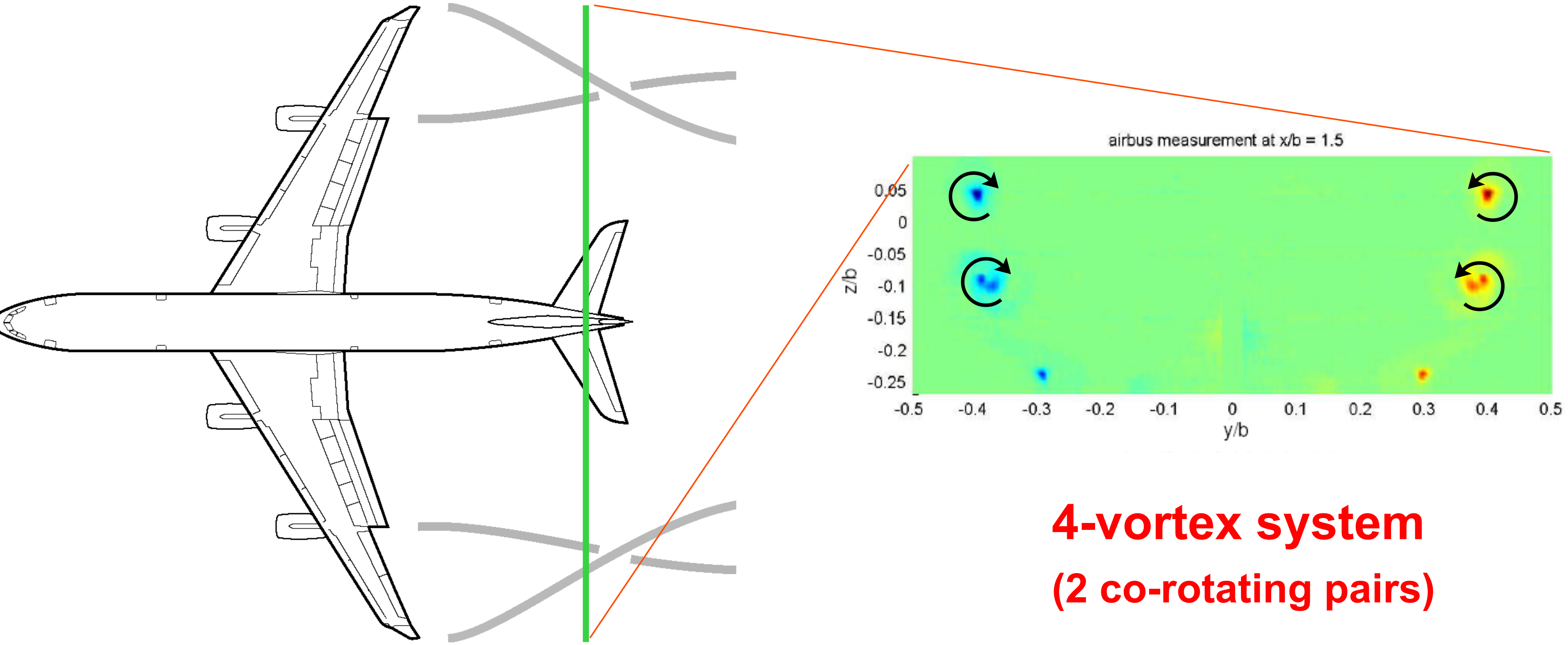
Vortex system in the wake of a civil aircraft

