Jamming & Flow

Martin van Hecke, Leiden

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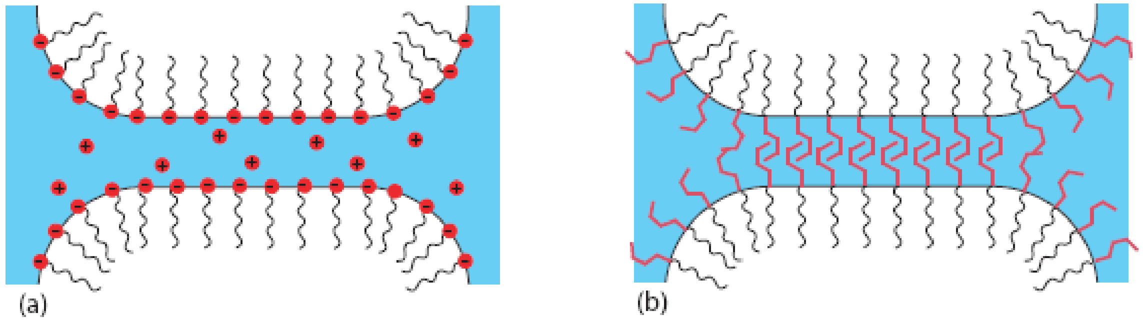
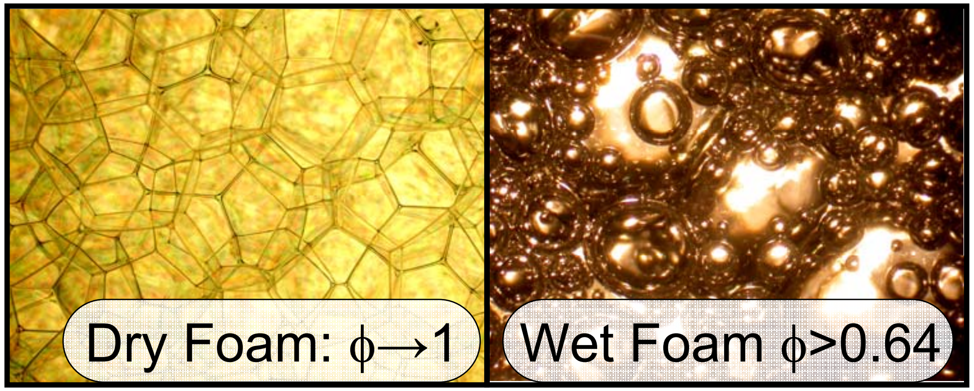
Nichol et al, PRL **104** 078302 (2010)

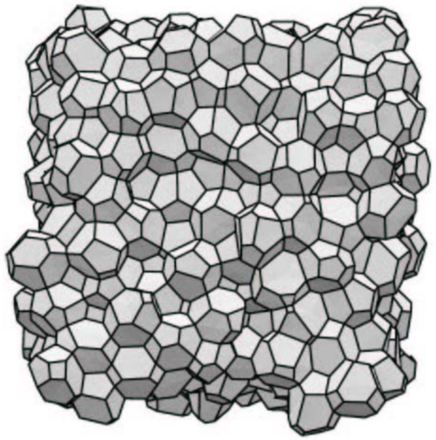
Foam

Gas (nitrogen) + Liquid (water & glycerol) + Surfactant (soap)

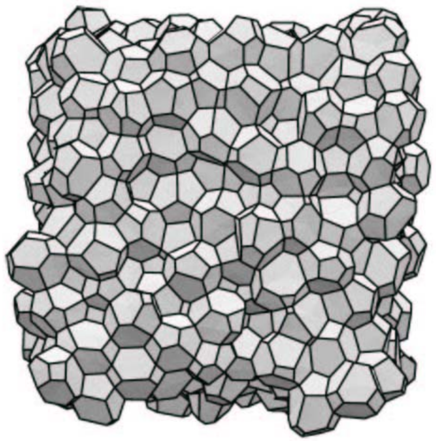


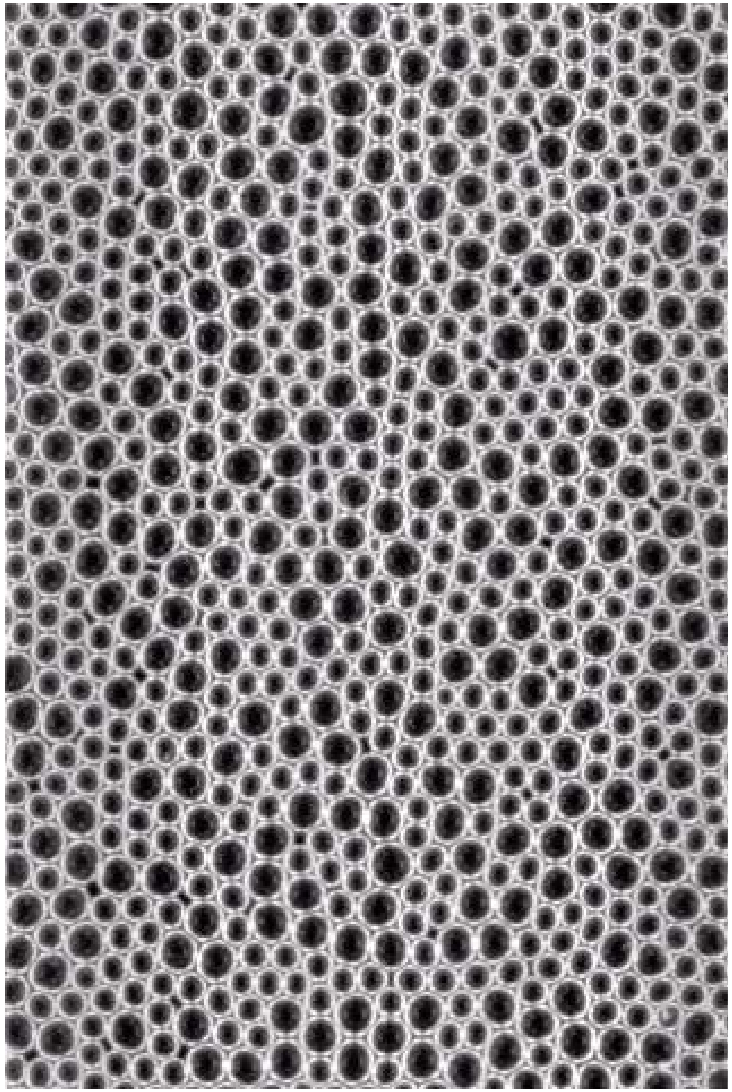


Jamming & Flow



Jamming & Flow





Martin van Hecke, Leiden

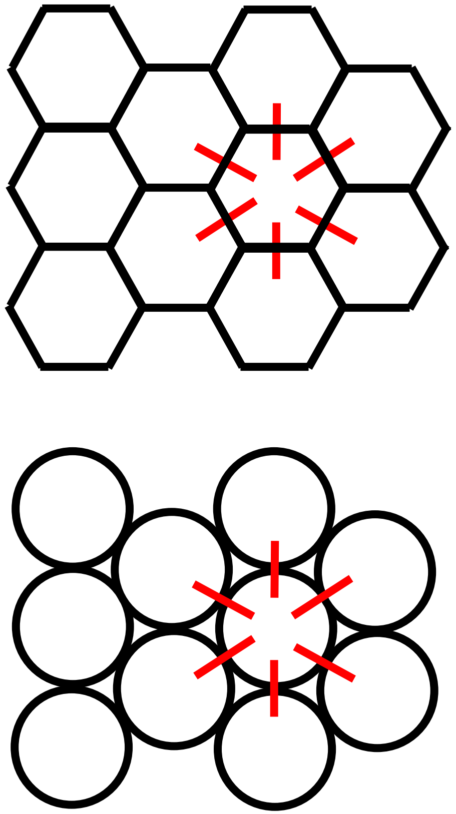
Jamming

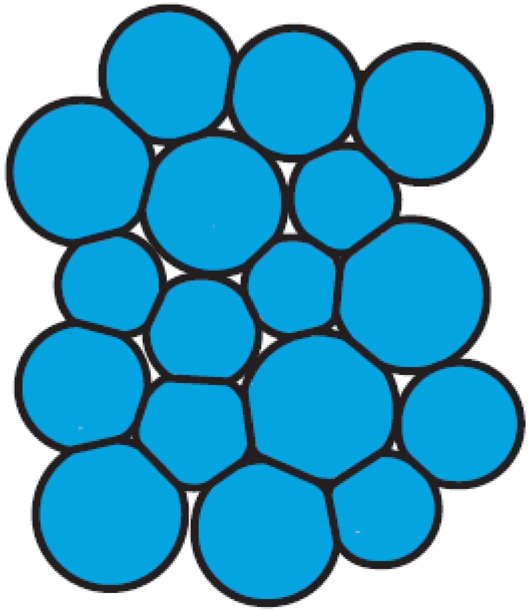
Soft materials: Hybrids, solid/liquid

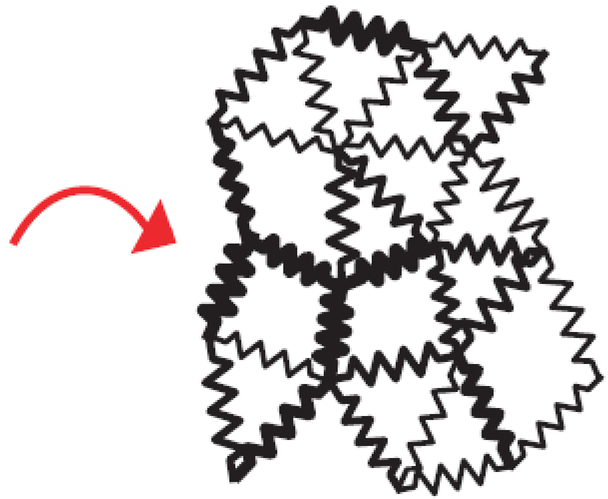
Weird solid: Elasticity of Foams

Weird liquid: Rheology of Foams

Disorder is Crucial!!!



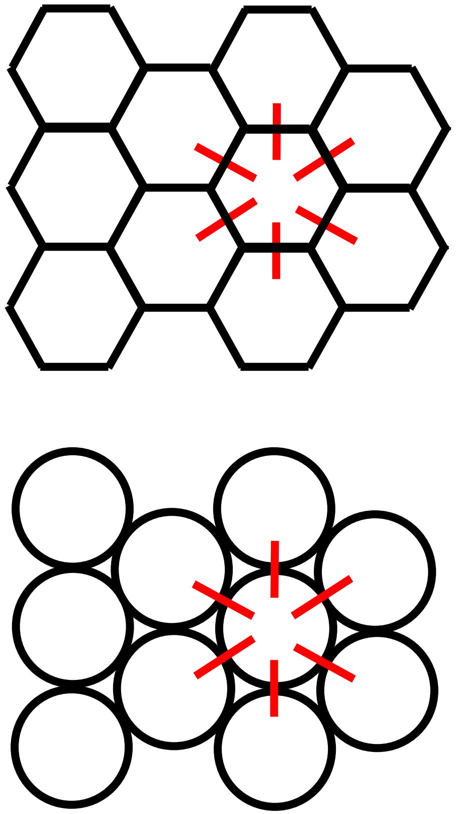




Linear Springs: k

A M Kraynik Annu. Rev. Fluid Mech. 20 325 (1988)
H M Princen J. Colloid Interface Sci. 91 160 (1983)
H M Princen and A D Kiss J. Colloid Interface Sci. 112 427 (1986)
Makse et al

Disorder is Crucial!!!



Unjamming at $\phi = \pi/(2\ \mathrm{sqrt}\ 3) \approx 0.91$

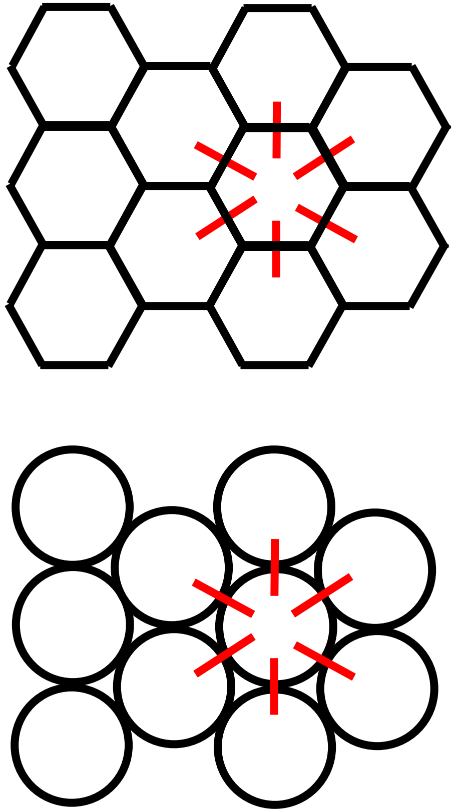
Z=6

Elasticity Independent of Wetness

Compression ~ Shear: K~G~k (2D)

A M Kraynik Annu. Rev. Fluid Mech. 20 325 (1988)
H M Princen J. Colloid Interface Sci. 91 160 (1983)
H M Princen and A D Kiss J. Colloid Interface Sci. 112 427 (1986)
Makse et al

Disorder is Crucial!!!



~~Unjamming at $\phi = \pi/(2 \text{ sqrt } 3) \approx 0.91$~~

~~Z=6~~

~~Elasticity Independent of Wetness~~

~~Compression ~ Shear~~

A M Kraynik Annu. Rev. Fluid Mech. 20 325 (1988)
H M Princen J. Colloid Interface Sci. 91 160 (1983)
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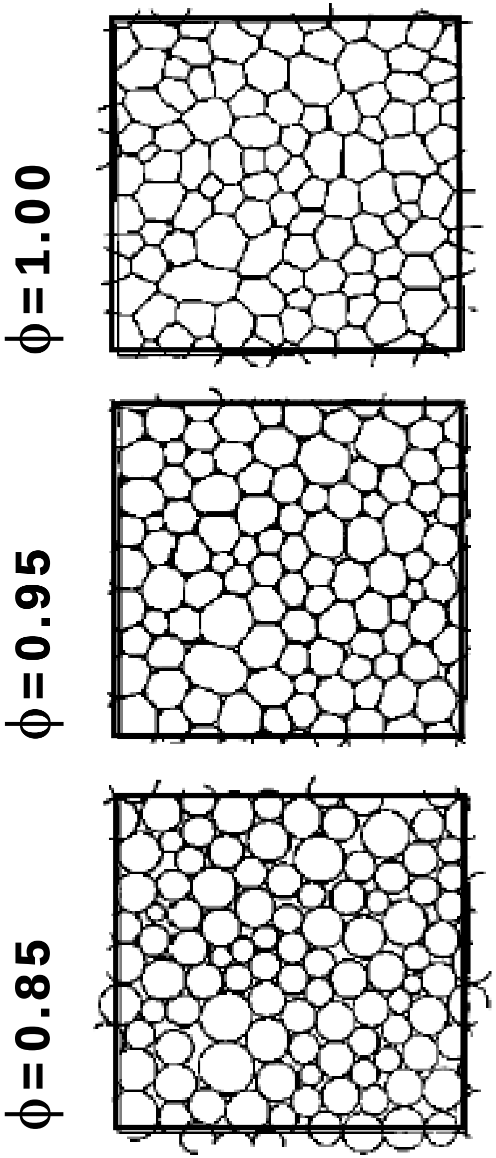
Makse et al

Disorder is Crucial!!!



Unjamming at $\phi \approx 0.84$

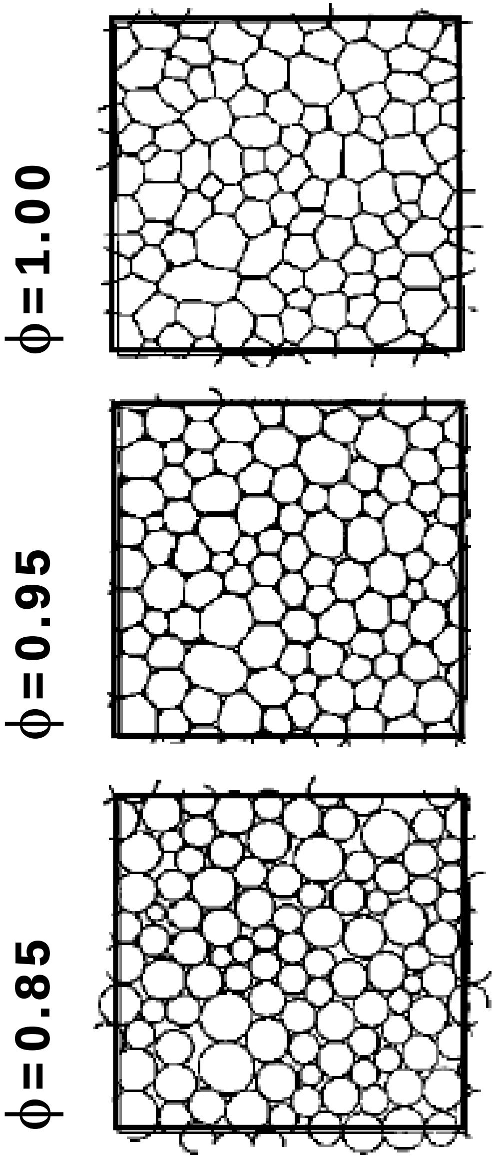
Z from 4 to 6

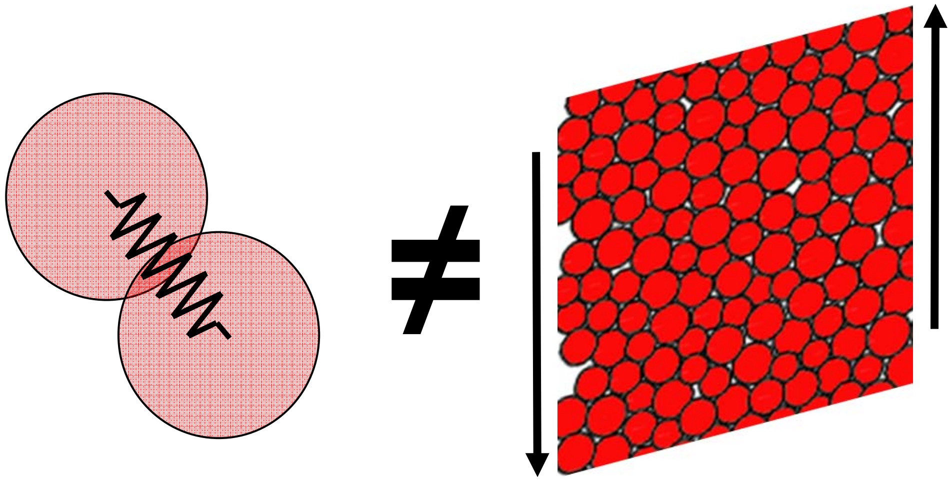
Elasticity Depends on Wetness

Compression ≠ Shear

F Bolton and D Weaire, Phys. Rev. Lett. 65 3449 (1990)

Disorder is Crucial!!!



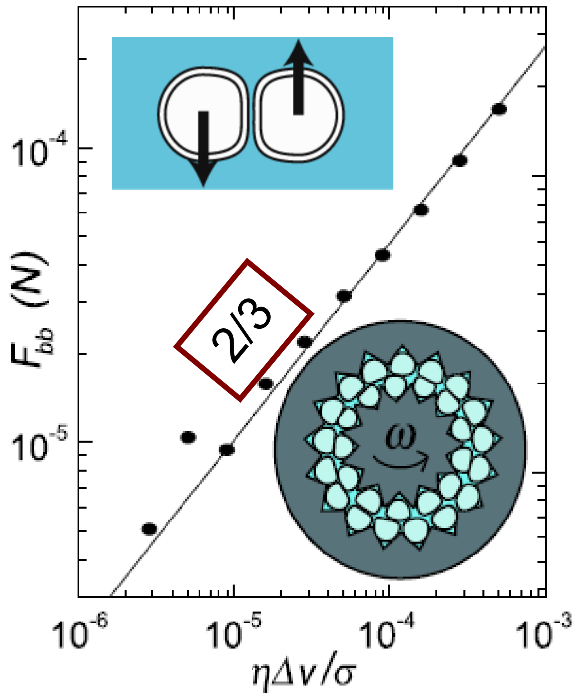


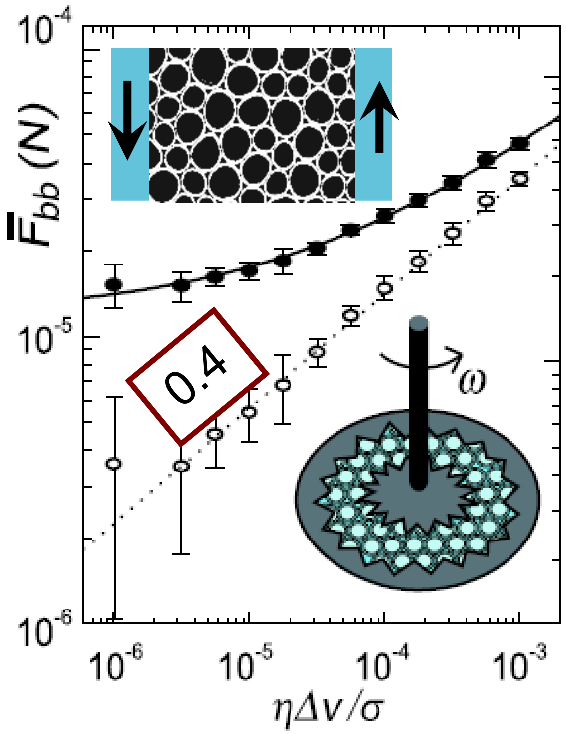
F Bolton and D Weaire, Phys. Rev. Lett. 65 3449 (1990)

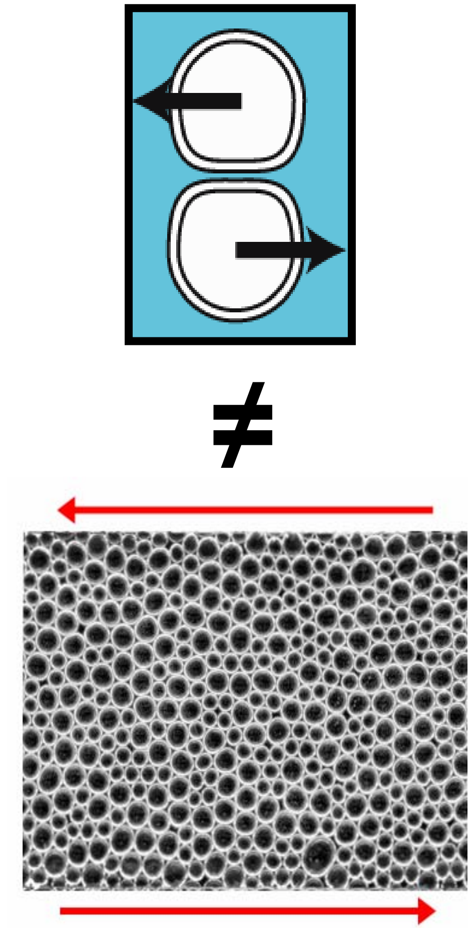
Flow and Disorder: Experiments

Bubble-Bubble
$F_{bb} \sim \Delta v^{2/3}$

Foam
$\bar{F}_{bb} \sim F_y + \Delta v^{0.4}$







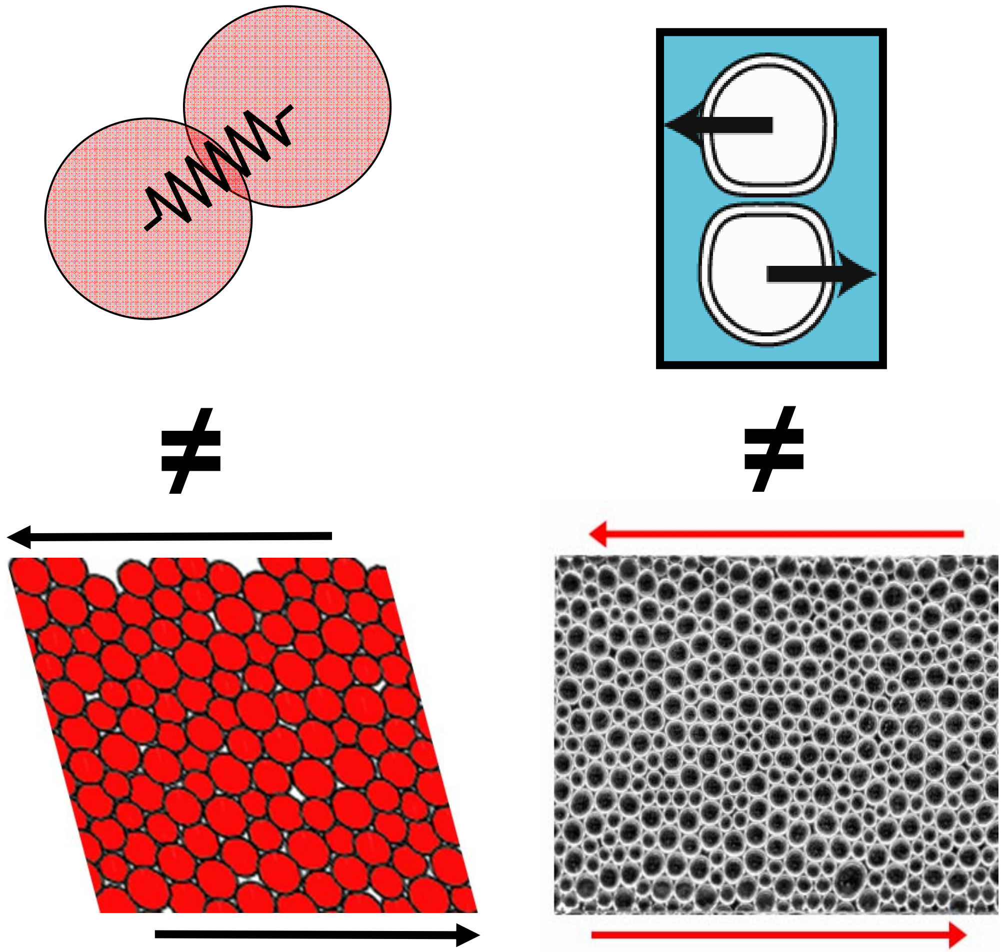
Gijs Katgert, Matthias Möbius & MvH, PRL 101 058301 (2008); PRE 79 066318 (2009)

Foams and Disordered Media

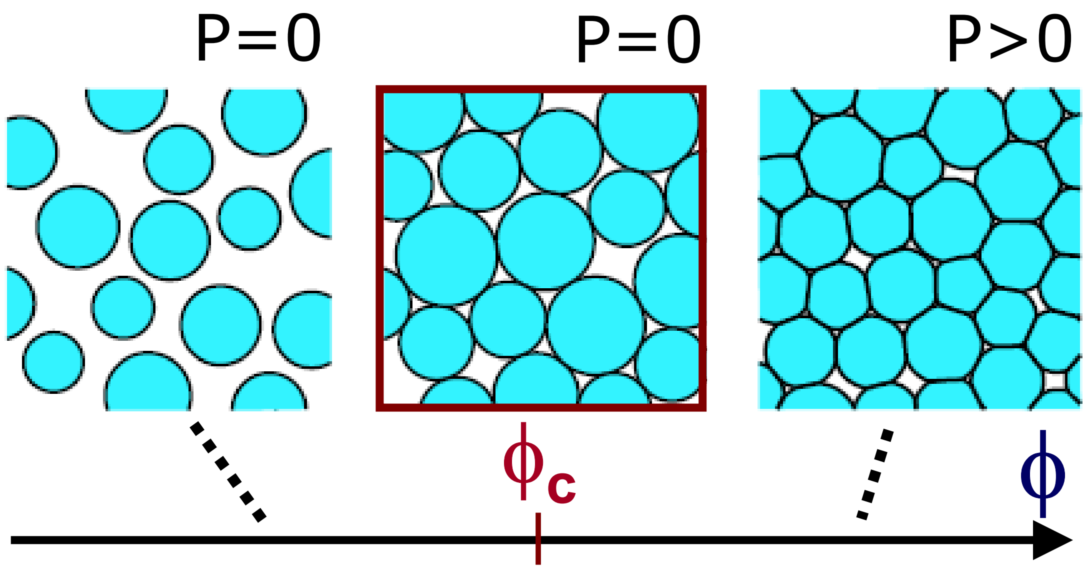
Solid/liquid

Weird solid

Weird liquid

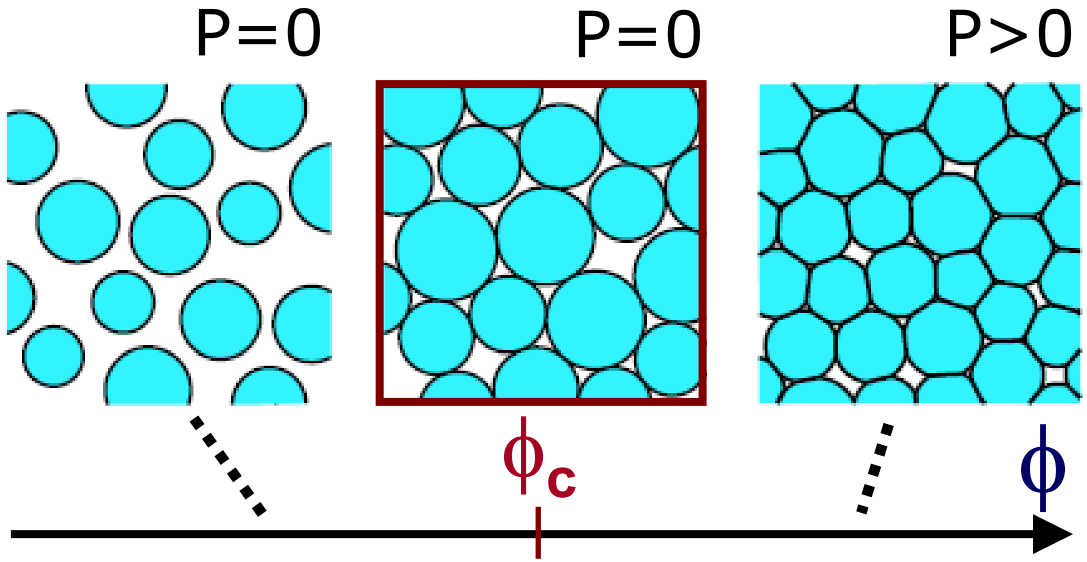


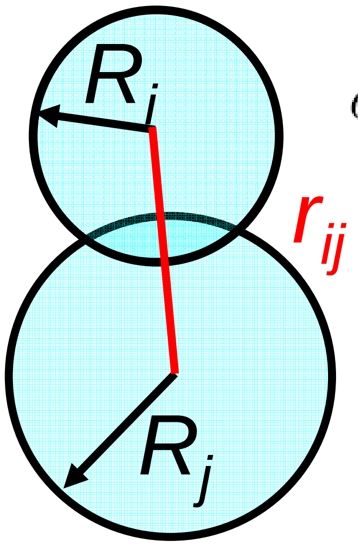
Simple Model for Disordered Jammed Matter



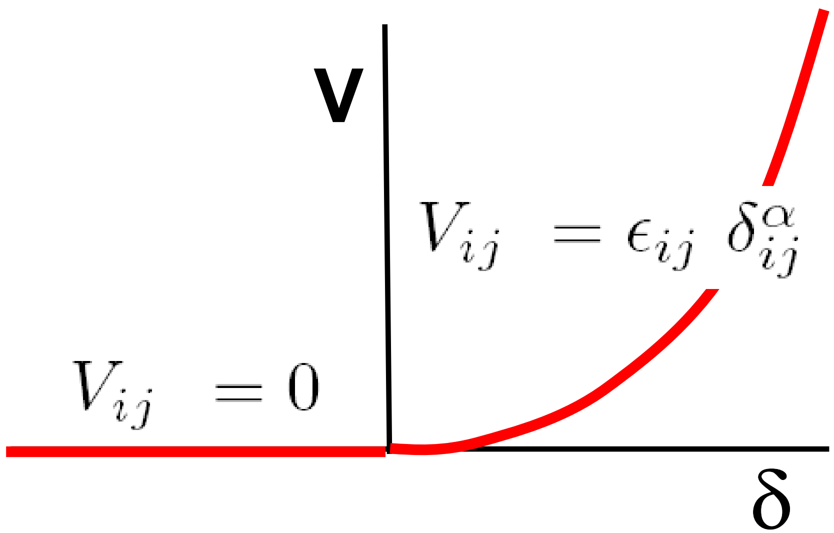
Disordered Packings
Purely Repulsive
Purely Mechanical (T=0)

Simple Model for Disordered Jammed Matter



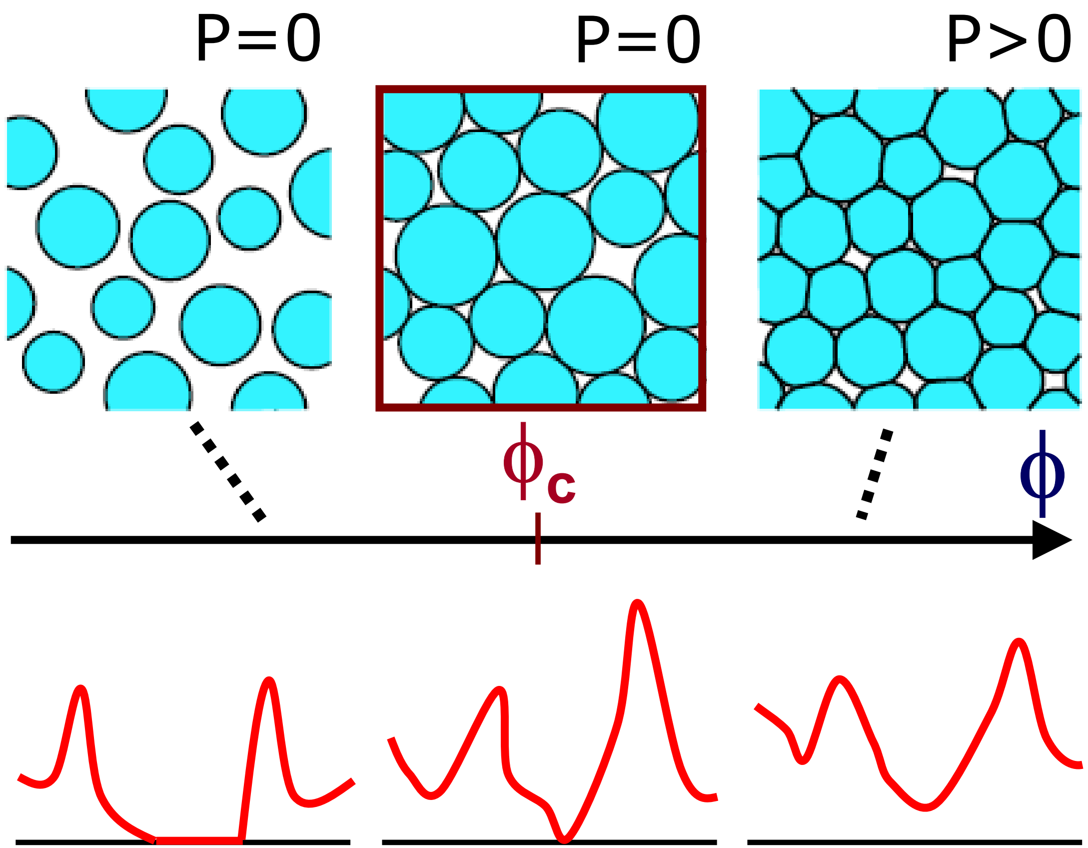


$$\delta_{ij} := 1 - \frac{r_{ij}}{R_i + R_j}$$



CS O'Hern et al, PRE 2003

Simple Model for Disordered Jammed Matter

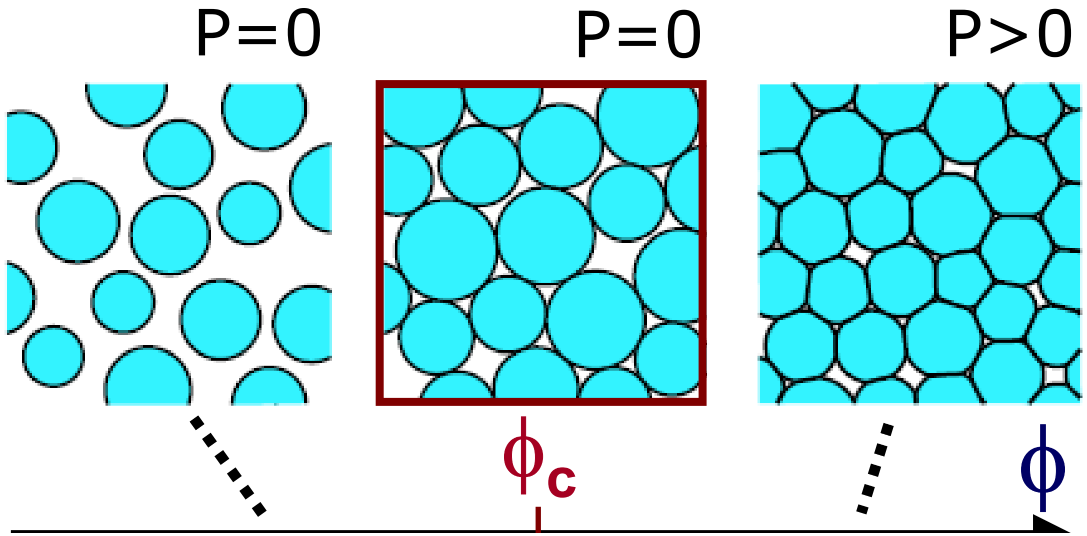


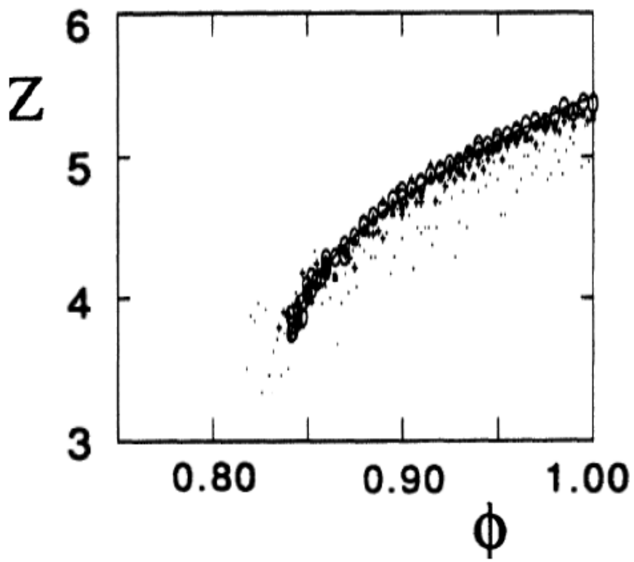
What are mechanical properties as function of P?