– typical take-off/landing configuration –



Vortex system in the wake of a civil aircraft

– typical take-off/landing configuration –

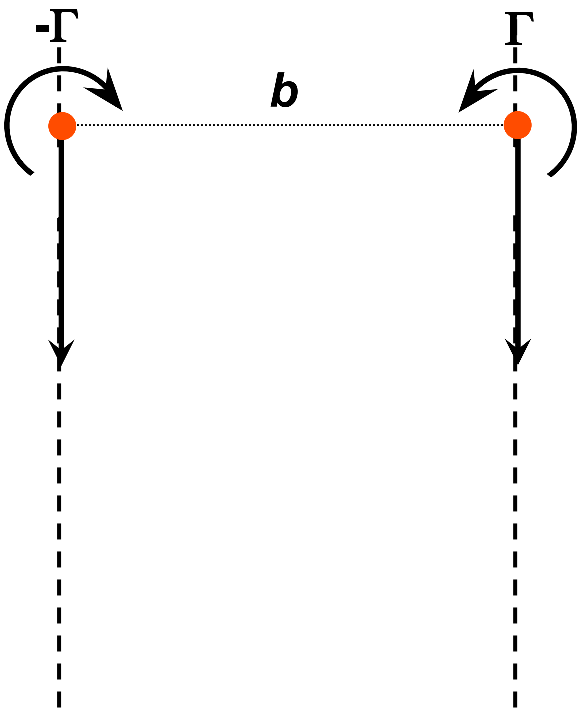


4-vortex system

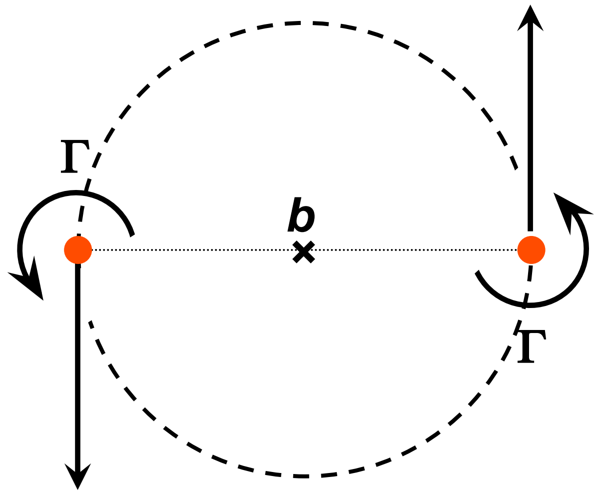
(2 co-rotating pairs)