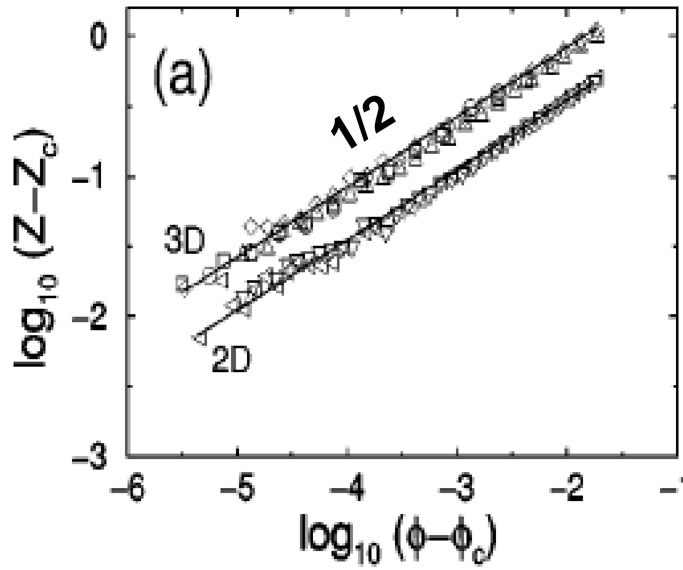
CS O'Hern et al, PRE 2003

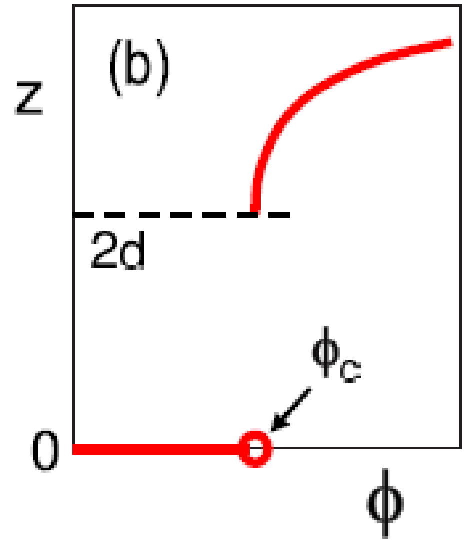
Scaling near Jamming: Contact Number

$z - z_c \sim \mathrm{sqrt}(\phi - \phi_c)$



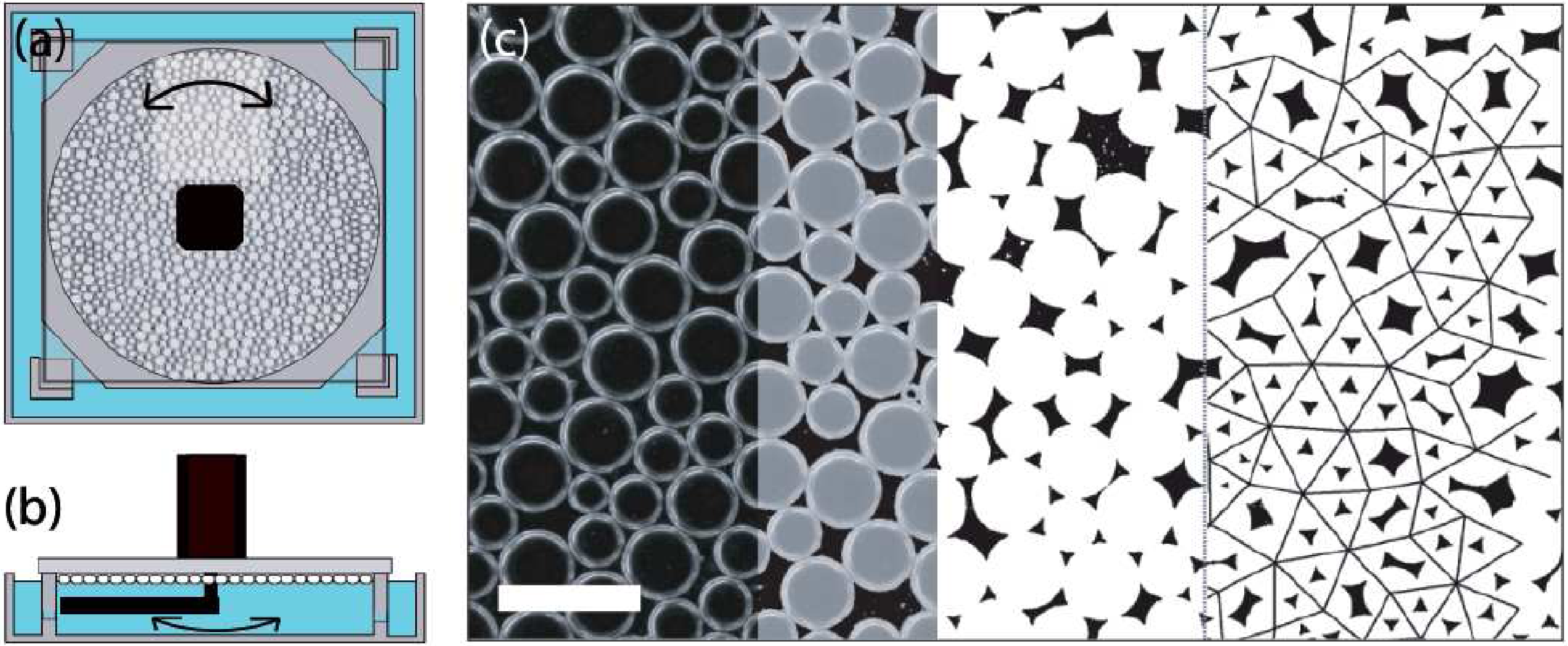






D Durian, PRL 1995; CS O'Hern et al, PRE 2003

Contact Number in 2D Foams

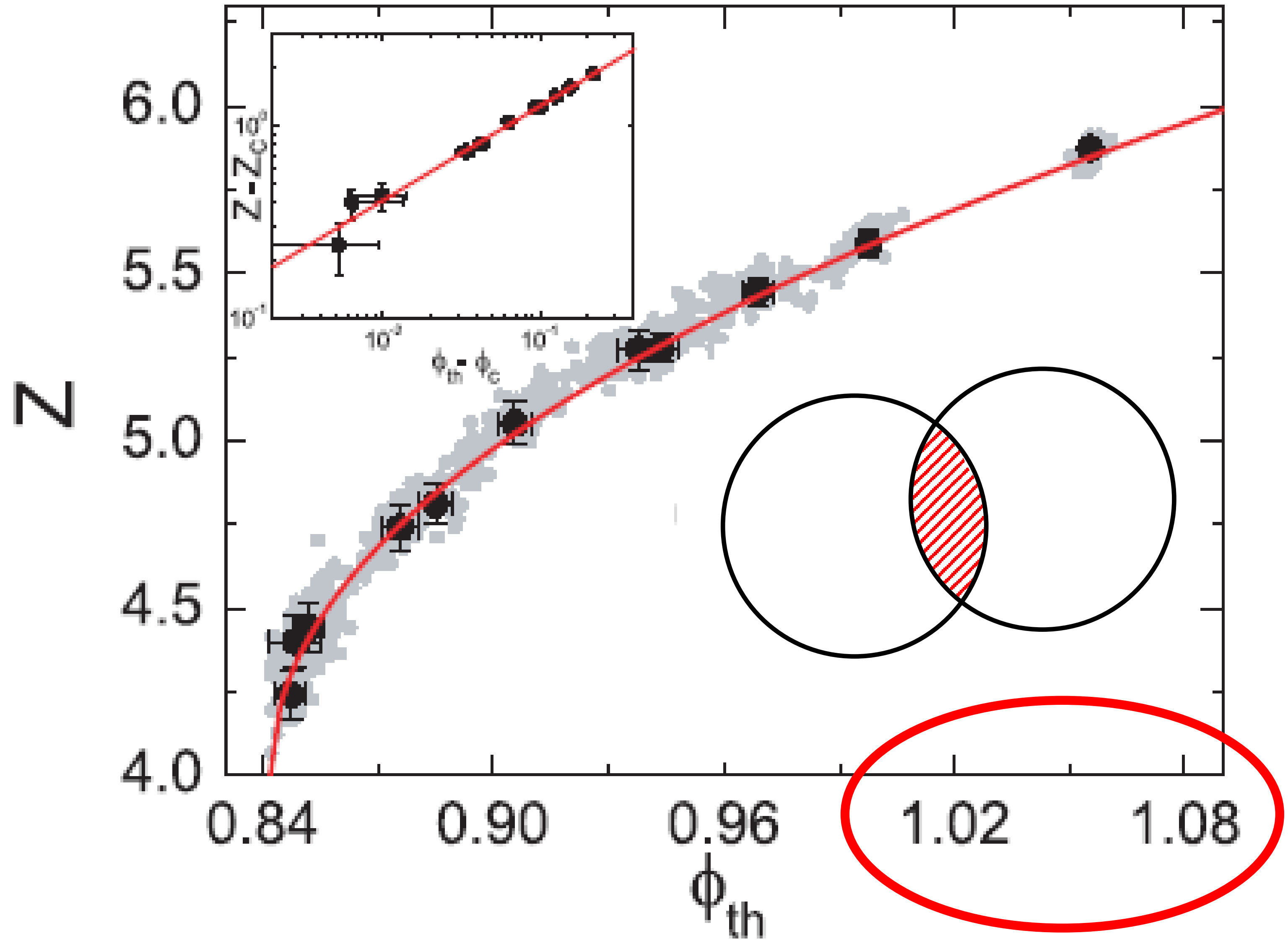


ϕ: Density

z: Contact number

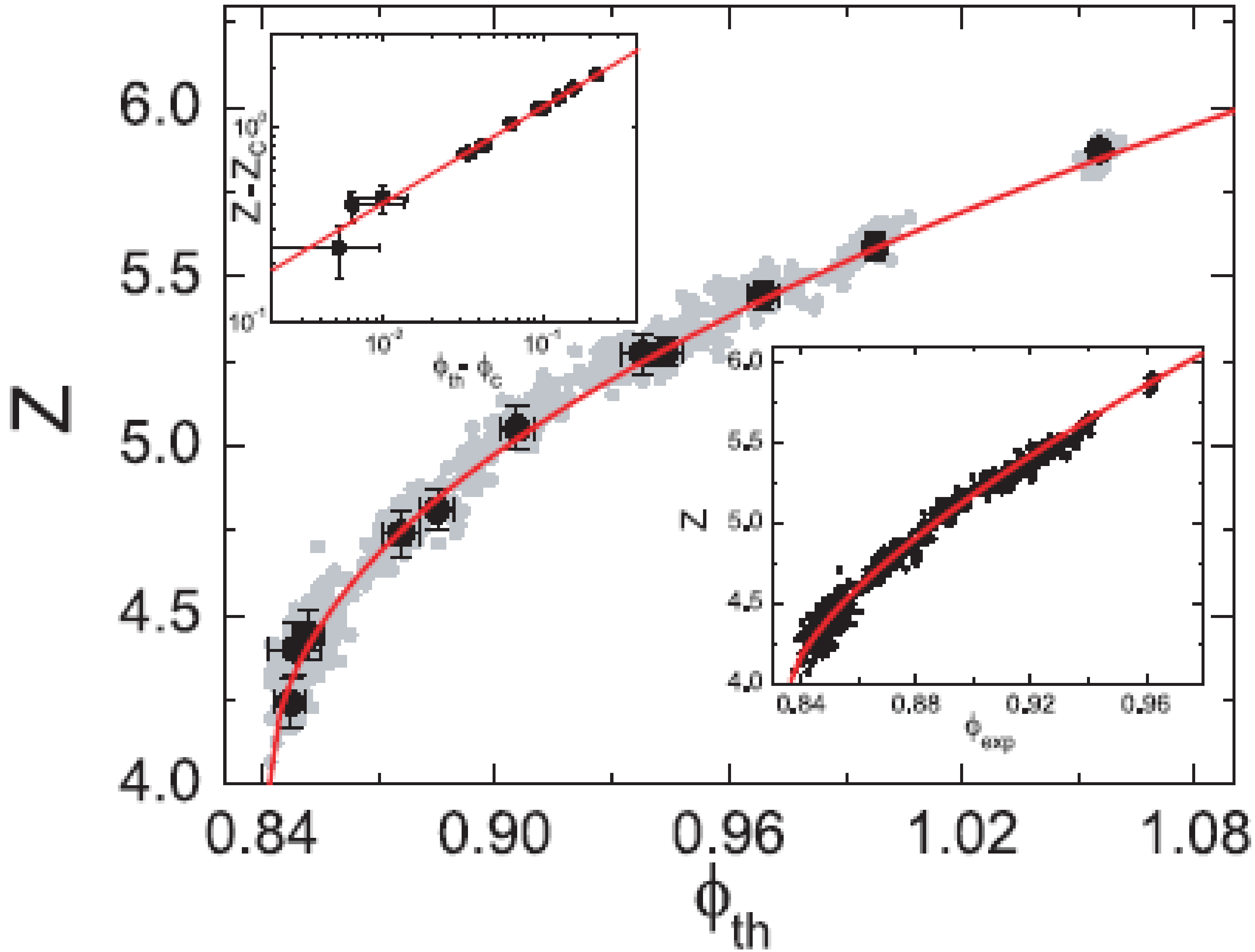
G Katgert and MvH, EPL 92 34002 (2010)

Contact Number in 2D Foams



G Katgert and MvH, EPL 92 34002 (2010)

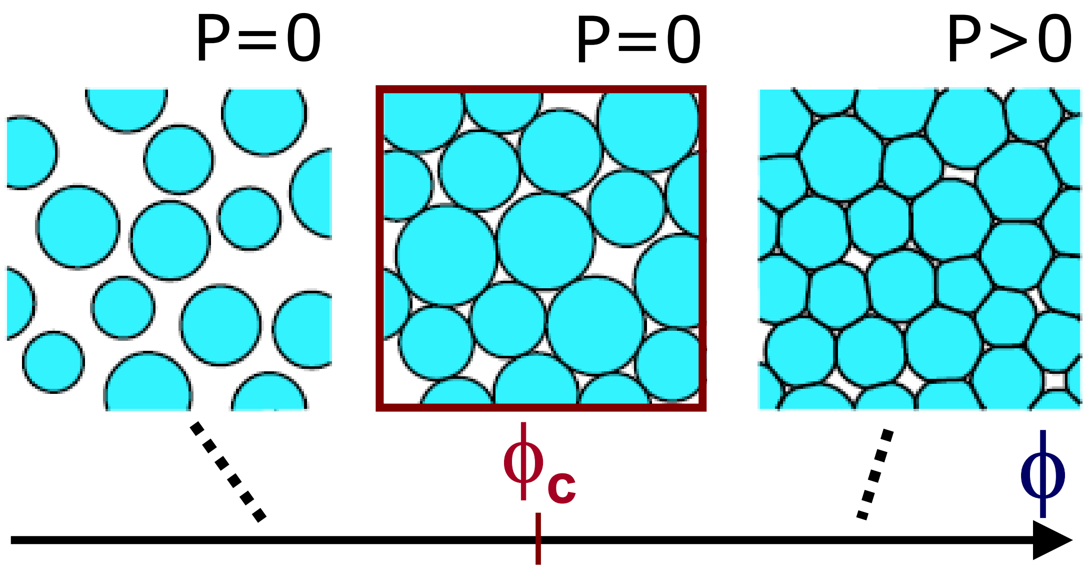
Contact Number in 2D Foams

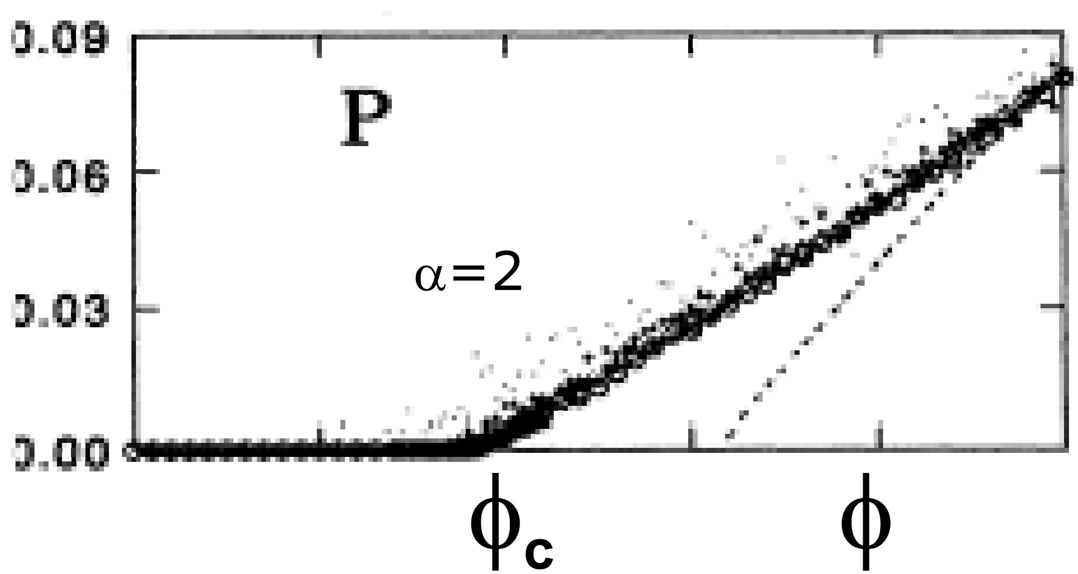


G Katgert and MvH, EPL 92 34002 (2010)

Scaling near Jamming: Pressure

$$P \sim (\phi - \phi_c)^{\alpha - 1}$$





D Durian, PRL 1995; CS O'Hern et al, PRE 2003

Scaling near Jamming: Pressure

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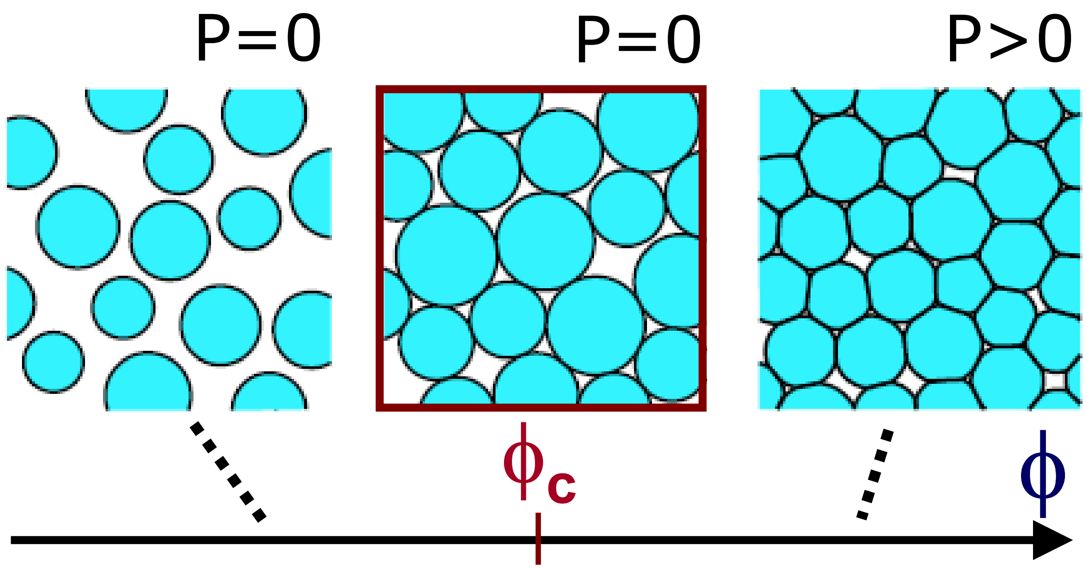
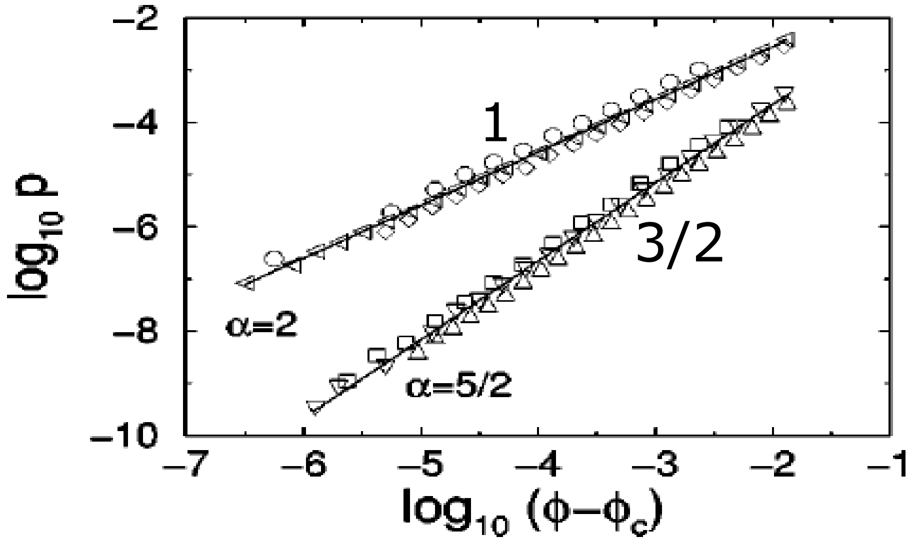
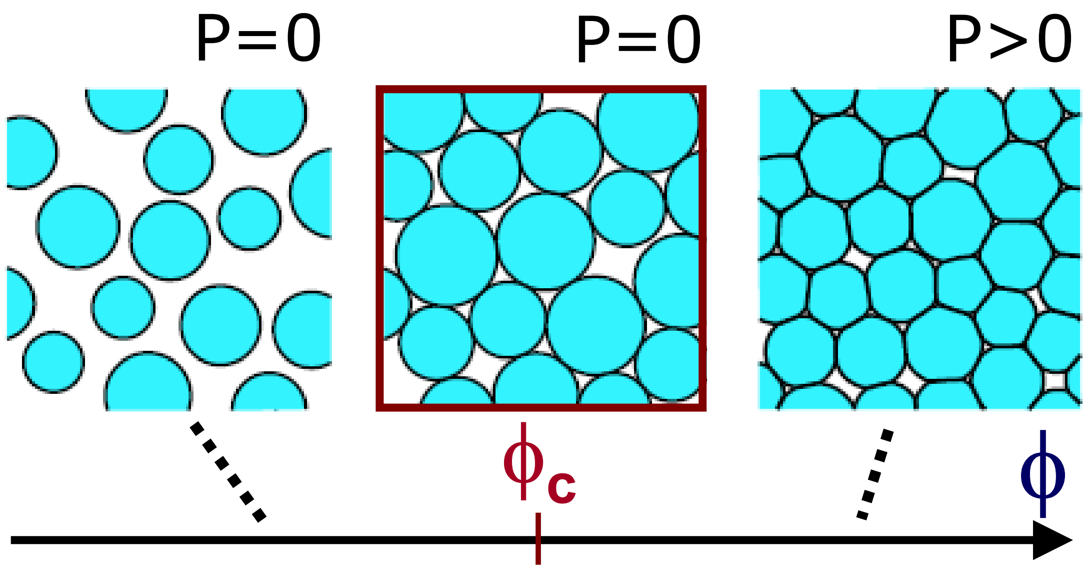


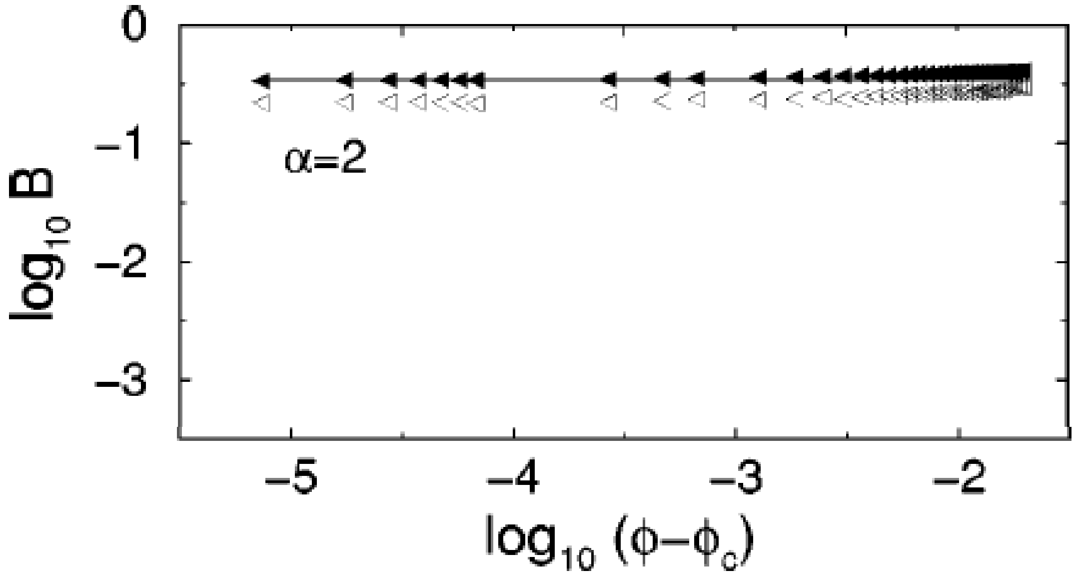


D Durian, PRL 1995; CS O'Hern et al, PRE 2003

Scaling near Jamming: Bulk Modulus

$$B \sim (\phi - \phi_c)^{\alpha - 2}$$

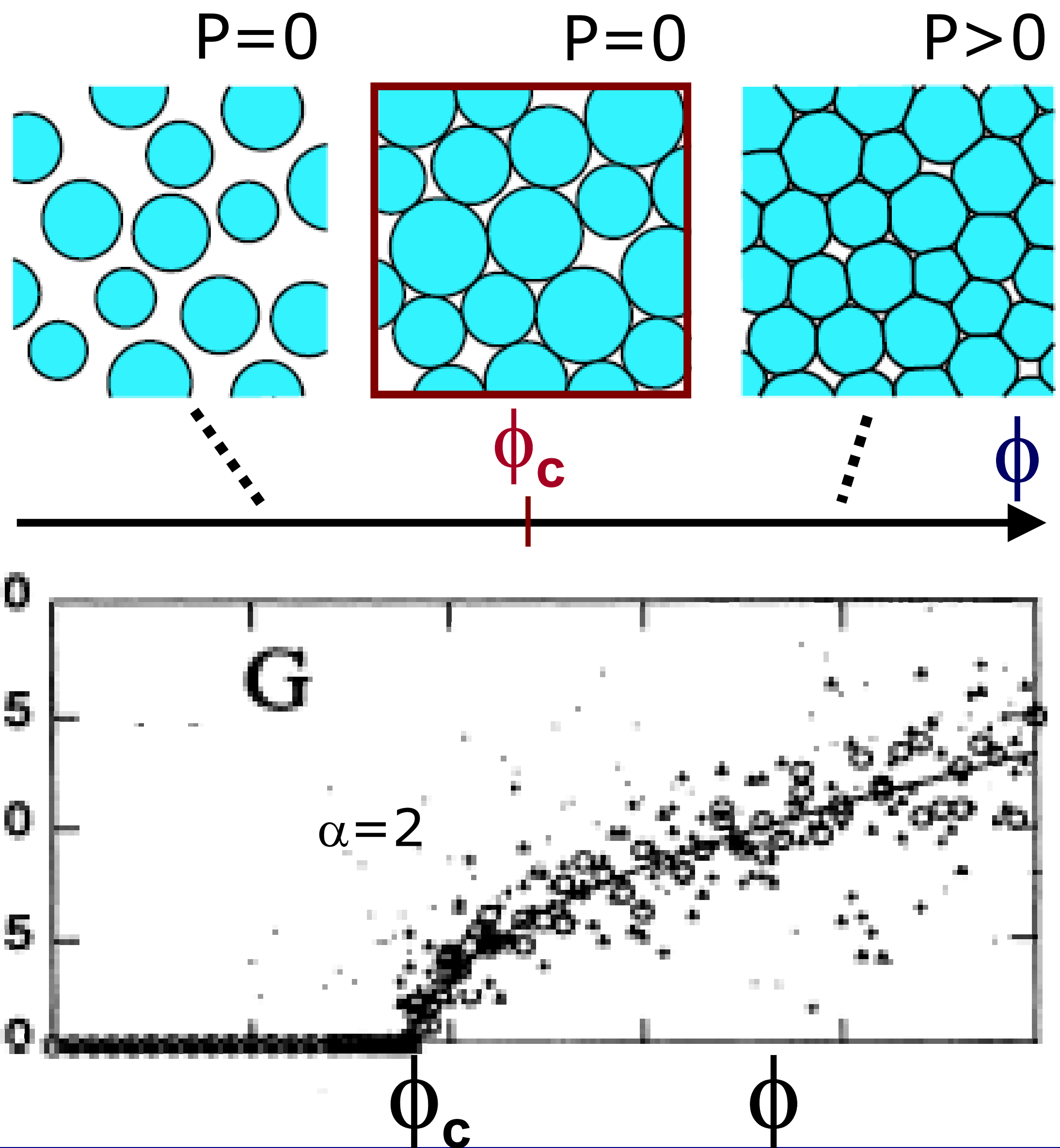




D Durian, PRL 1995; CS O'Hern et al, PRE 2003

Scaling near Jamming: Shear Modulus

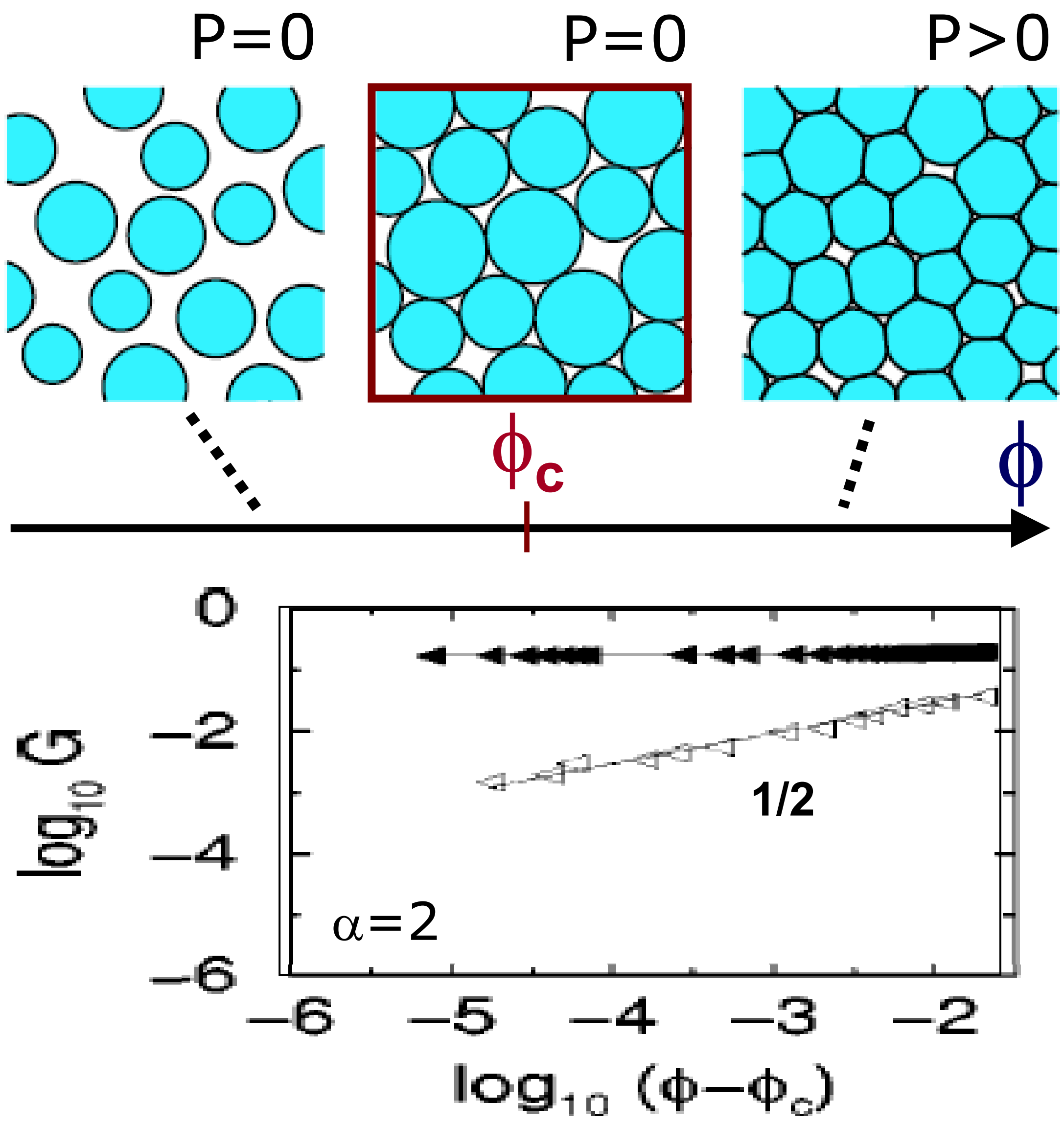
G ~ ???



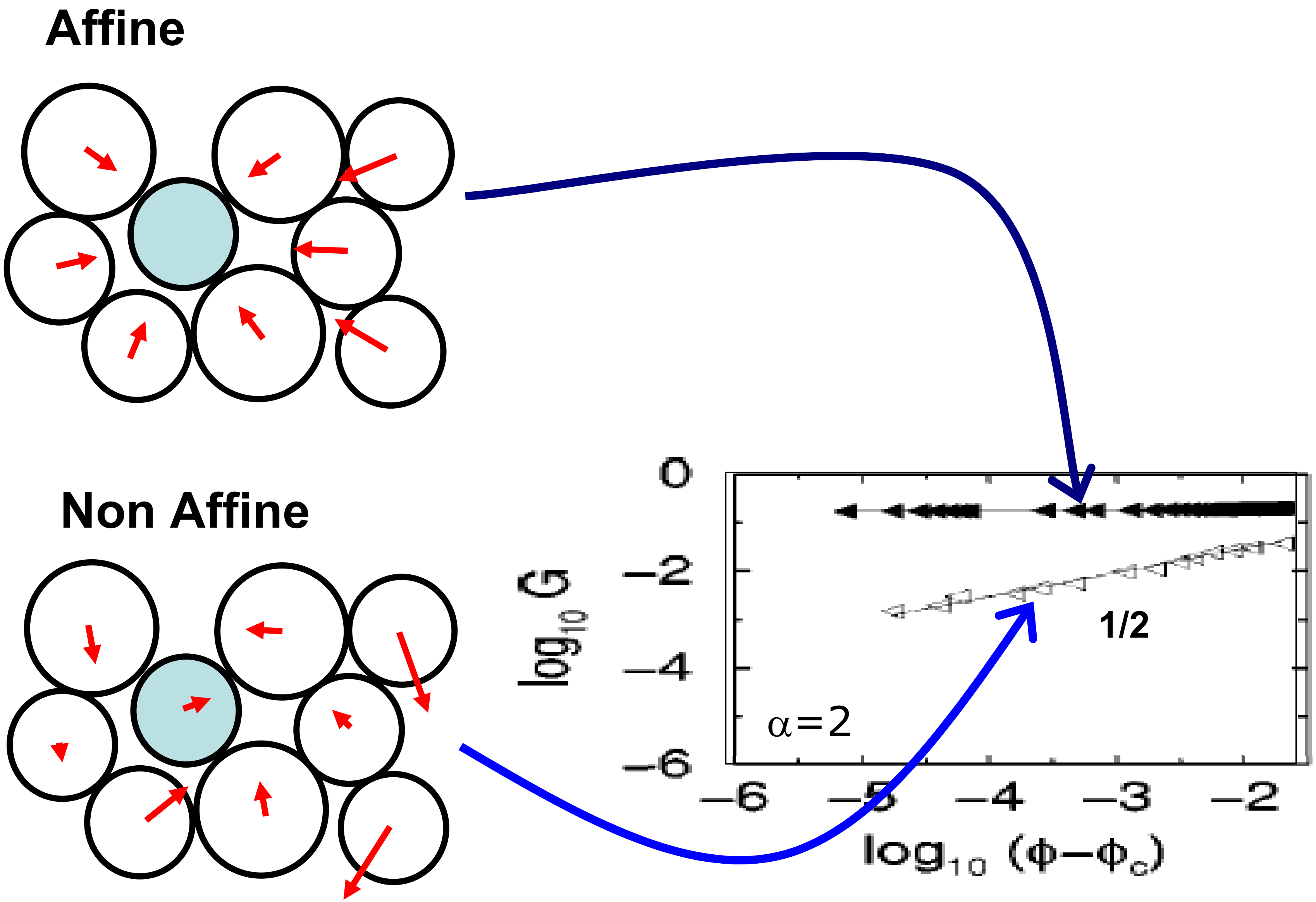
D Durian, PRL 1995; CS O'Hern et al, PRE 2003

Scaling near Jamming: Shear Modulus

$$G \sim (\phi-\phi_c)^{\alpha-3/2}$$

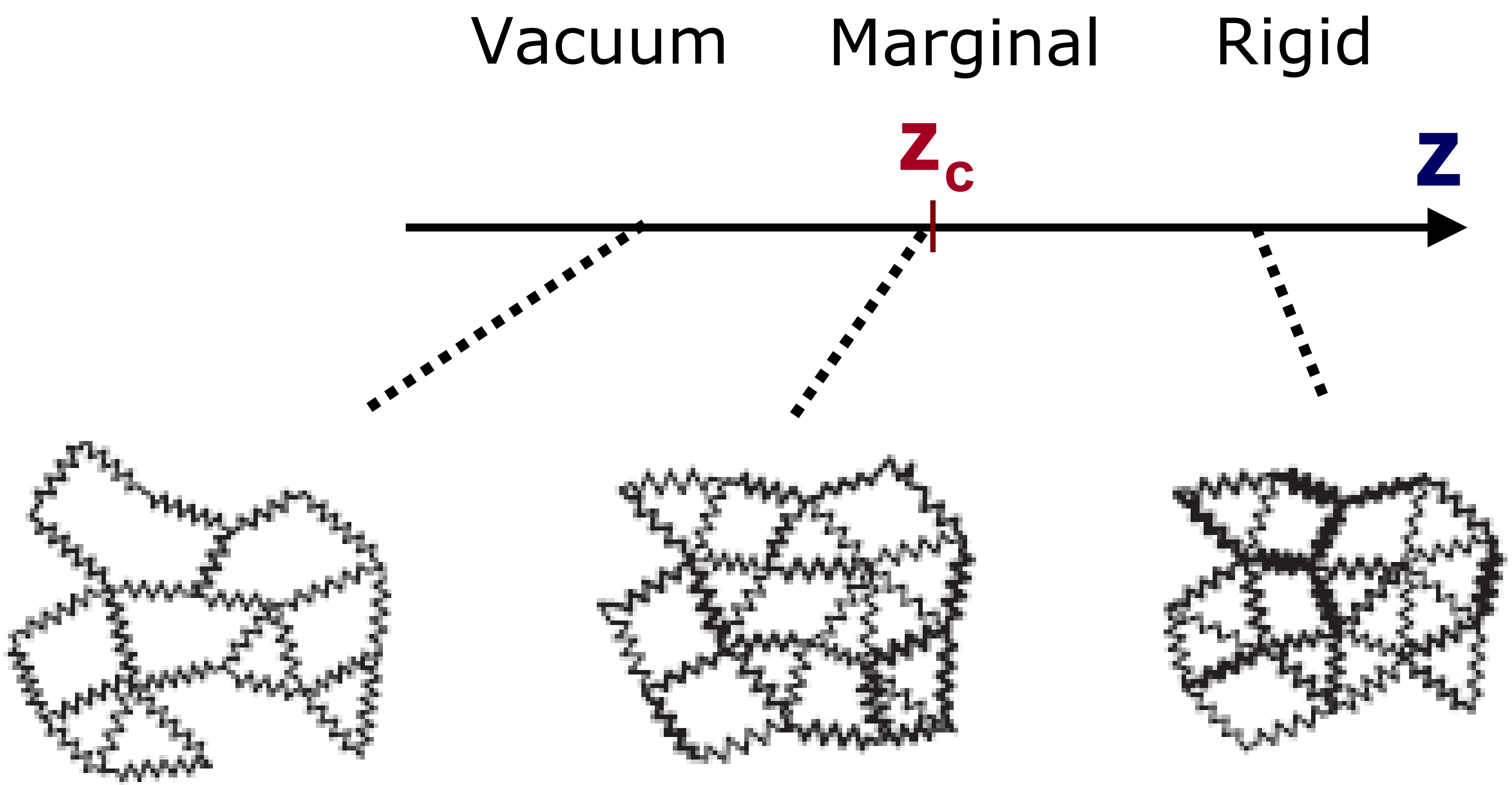


Scaling near Jamming: Shear Modulus



D Durian, PRL 1995; CS O'Hern et al, PRE 2003

Floppy Networks

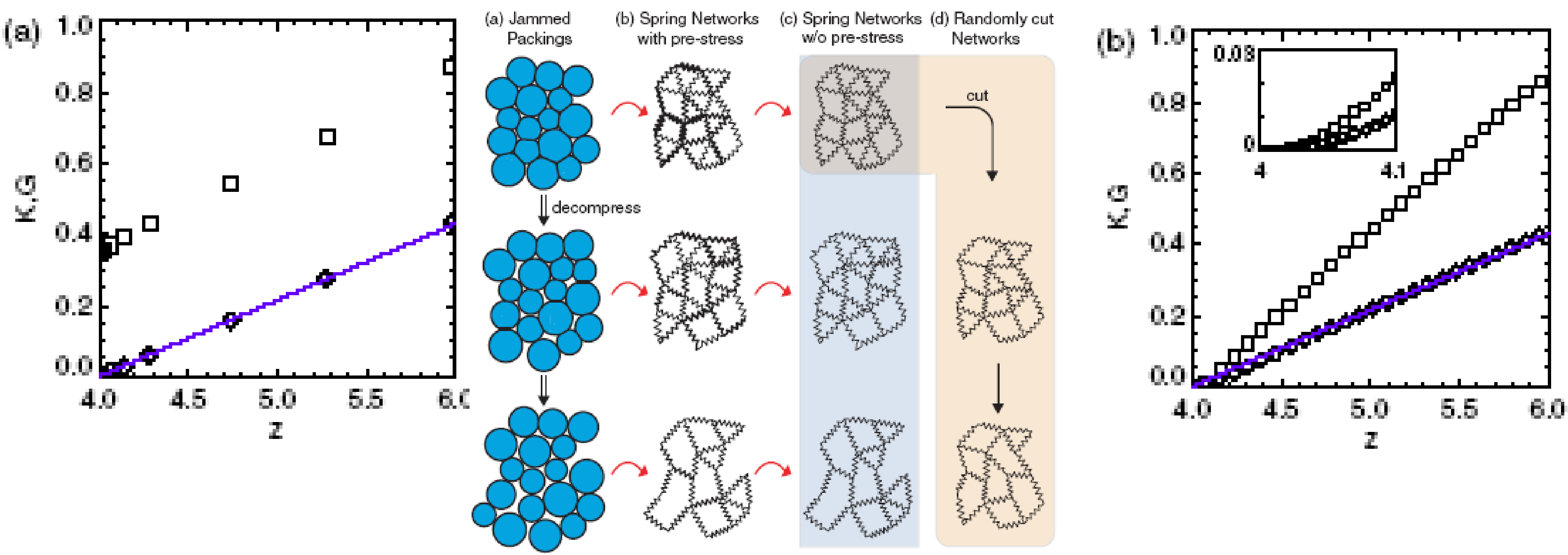


Rigidity vanishes when $z \rightarrow z_c$

$$\mathbf{G \sim (\phi-\phi_c)^{\alpha-3/2} \sim (\phi-\phi_c)^{\alpha-2}\, \Delta z \sim k\, \Delta z}$$

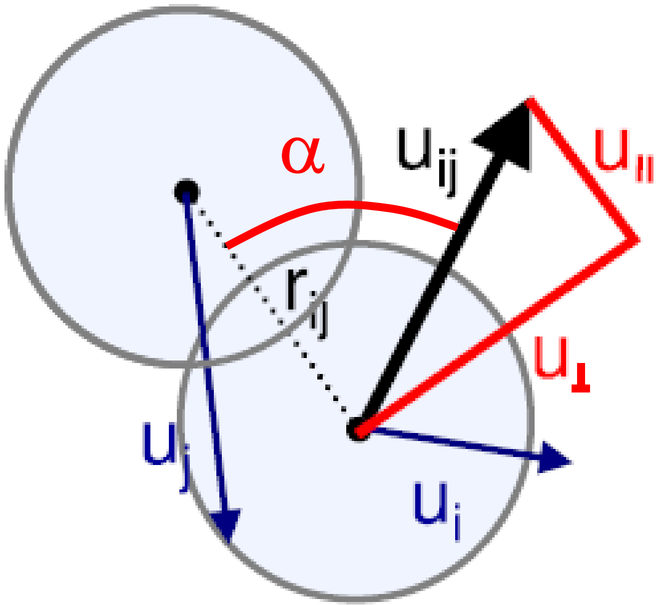
$$\mathbf{B \sim \qquad k??}$$

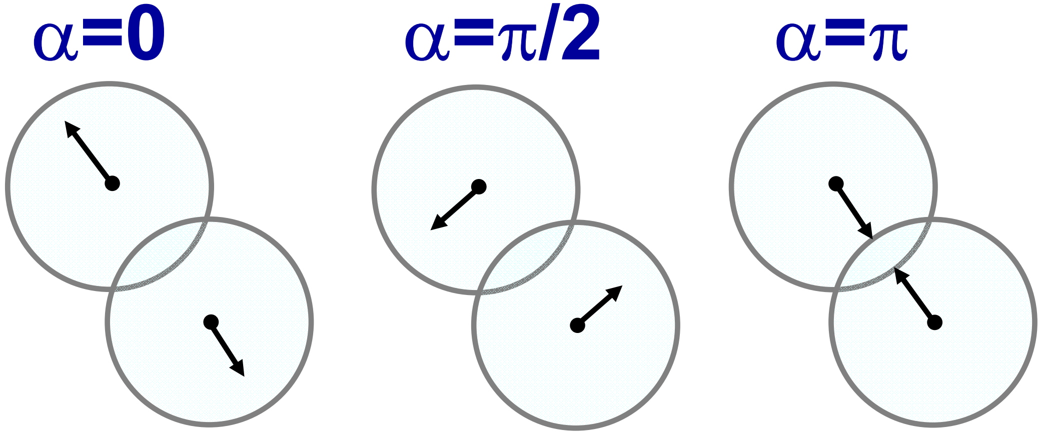
Elasticity Random Networks



Compression Jammed Packings is Special!