Dynamics of two point vortices
- same circulation -

counter-rotating

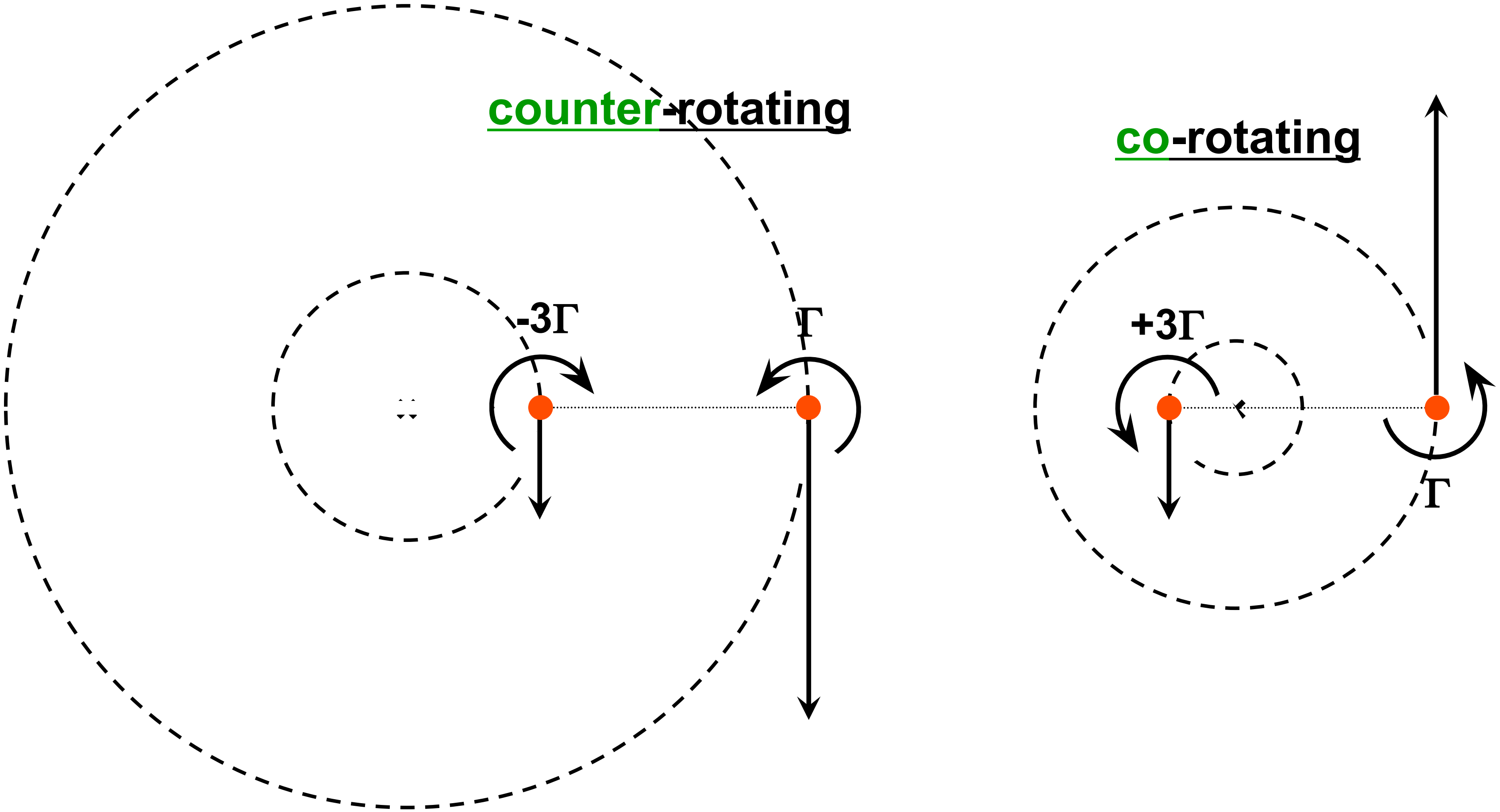


co-rotating



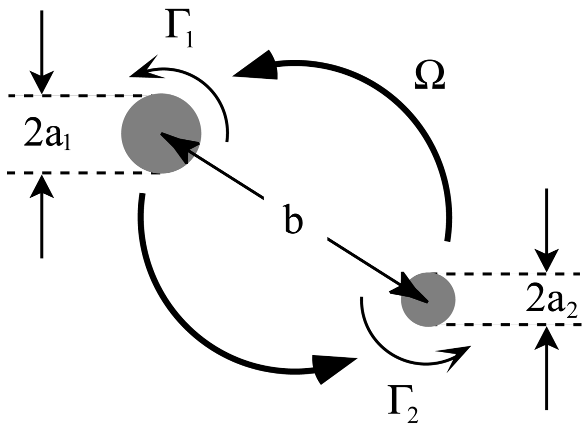
mutually induced velocity:
$|V| = \Gamma / 2\pi b$

Dynamics of two point vortices
- different circulations -



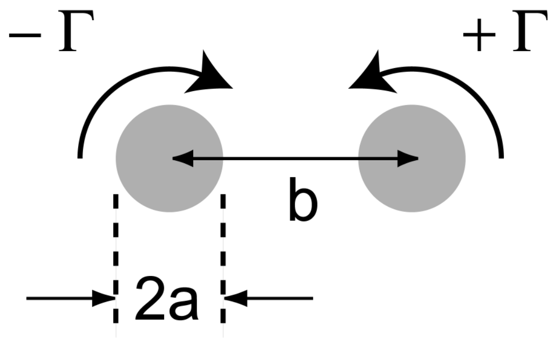
Vortex pair parameters

co-rotating



- vortex separation *b*
- angular freq. $\Omega = (\Gamma_1 + \Gamma_2)/2\pi b^2$
- non-dim. time $t^* = t\,\Omega\,/\,2\pi$

counter-rotating



- vortex separation *b*
- descent speed $V = \Gamma\,/\,2\pi b$
- non-dim. time $t^* = t\,V\,/\,b$