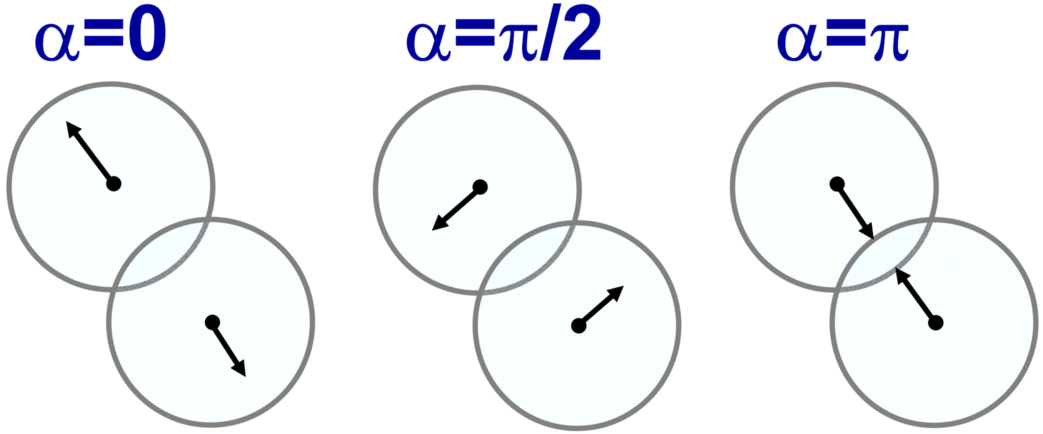
Local Probe: Relative Displacements

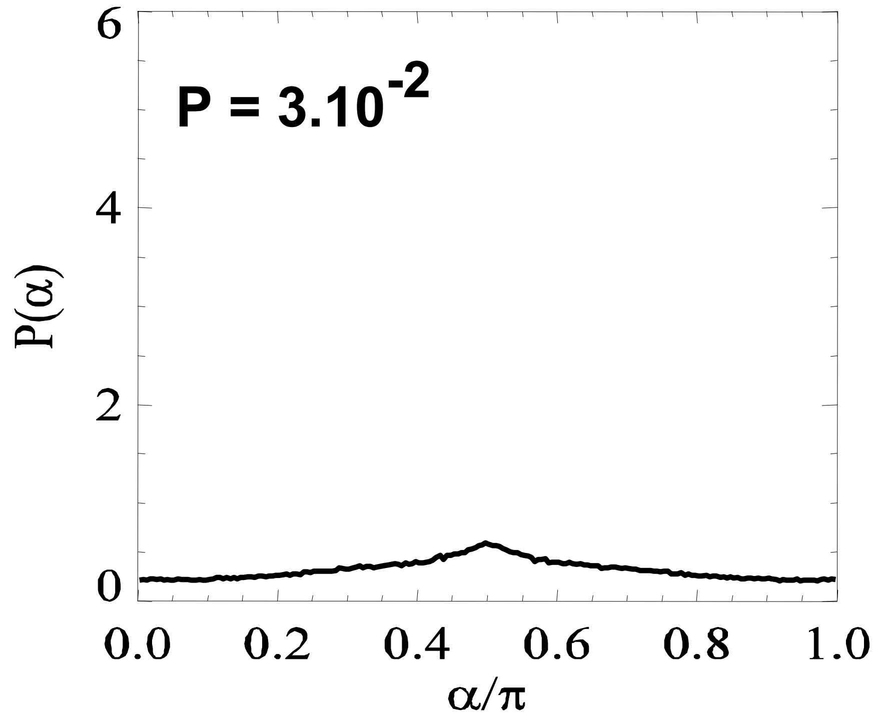


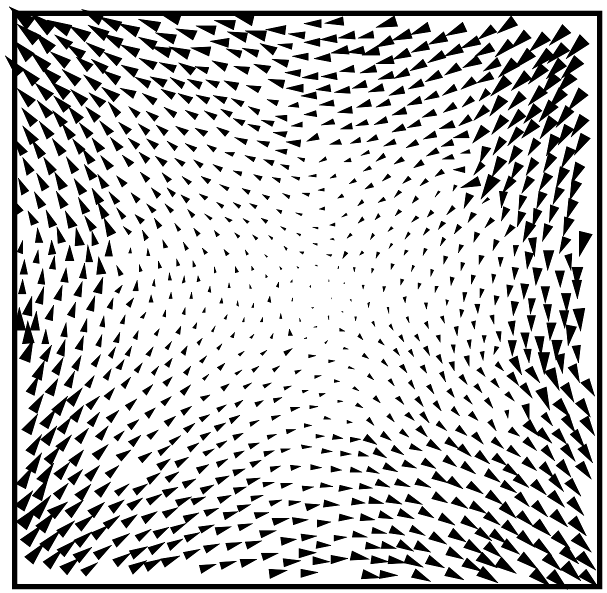


$$P(\alpha), \quad P(u_{//}), \quad P(u_{perp})$$

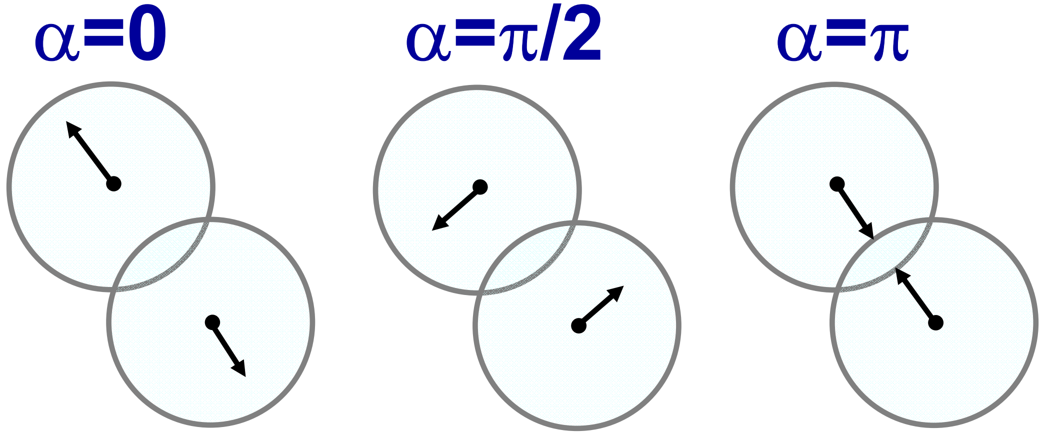
P(α): Shear

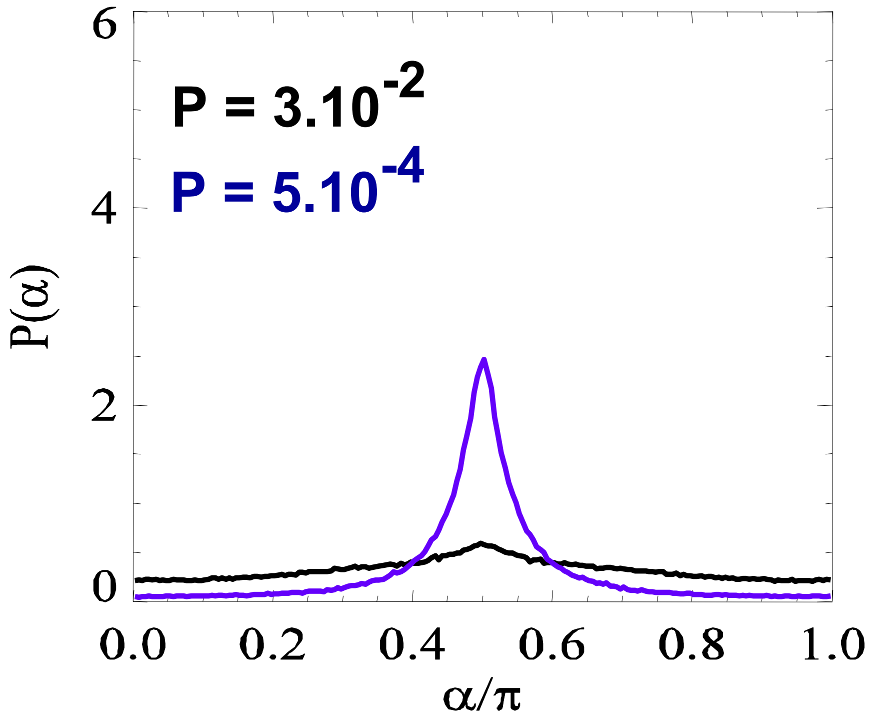


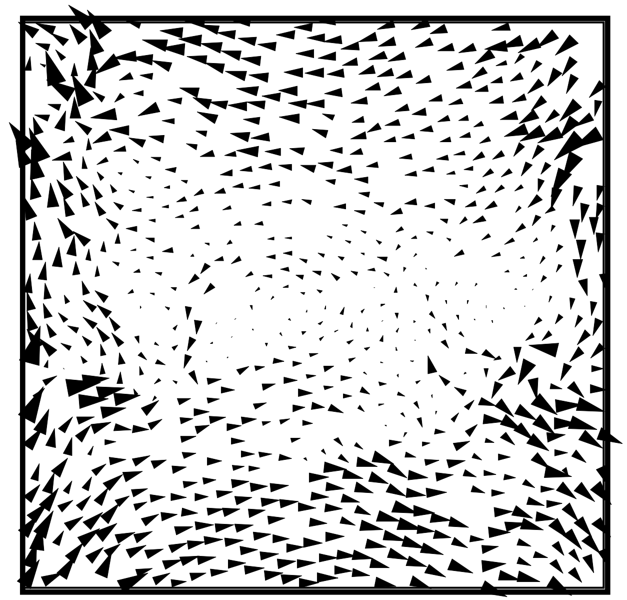




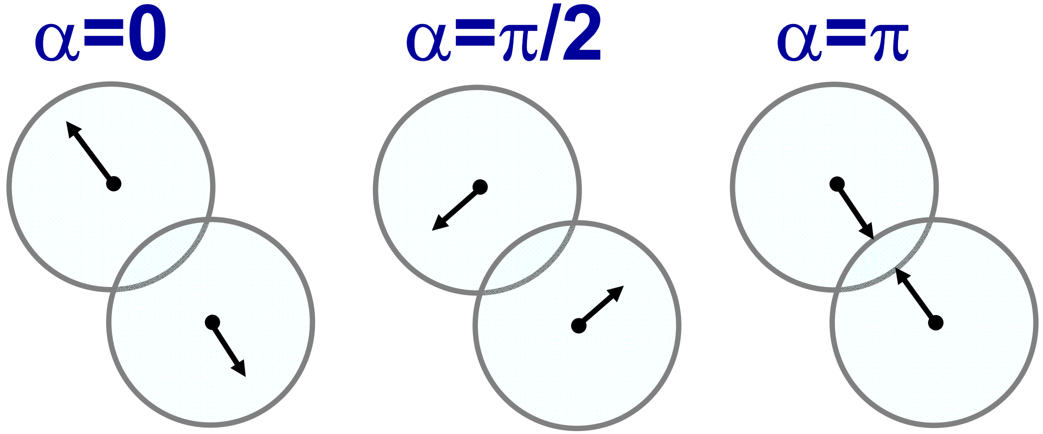
P(α): Shear

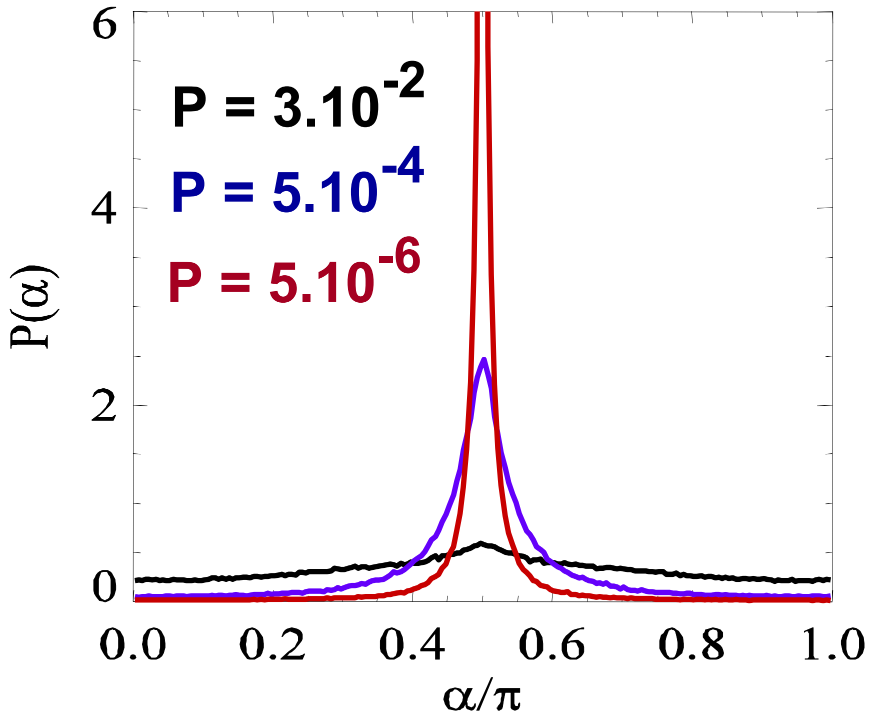


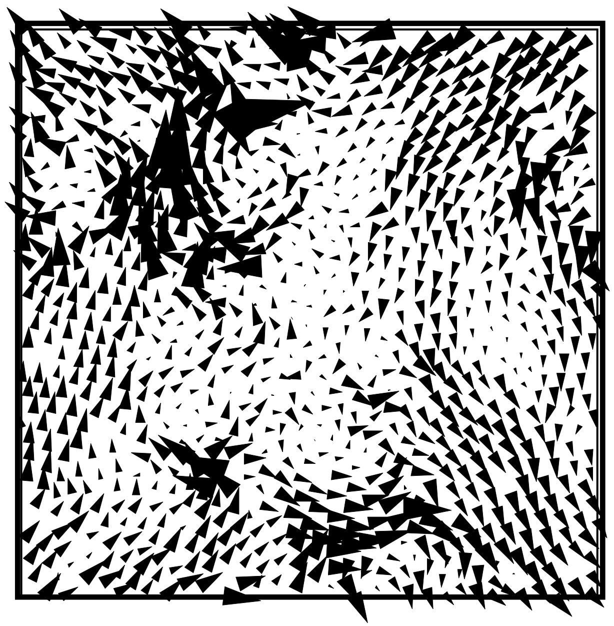




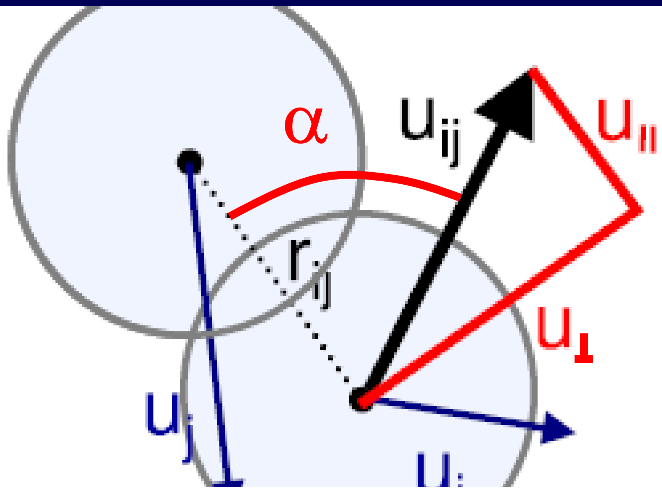
P(α): Shear

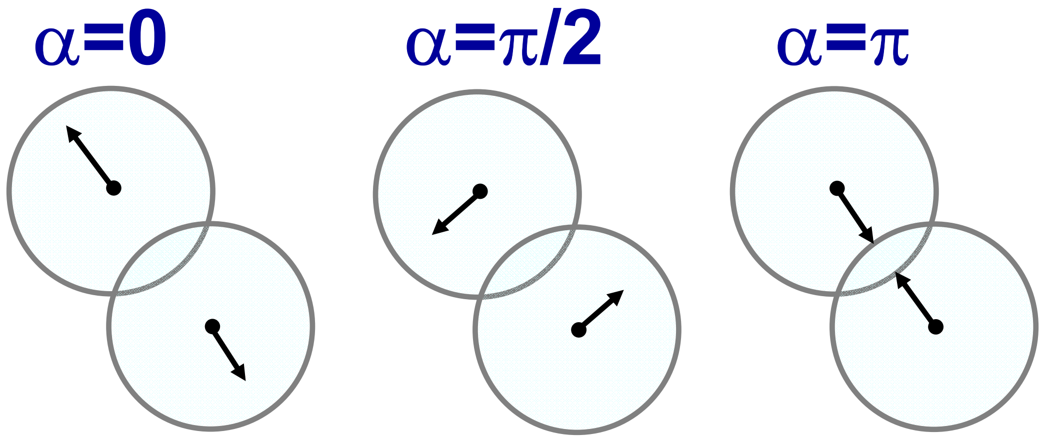


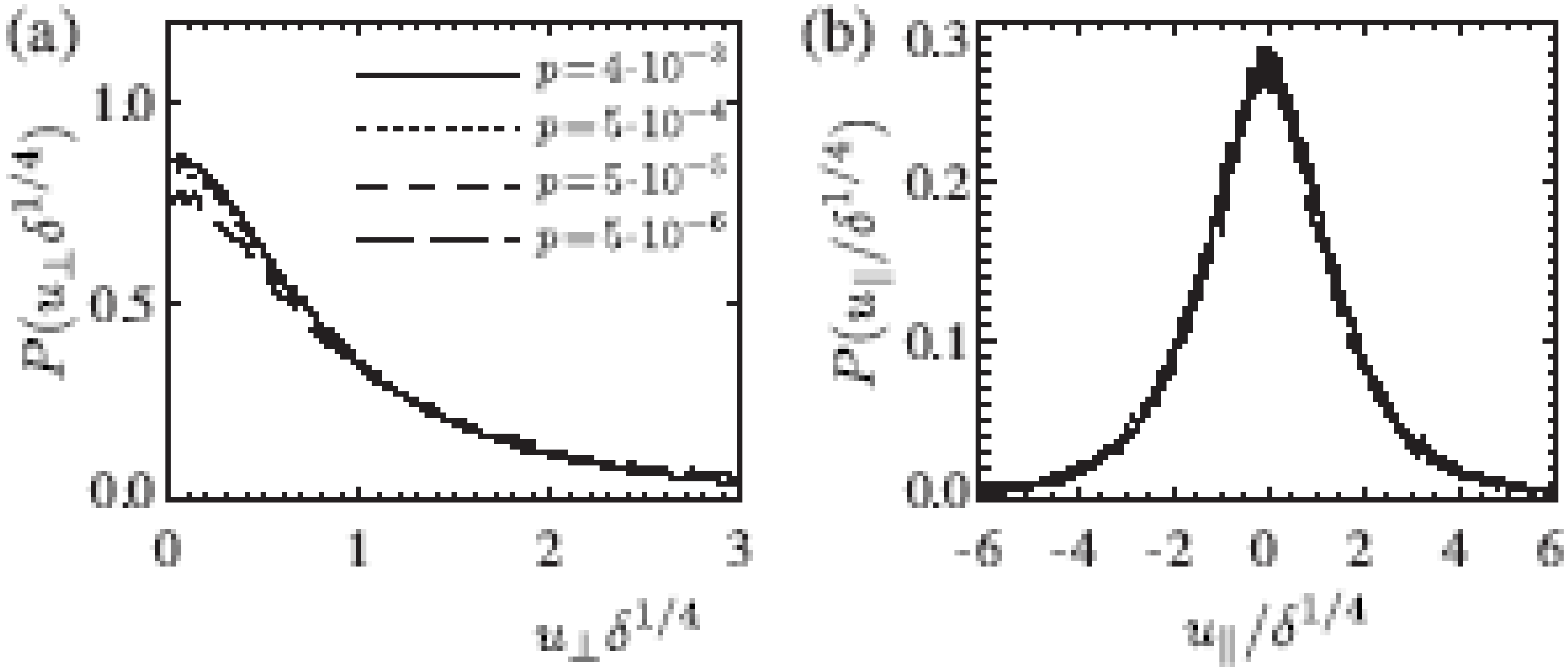




P(u): Shear

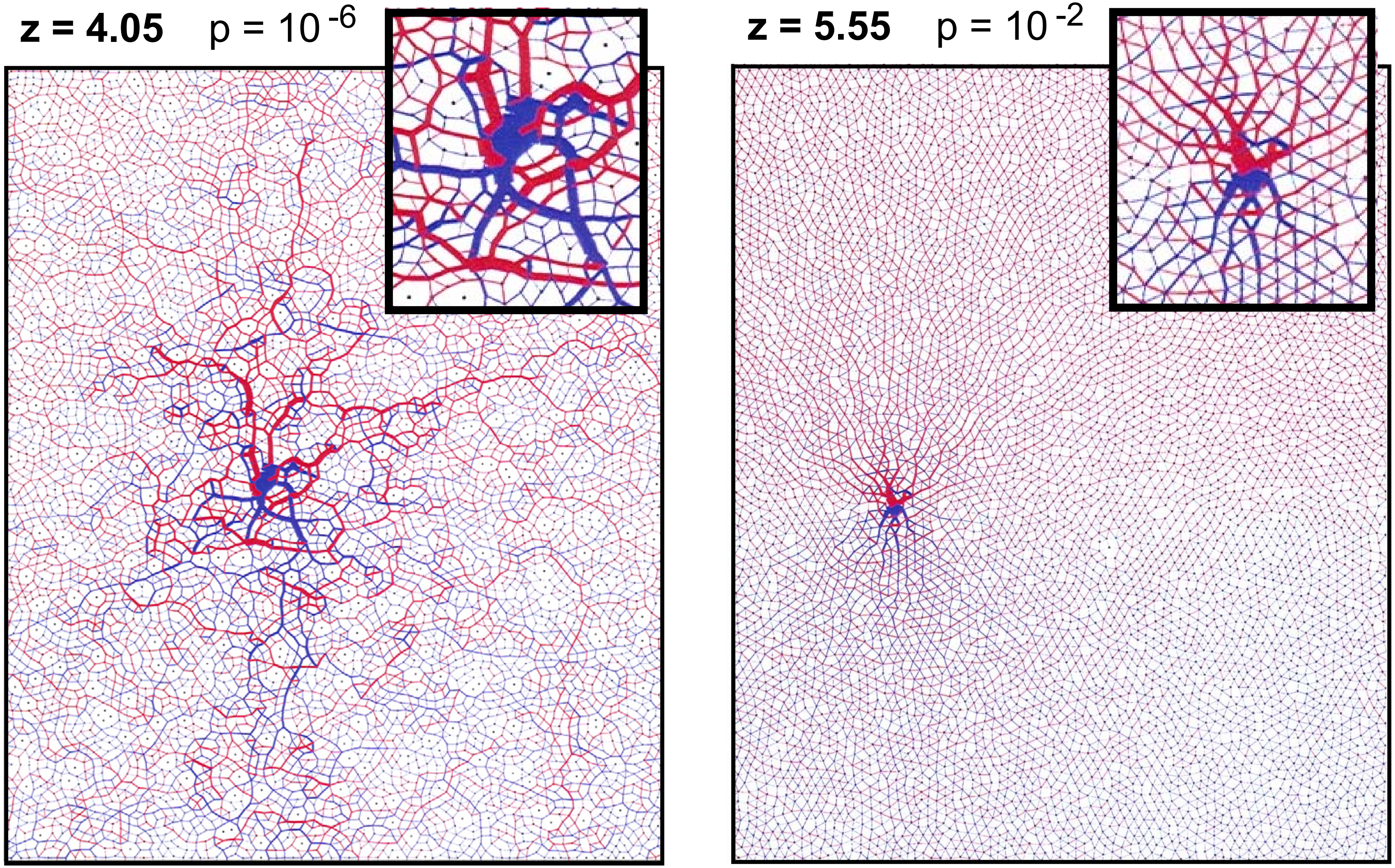

α
u_{ij}
$u_{||}$
r_{ij}
$u_{\perp}$
u_j


α=0
α=π/2
α=π


(a)
(b)
$P(u_{\perp}\delta^{1/4})$
$u_{\perp}\delta^{1/4}$
$P(u_{||}/\delta^{1/4})$
$u_{||}/\delta^{1/4}$
0.0
0.5
1.0
0.1
0.2
0.3
0
1
2
3
-6
-4
-2
4
6

Point Forcing: Diverging Lengthscale

z = 4.05 $p = 10^{-6}$ **z = 5.55** $p = 10^{-2}$



Ellenbroek et al, PRL **97** 258001 (2006) / PRE **80** 061307 (2009)

Jamming

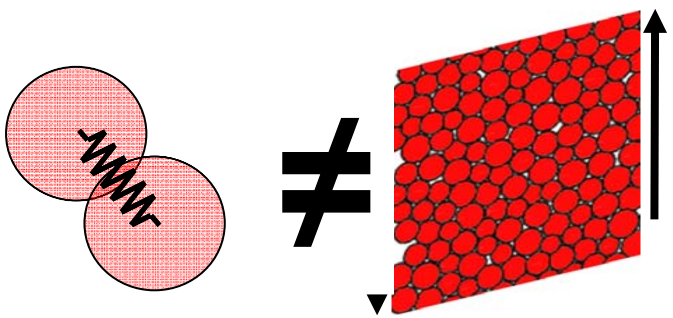
Simple model: solid/vacuum

Marginal point: weird

Scaling away from marginal point:

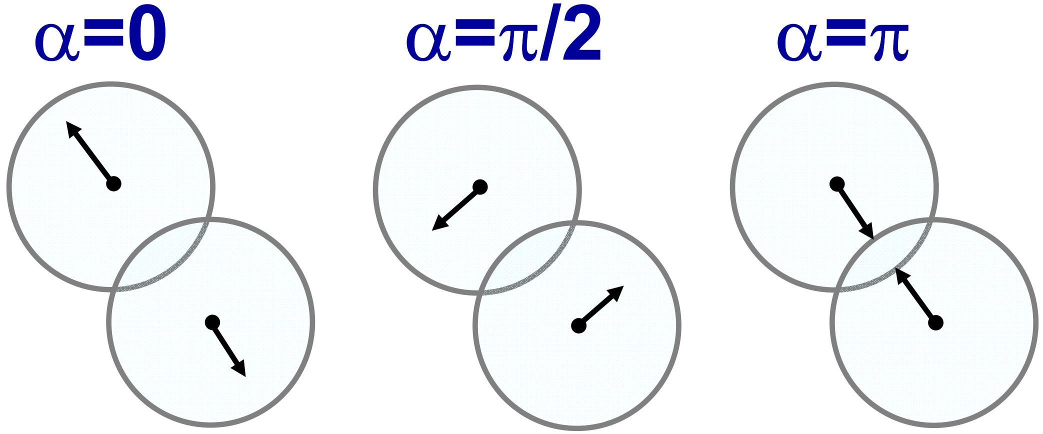
G ~ k sqrt(dϕ)

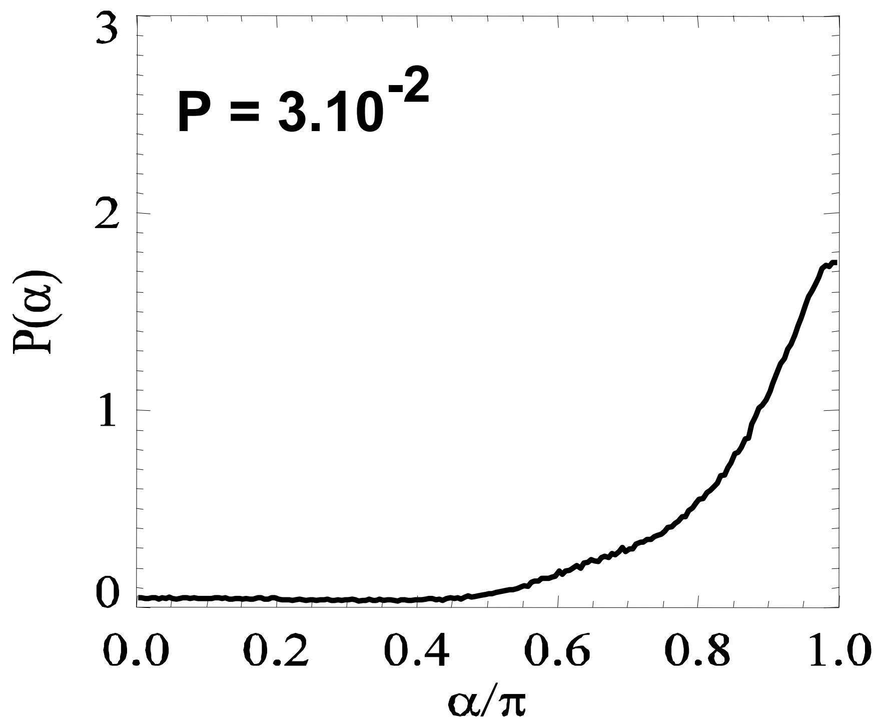
Non-affinity:

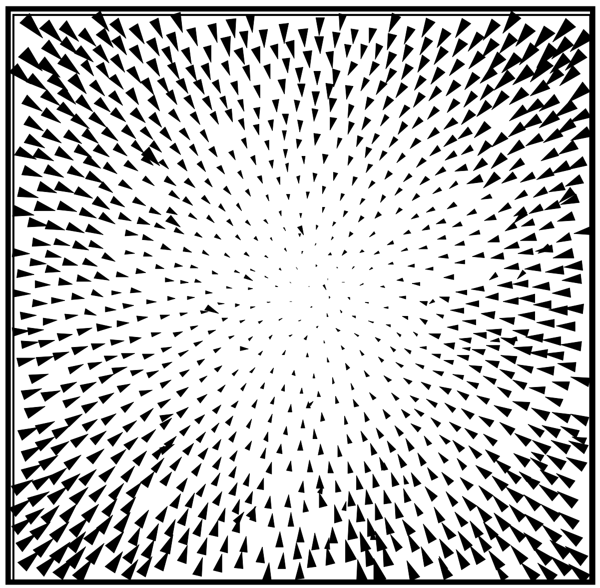


Tomorrow: Flow near Jamming

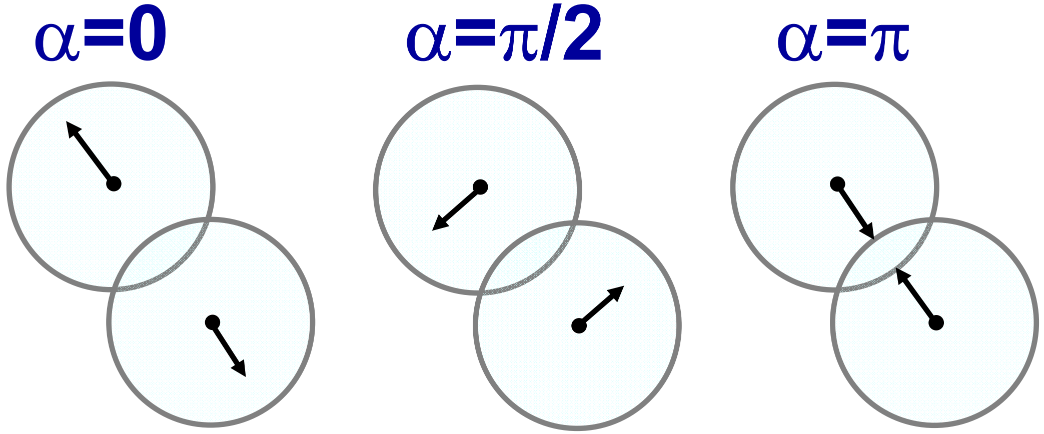
P(α): Compression

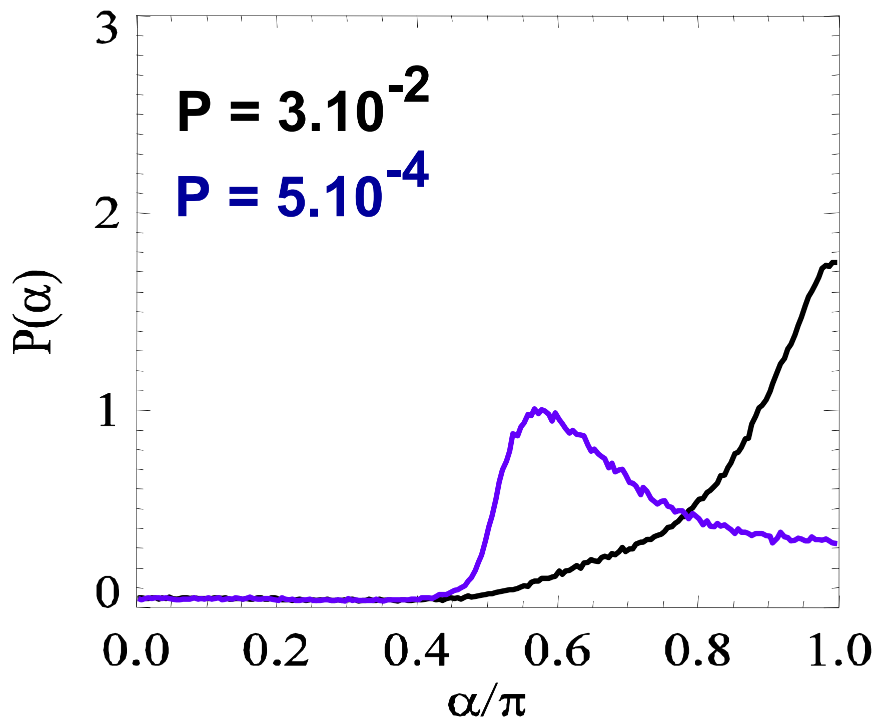


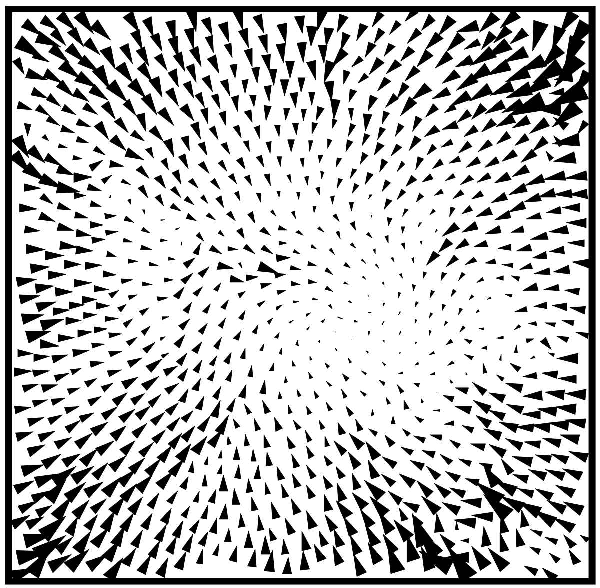




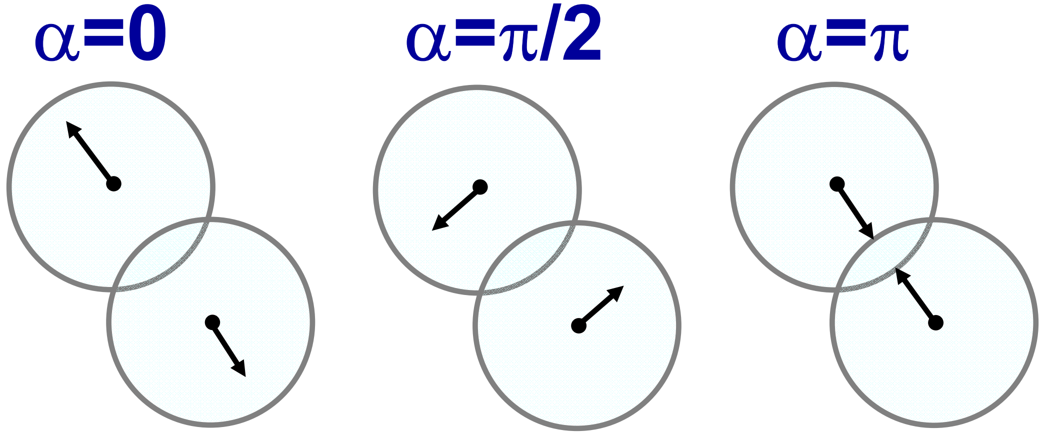
P(α): Compression

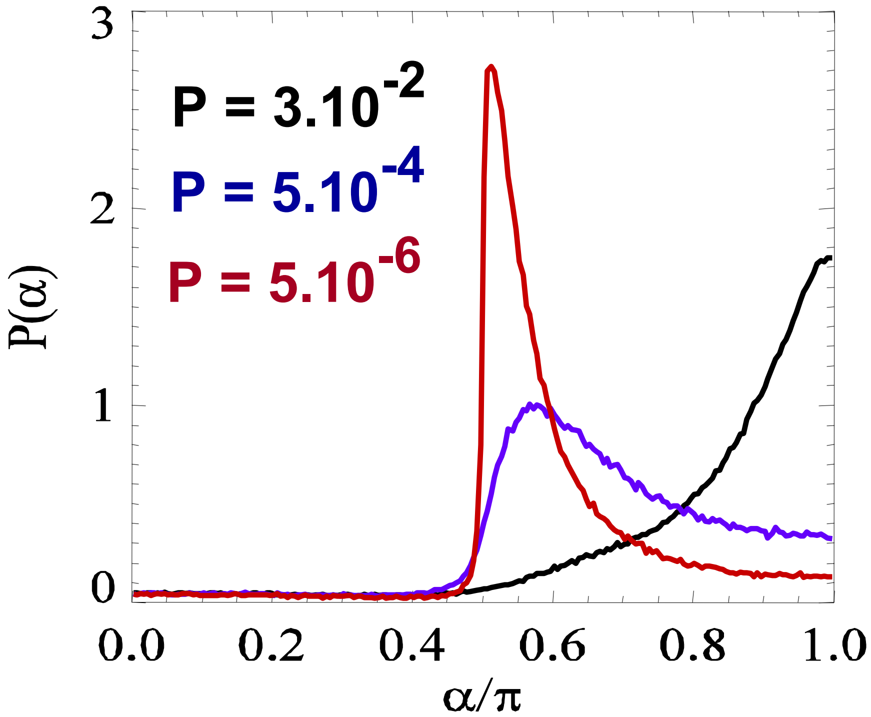




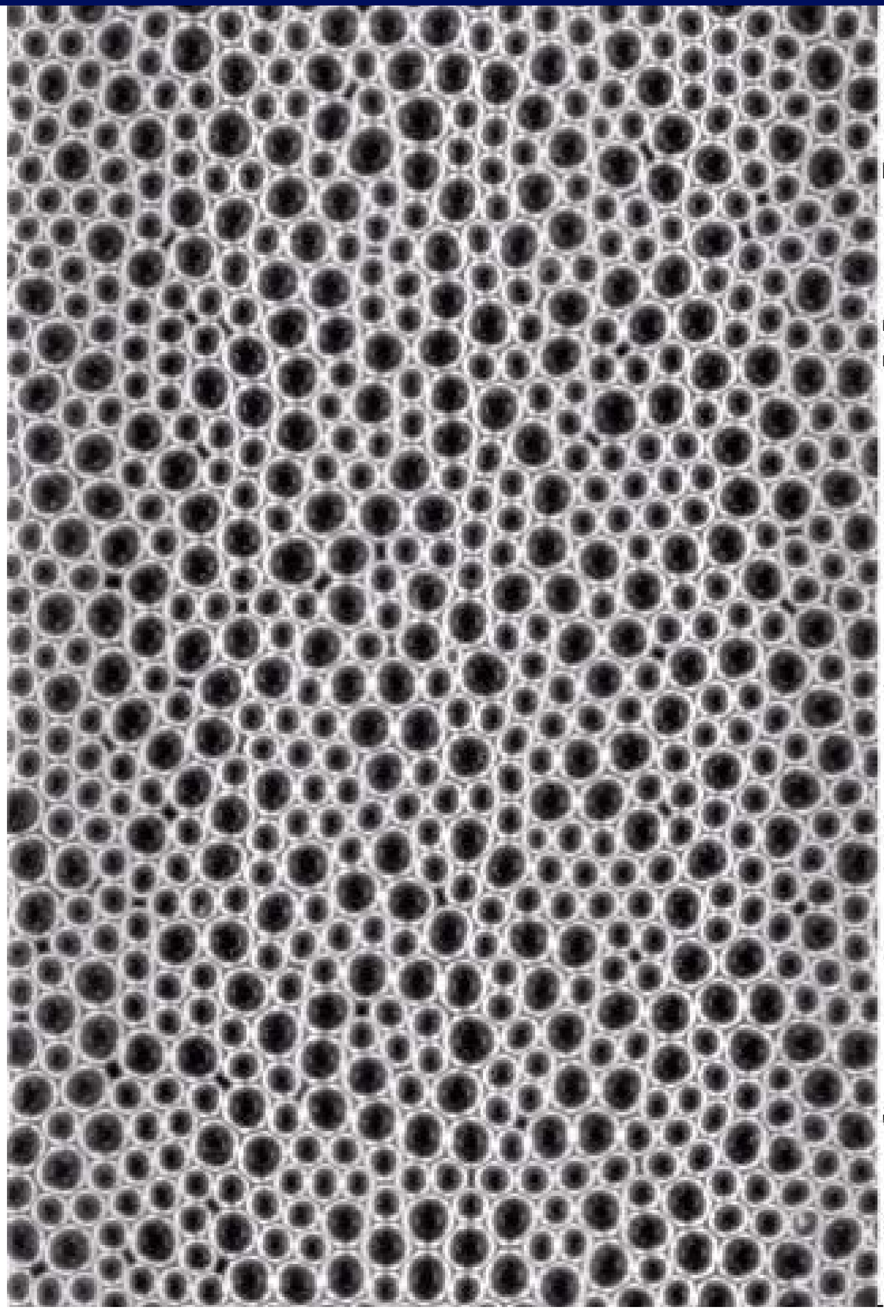


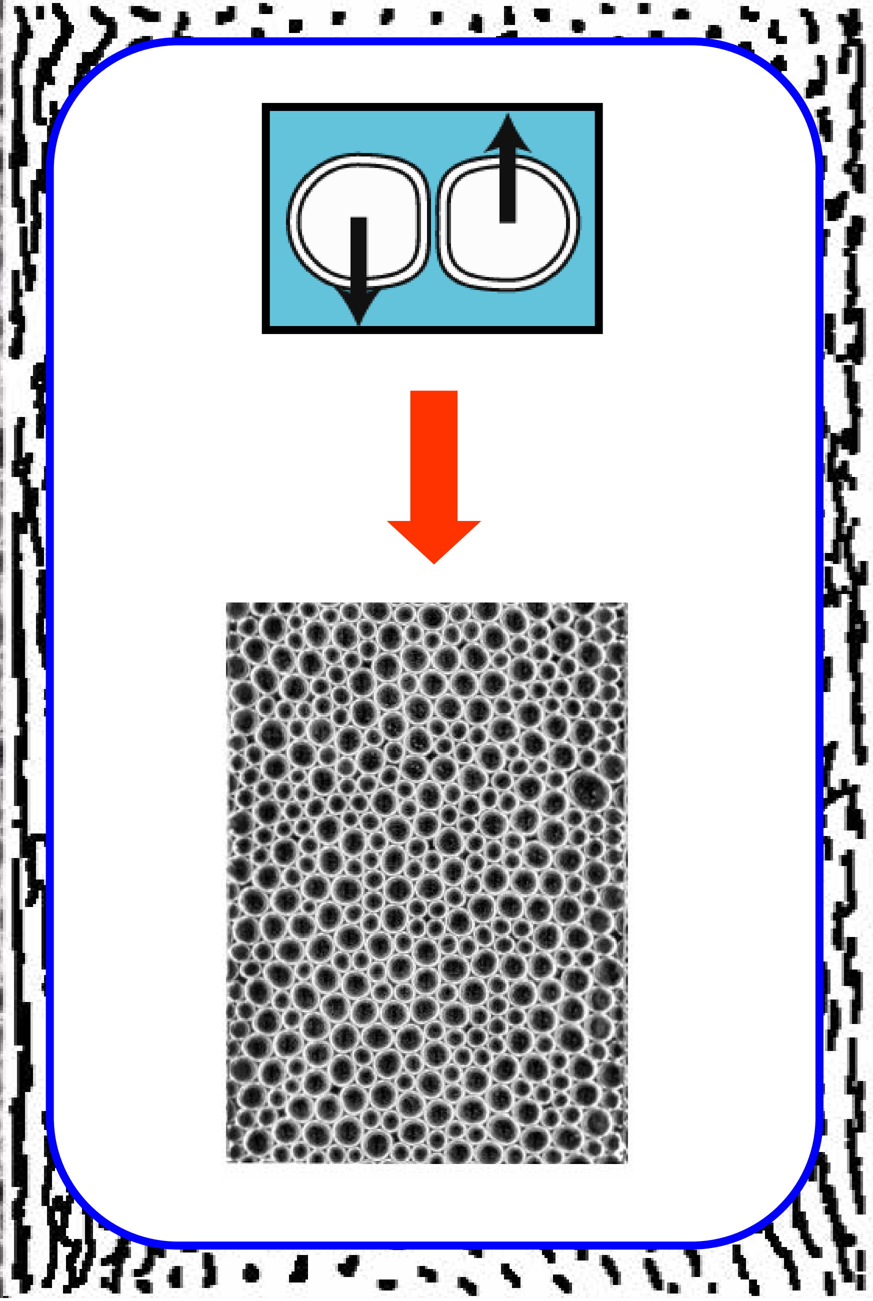
P(α): Compression





Flow of Foams: the Jamming Perspective





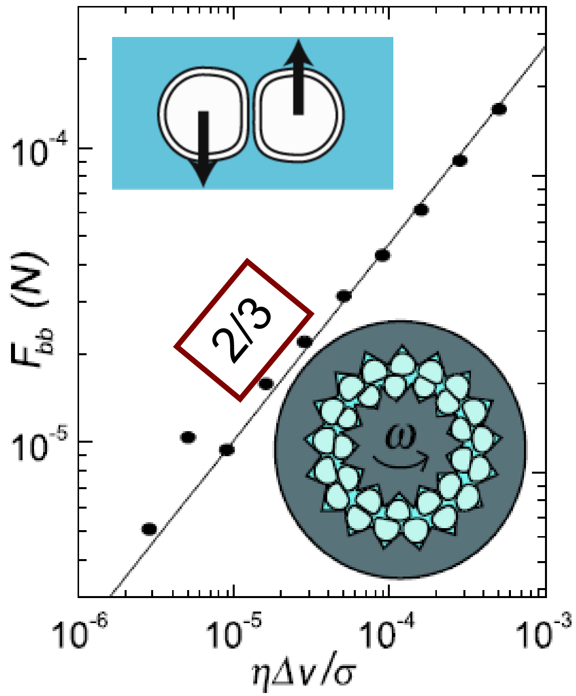
Flow and Disorder: Experiments

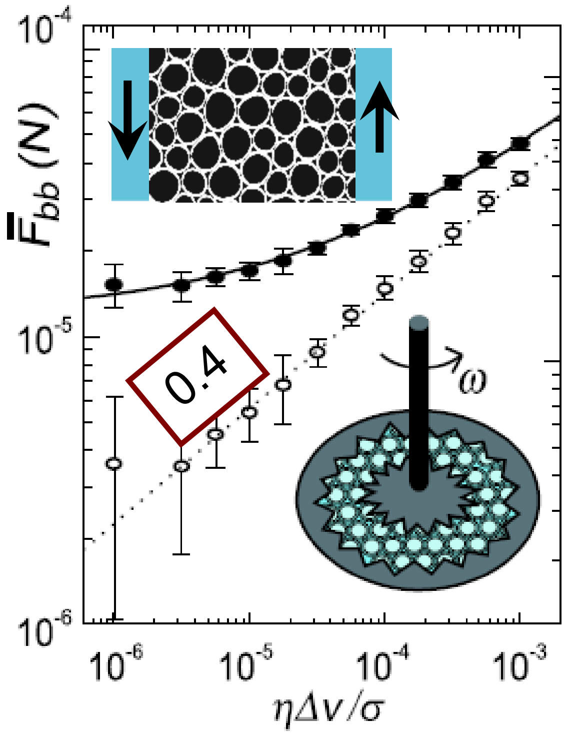
Bubble-Bubble

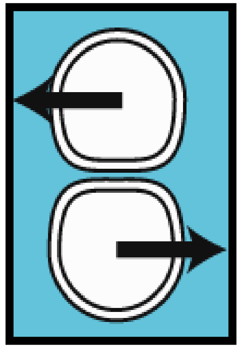
$F_{bb} \sim \Delta v^{2/3}$

Foam

$\bar{F}_{bb} \sim F_y + \Delta v^{0.4}$







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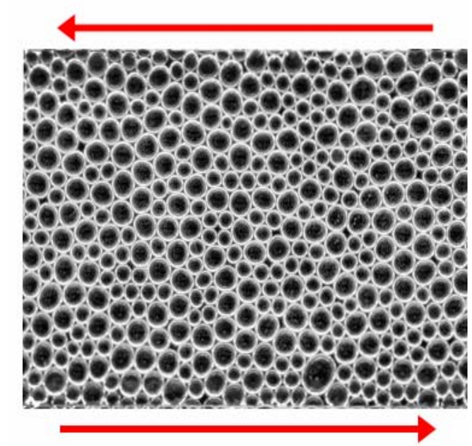
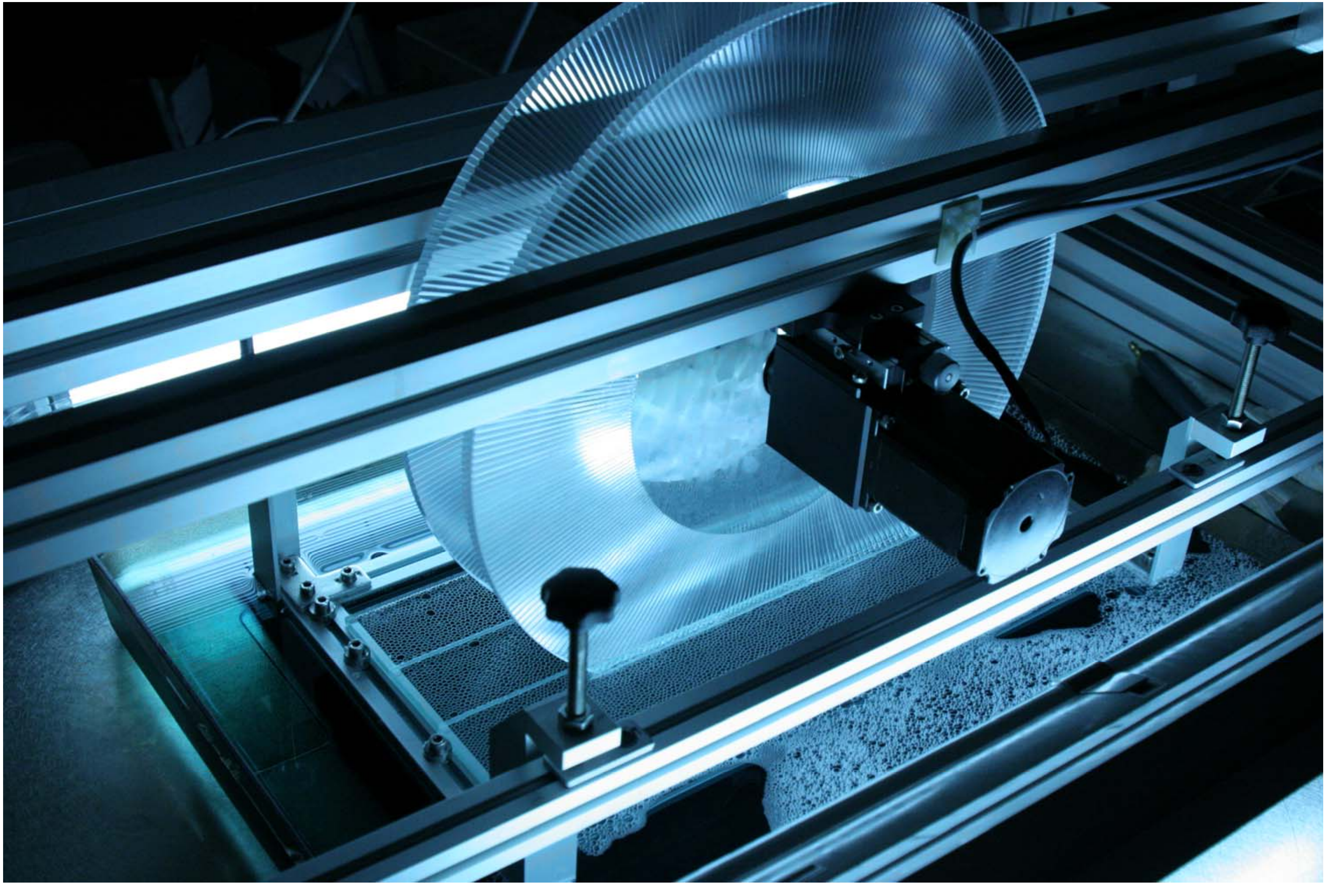
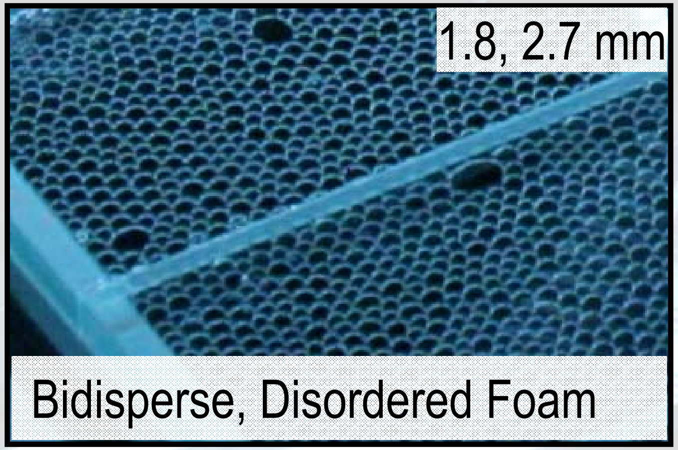