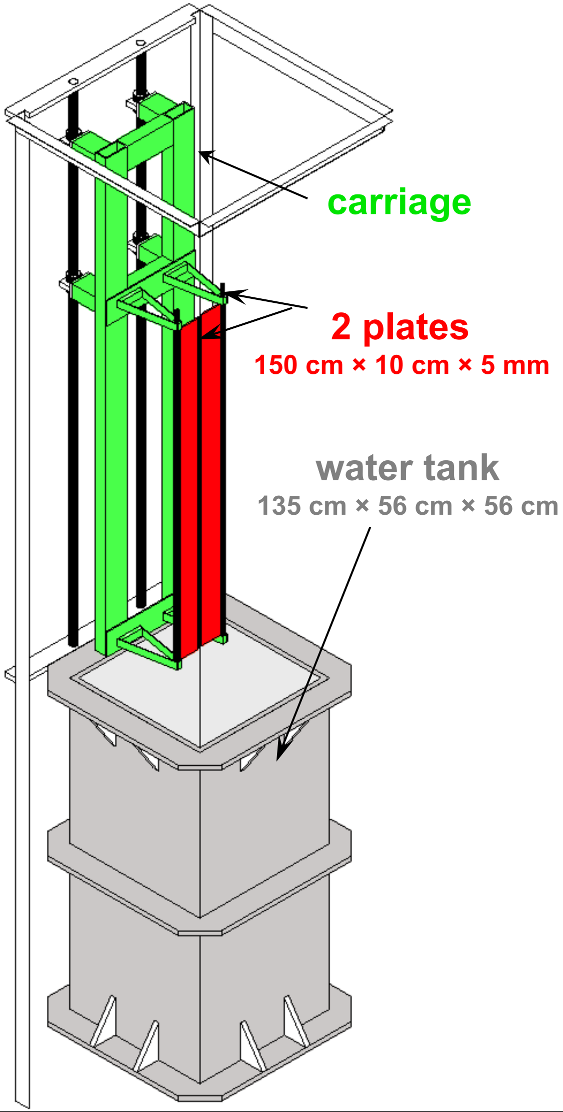
rate of strain induced by one vortex on the other: $\varepsilon = \Gamma\,/\,2\pi b^2$ $(\varepsilon^* = a^2/b^2)$

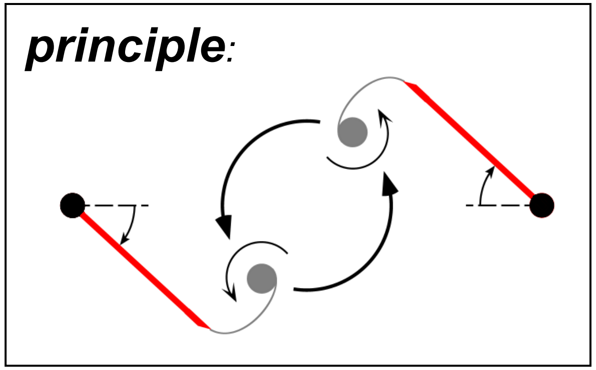
Overview

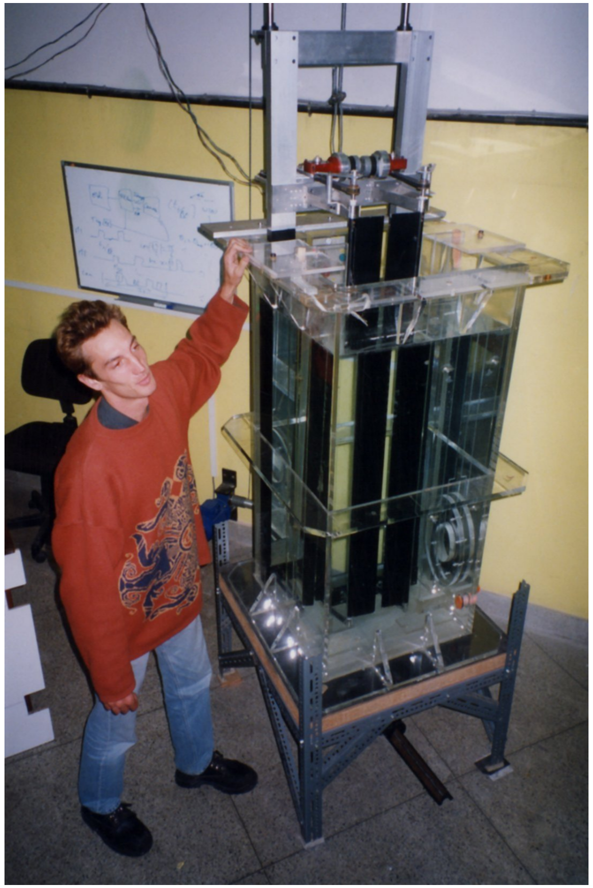
- **Basic elements of vortex dynamics and wing wakes**
  - Vorticity/circulation, vortices, lifting surface, wake vortex systems
- **Merging of co-rotating vortices**
- **Three-dimensional instabilities**
  - Long wavelength (Crow instability)
  - Medium wavelength
  - Short wavelength (elliptic instability)
- **Vortex reconnection**
- **Meandering**
- **Pairing instability of helical vortices**