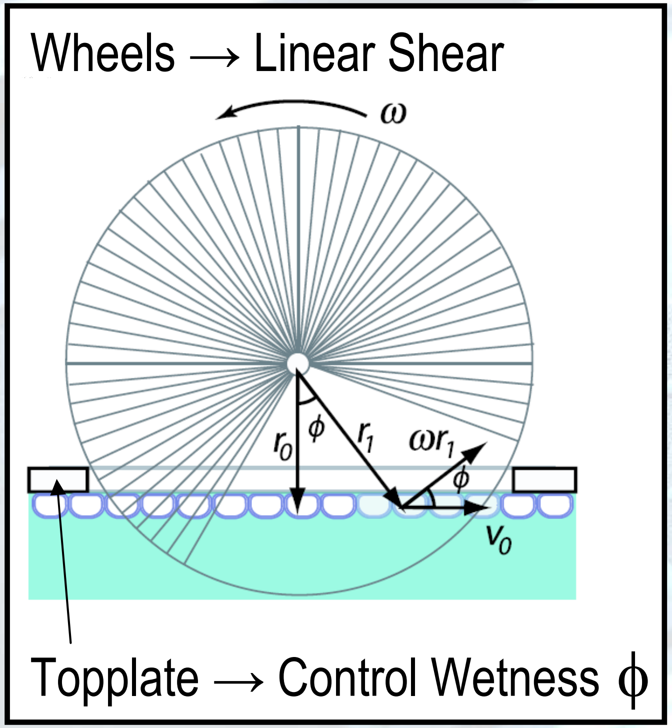
Gijs Katgert, Matthias Möbius & MvH, PRL 101 058301 (2008); PRE 79 066318 (2009)

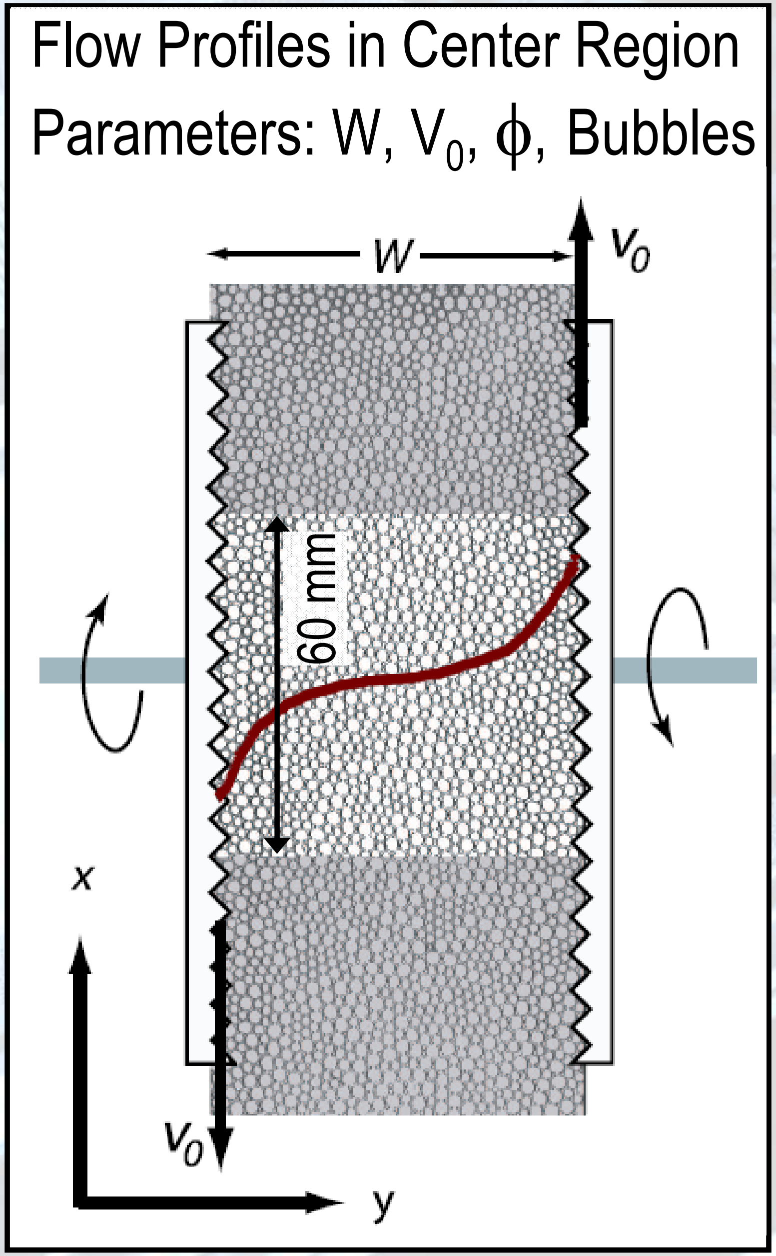
Setup



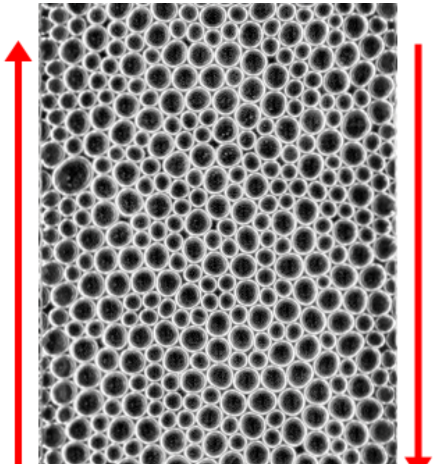
Setup

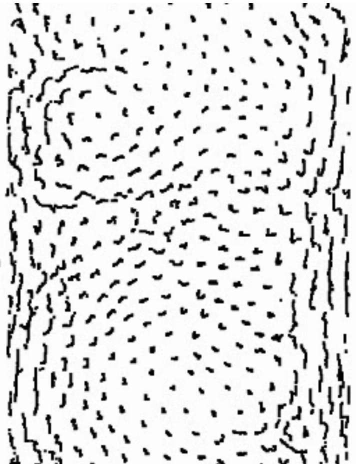

1.8, 2.7 mm
Bidisperse, Disordered Foam

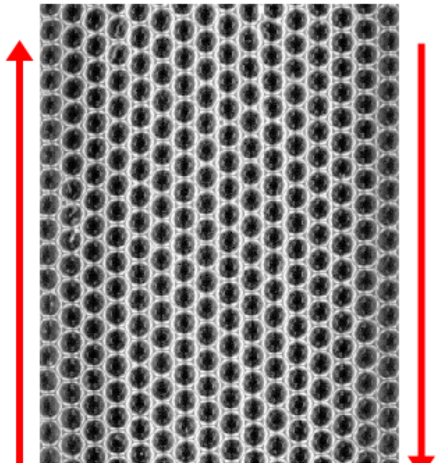

Wheels → Linear Shear
ω
r0
ϕ
r1
ωr1
ϕ
v0
Topplate → Control Wetness ϕ

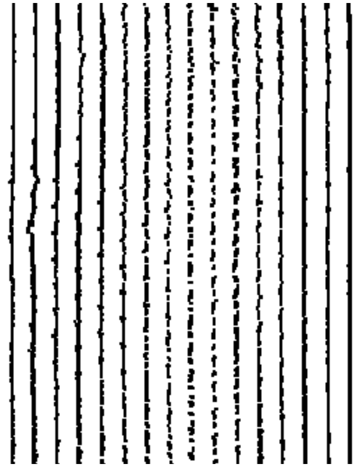

Flow Profiles in Center Region
Parameters: W, V0, ϕ, Bubbles
W
v0
60 mm
x
v0
y

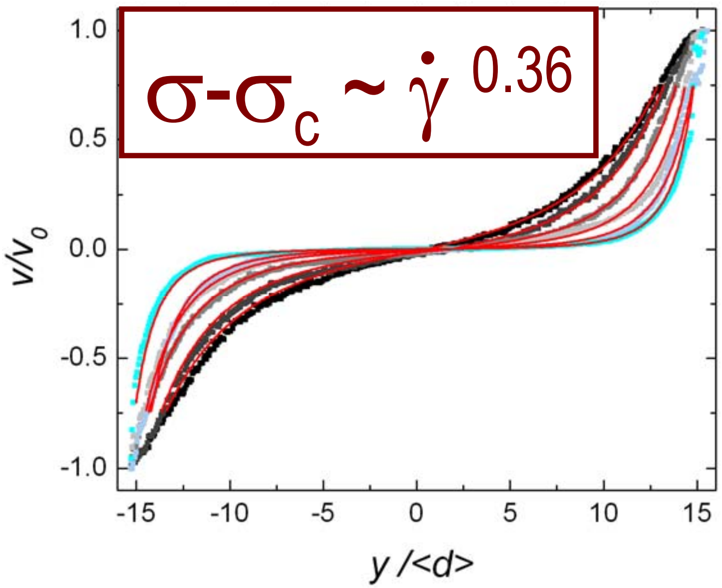
Flow and Disorder: Experiments

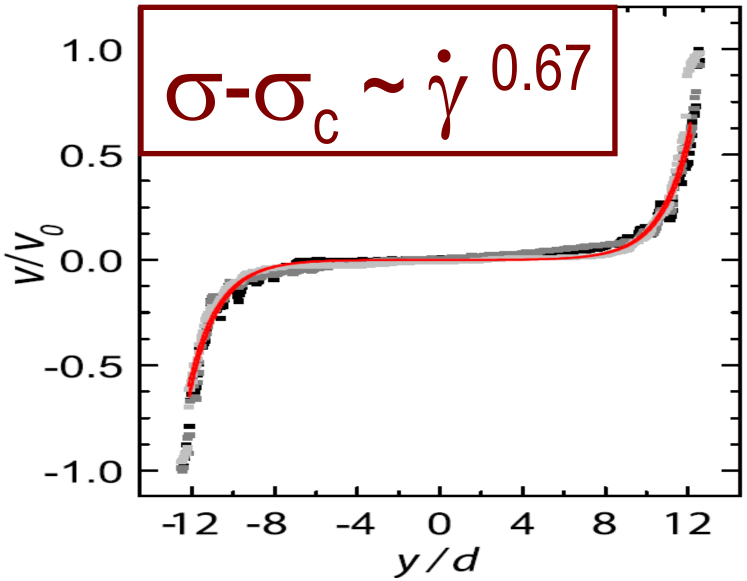






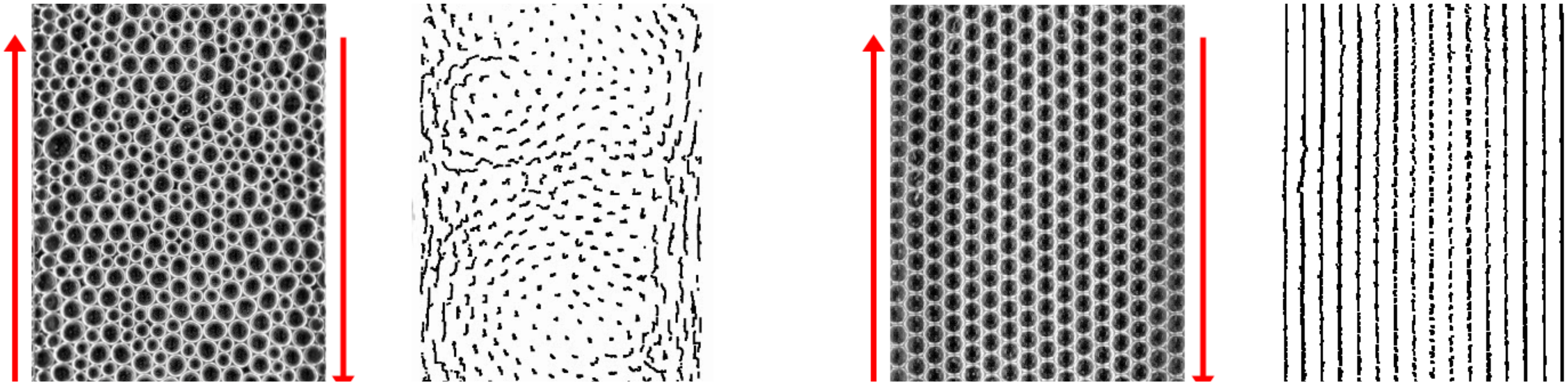






Gijs Katgert, Matthias Möbius & MvH, PRL 101 058301 (2008); PRE 79 066318 (2009)

Flow and Disorder: Experiments



≠ =

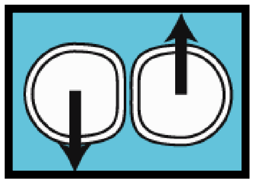
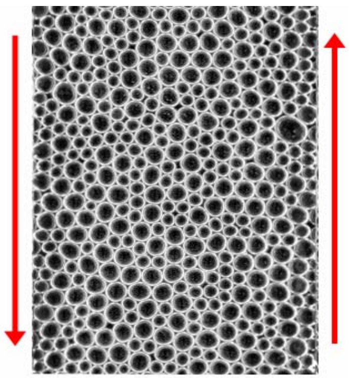
Gijs Katgert, Matthias Möbius & MvH, PRL 101 058301 (2008); PRE 79 066318 (2009)

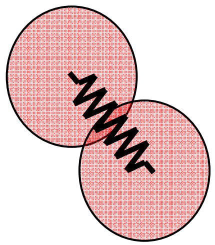
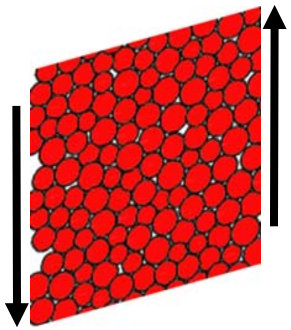
Jamming and Rheology: Commonalities

Strong Fluctuations Govern Elasticity Near Jamming …..

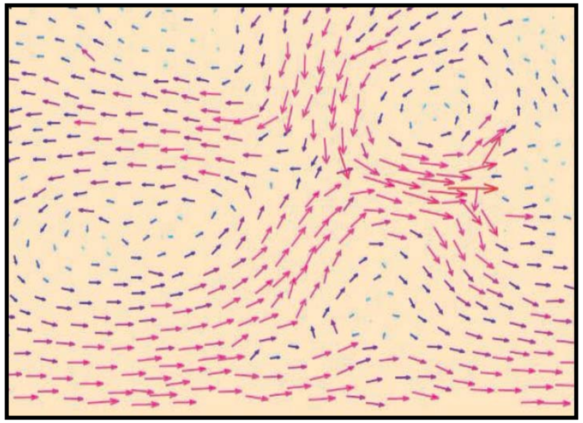
….. So What Governs Anomalous Rheology?

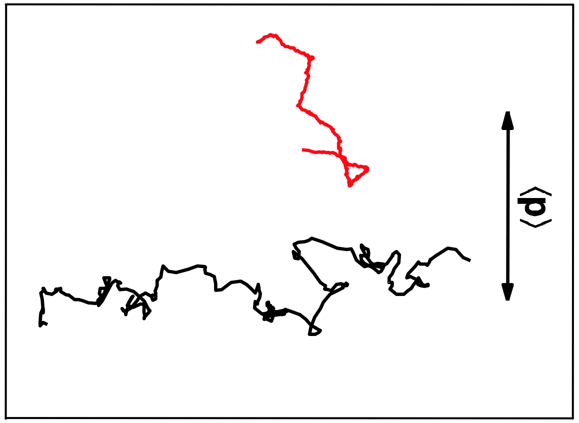
Anomalously Strong Fluctuations! ~~$\Delta \mathbf{v} \sim \dot{\gamma}$~~

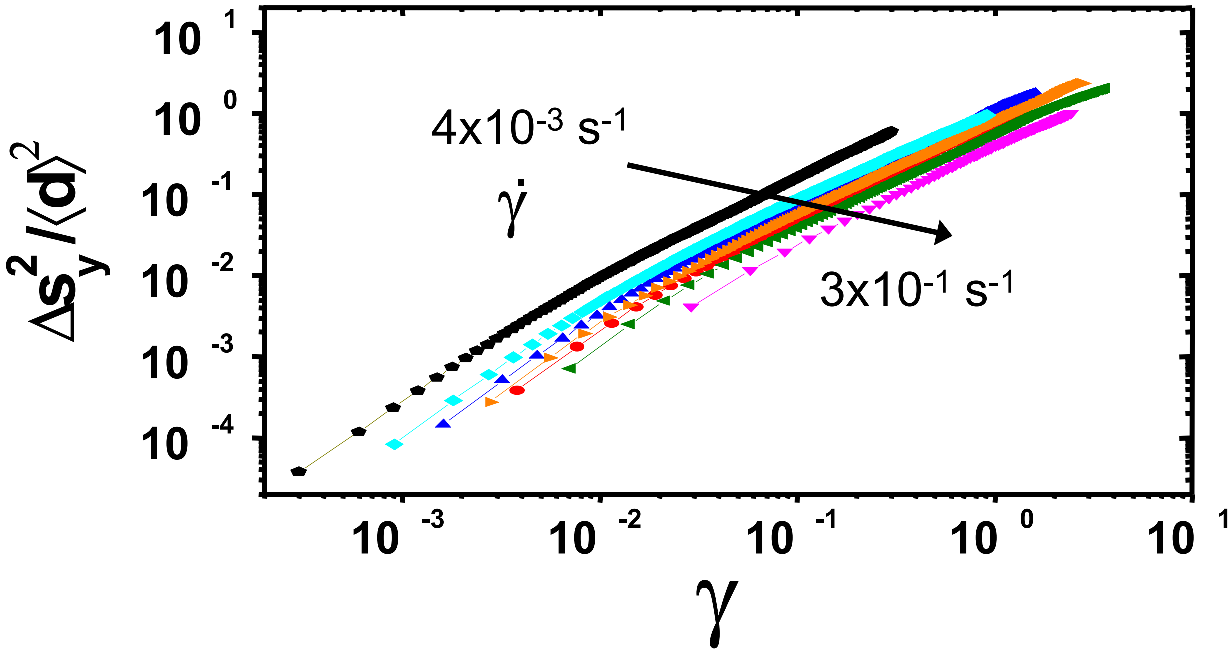
 $\neq$ 

 $\neq$ 

Anomalous Fluctuations: Experiments







M Moebius, G. Katgert, MvH EPL **90** 44003 (2010)

Microscopic Model

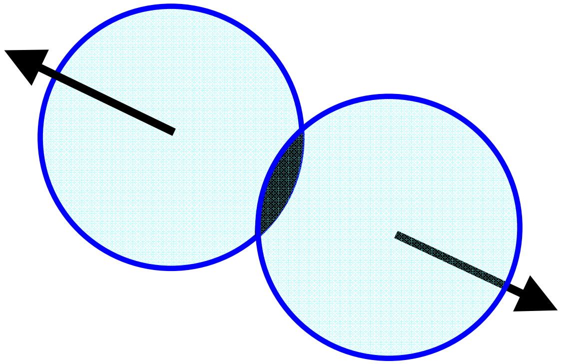
Connect Jamming, Fluctuations & Rheology

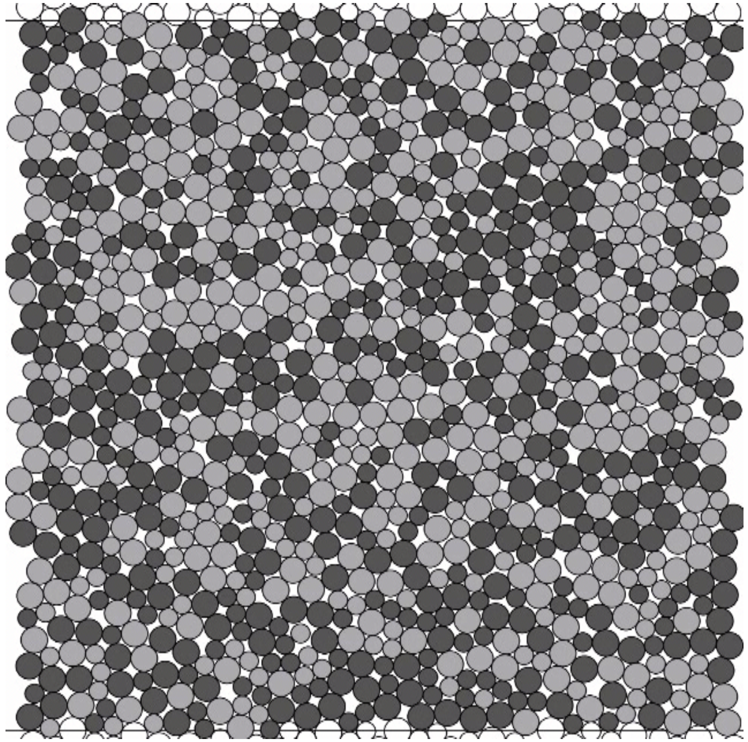
BP Tighe et al, PRL **105** 088303 (2010)