Co-rotating vortex pairs

Experimental set-up

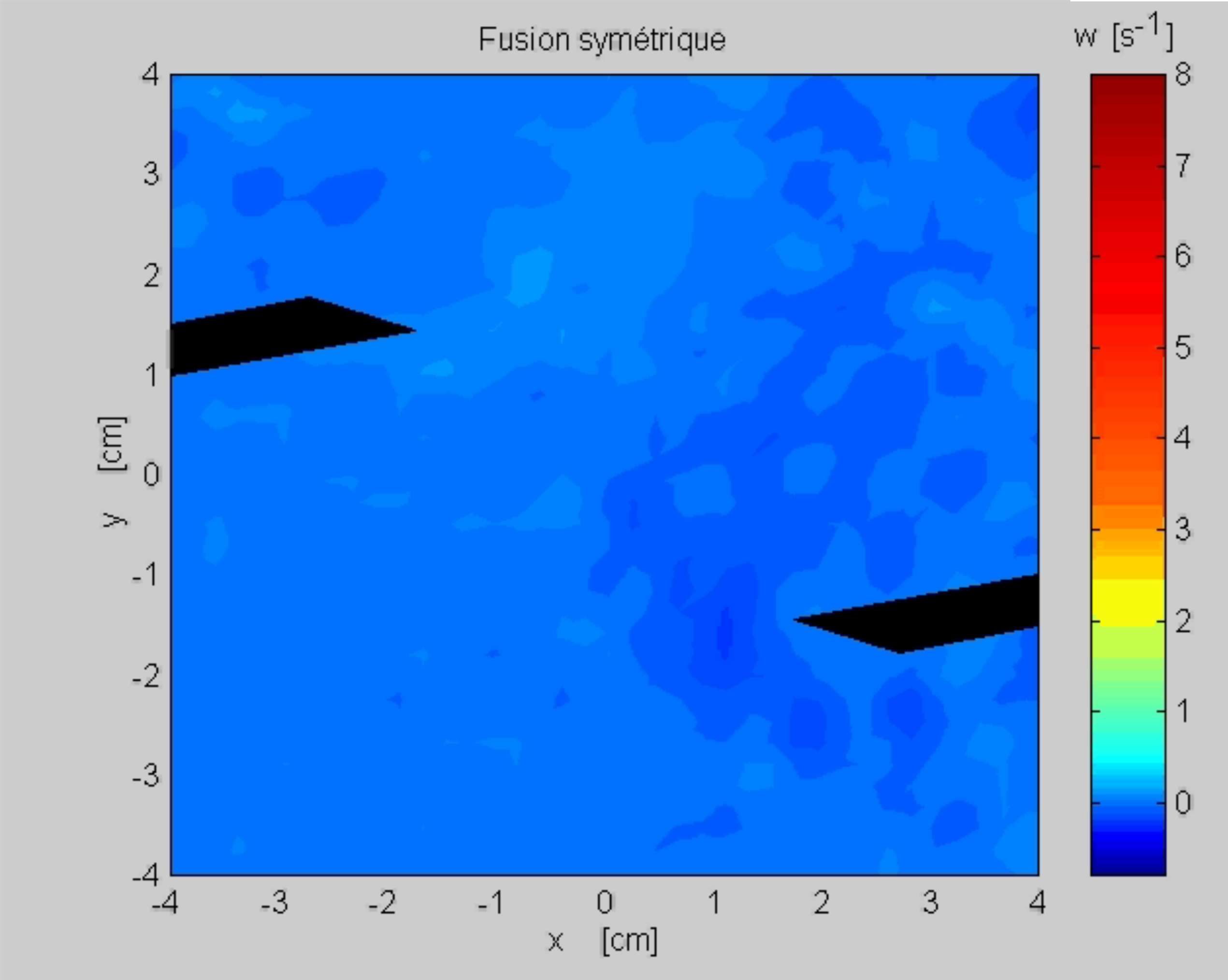






2D merging (*Re* = 500–2500)

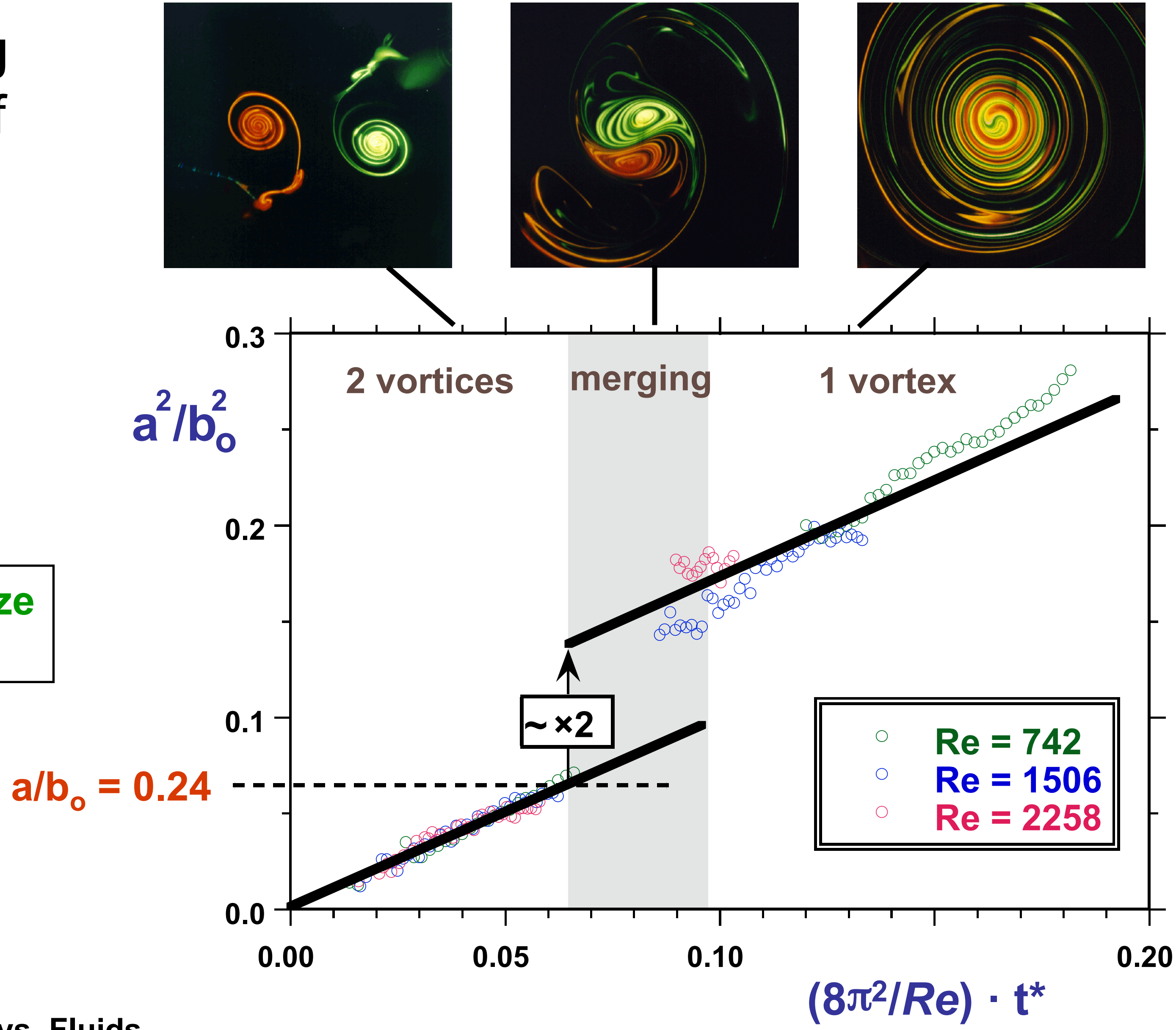
dye visualisation



vorticity from PIV

2D merging
(evolution of core size)

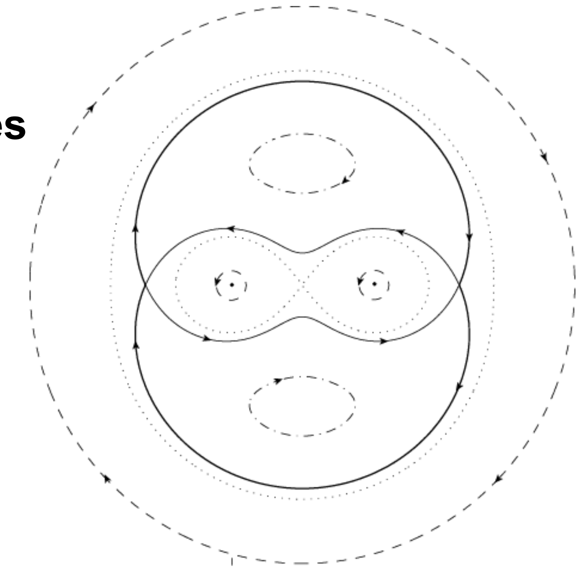
critical core size for merging



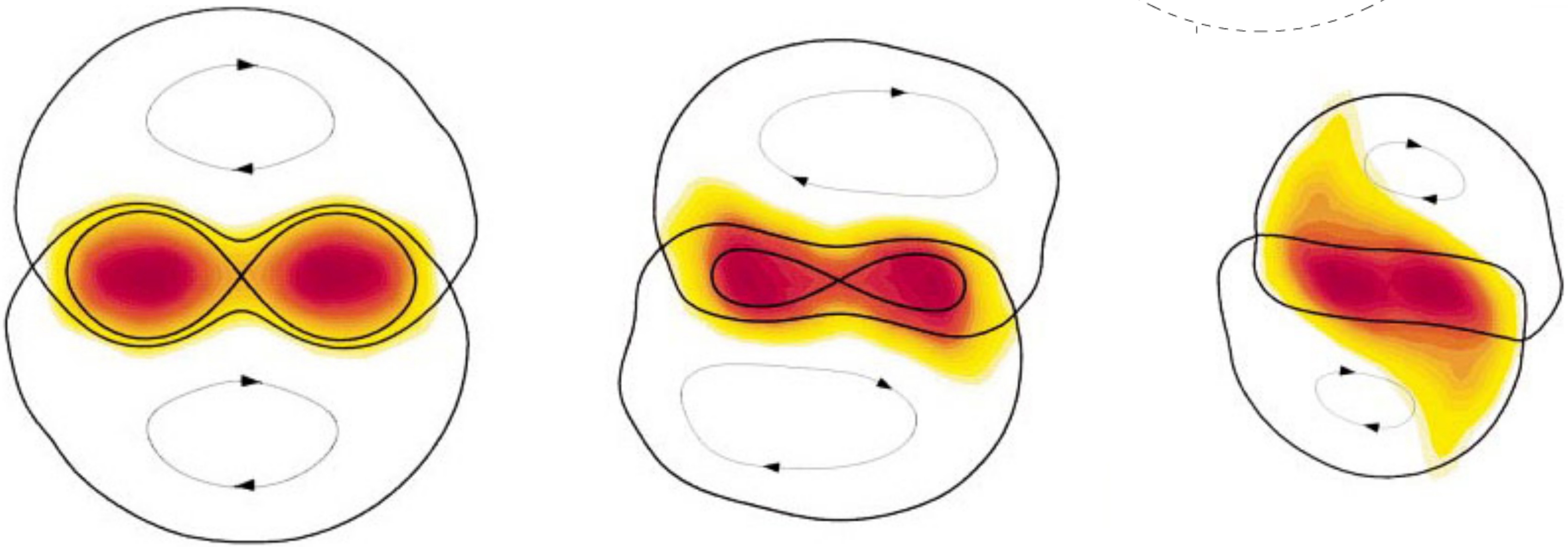
Meunier *et al.* (2002), Phys. Fluids

The mechanism of merging

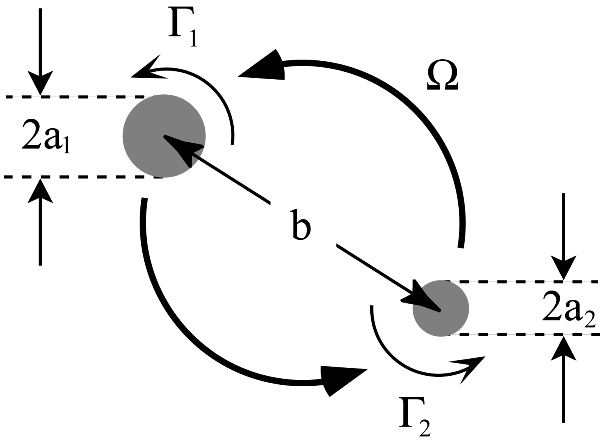
streamlines of two co-rotating point vortices
In the frame rotating with the pair



evolution of vorticity and streamlines
(Cerretelli & Williamson 2003)



Asymmetric 2D merger



example:

$\Gamma_2 / \Gamma_1 = 0.25$; $a_2 / a_1 = 0.5$