Microscopic Model

Elastic

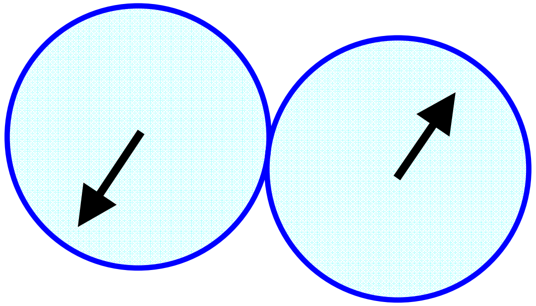
$F_e \sim \delta$





Viscous

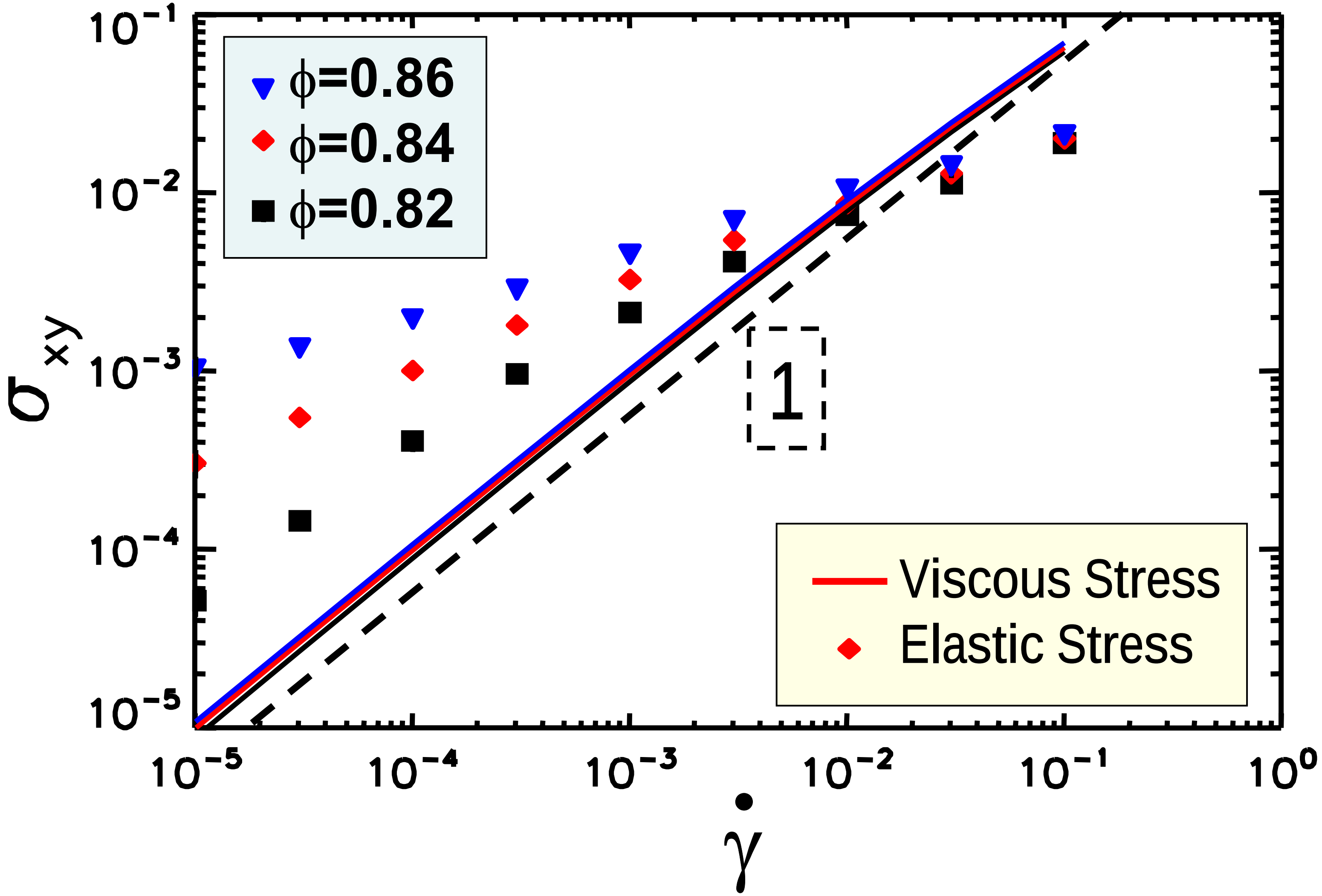
$F_v \sim (\Delta v)$



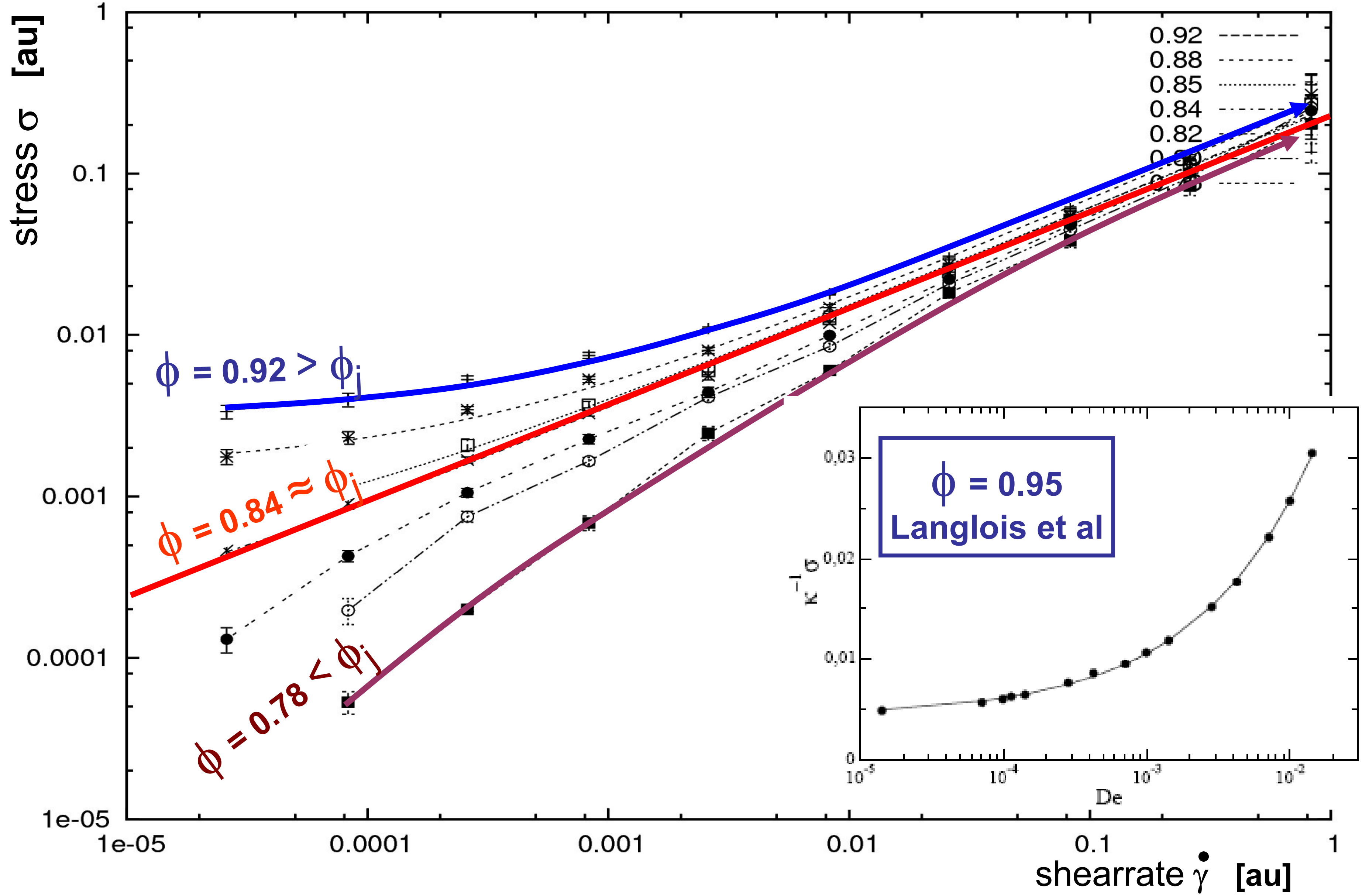
No inertia, force balance at all times

Doug Durian 1995 (Bubble Model), Olsson & Teitel, Langlois, Tighe, ...

Rheology: Viscous vs Elastic Stresses



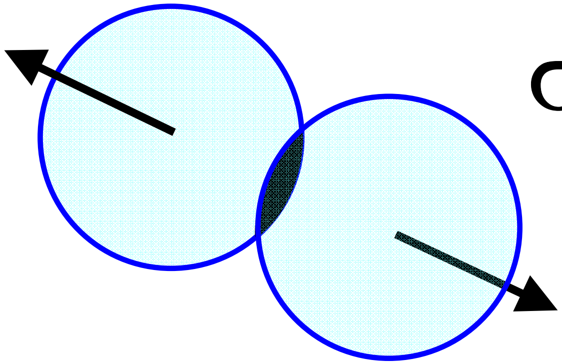
Rheology: Elastic Stresses



Microscopic Model

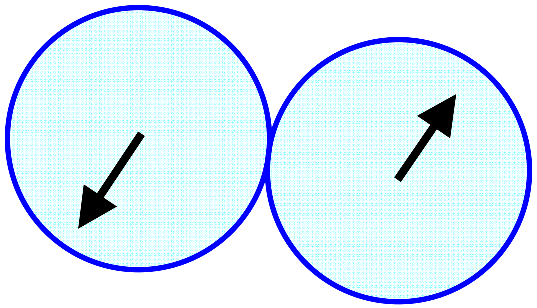
Elastic

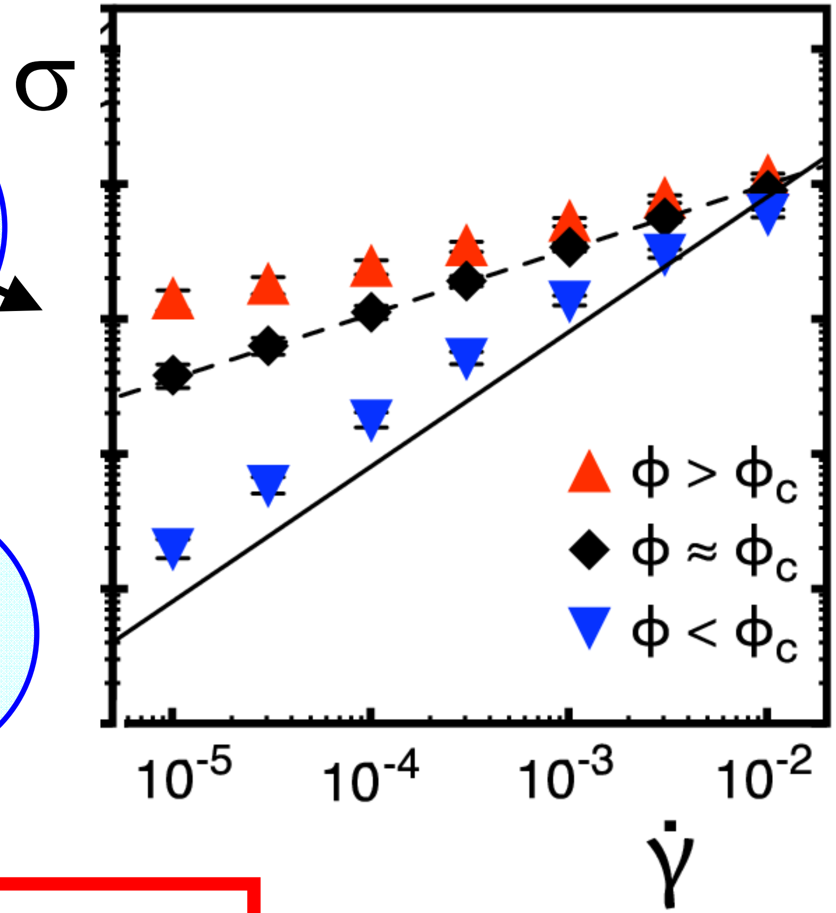
$F_e \sim \delta$



Viscous

$F_v \sim (\Delta v)$



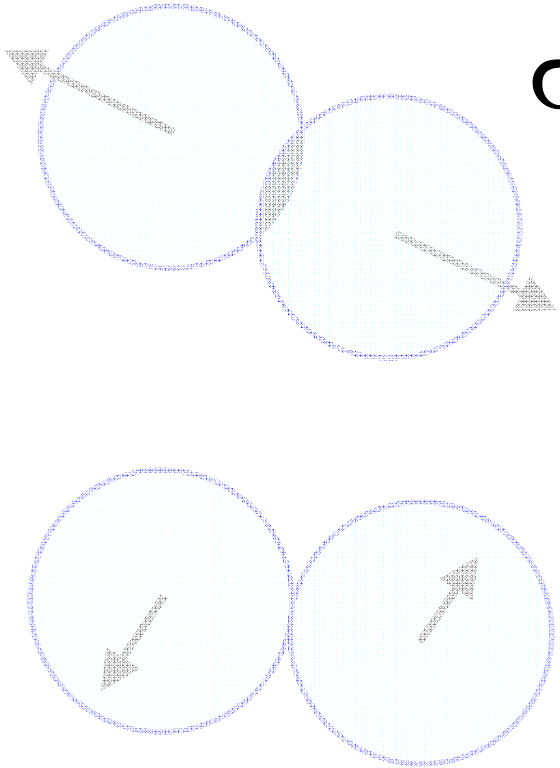


Control $\Delta\phi$, $\dot{\gamma}$ Measure σ

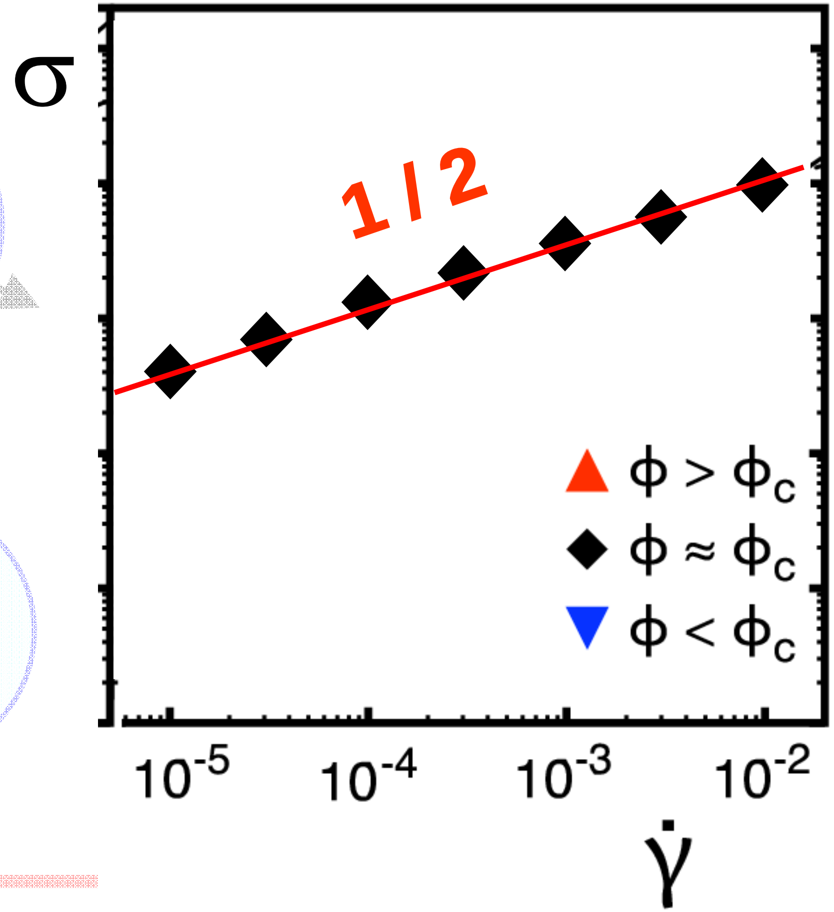
BP Tighe et al, PRL **105** 088303 (2010)

Microscopic Model

Elastic
$F_e \sim \delta$



Viscous
$F_v \sim (\Delta v)$

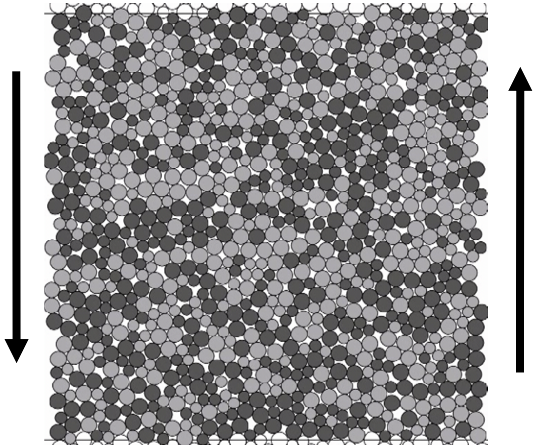


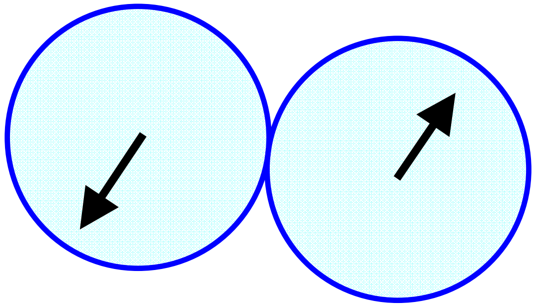
Control $\Delta\phi$, γ Measure σ

BP Tighe et al, PRL **105** 088303 (2010)

Energy Balance: Anomalous Fluctuations

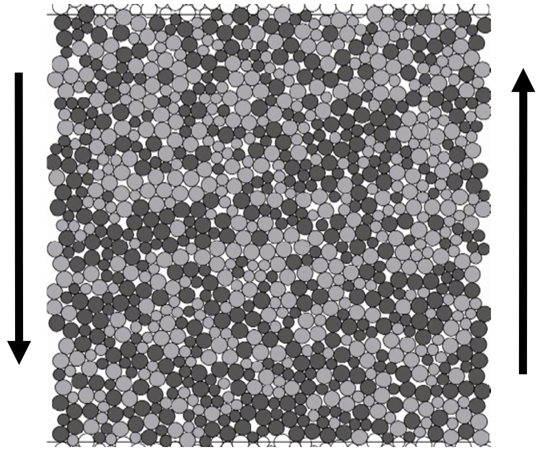
Power in = Power out

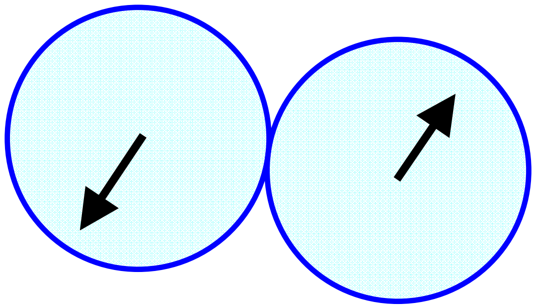




Energy Balance: Anomalous Fluctuations

$$\sigma \, \dot{\gamma} \quad \sim \quad \langle \Delta v^2 \rangle$$





Ono IK, Tewari S, Langer SA, Liu AJ, PRE 67 061503 (2003)

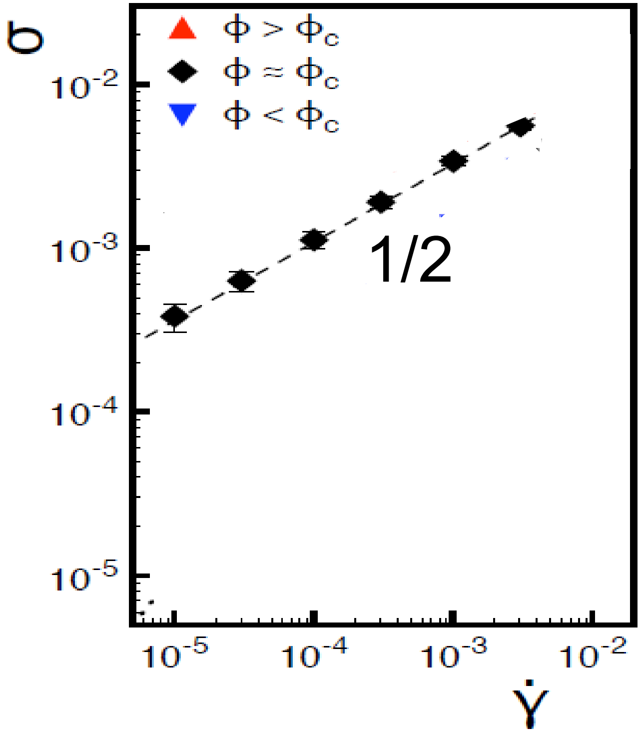
Energy Balance: Anomalous Fluctuations

$\sigma \dot{\gamma} \sim \langle \Delta v^2 \rangle$

Trivial Rheology ↔ Trivial Fluctuations

$\sigma \sim \dot{\gamma}$ $\langle \Delta v^2 \rangle \sim \dot{\gamma}^2$

BUT...



Energy Balance: Anomalous Fluctuations

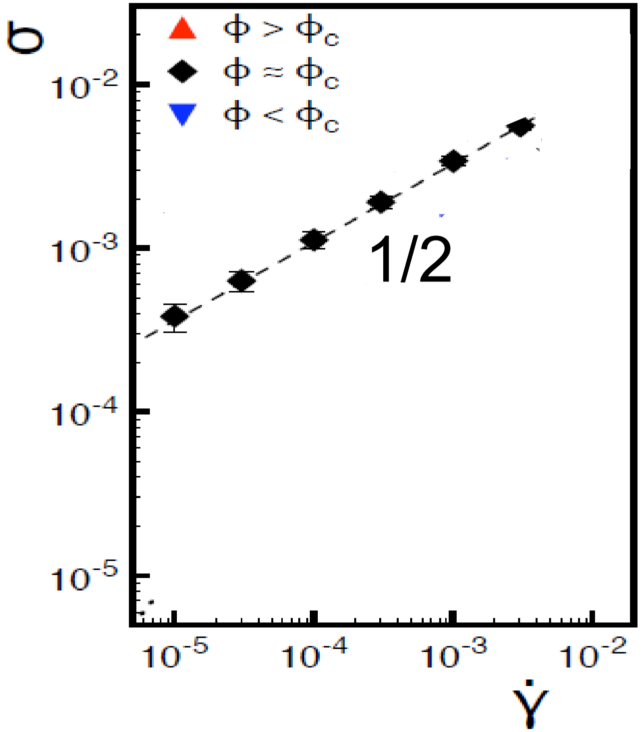
$\sigma \dot{\gamma} \quad \sim \quad <\Delta v^2>$

Trivial Rheology $\leftrightarrow$ Trivial Fluctuations

$\sigma \sim \dot{\gamma}$ $\quad$ $<\Delta v^2> \sim \dot{\gamma}^2$

$\dot{\gamma}^{3/2} \sim <\Delta v^2>$

$\Delta v/\dot{\gamma} \sim \dot{\gamma}^{-1/4}$

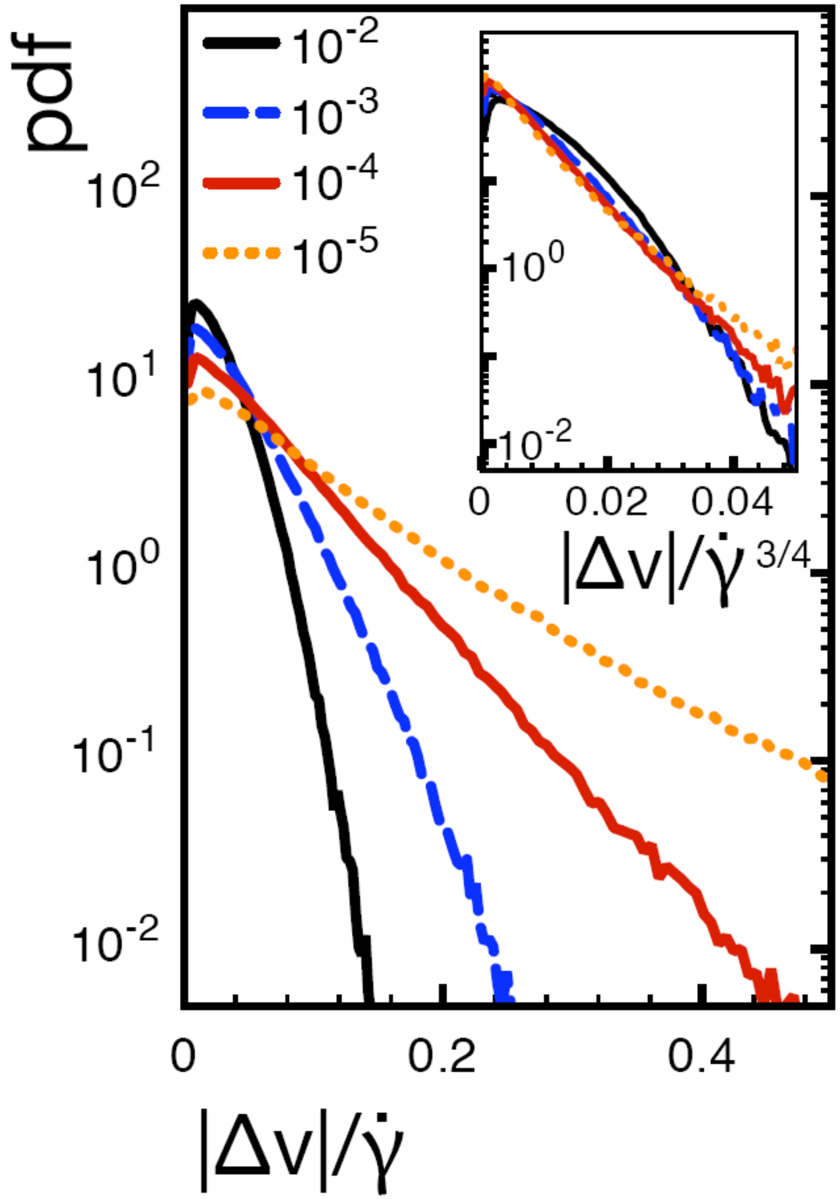


Energy Balance: Anomalous Fluctuations

$\sigma \dot{\gamma} \sim \langle \Delta v^2 \rangle$

$\dot{\gamma}^{3/2} \sim \langle \Delta v^2 \rangle$

$\Delta v/\dot{\gamma} \sim \dot{\gamma}^{-1/4}$



Energy Balance: Anomalous Fluctuations

$$\sigma\,\dot{\gamma} \sim \langle \Delta v^2 \rangle$$

Scaling Model for Rheology near Jamming

Energy Balance → Fluctuations

→ Characteristic Strain

→ Characteristic Stress

Ingredients:
Scaling of G,
Nonlinear elasticity near jamming (Wyart PRL 2008),
…

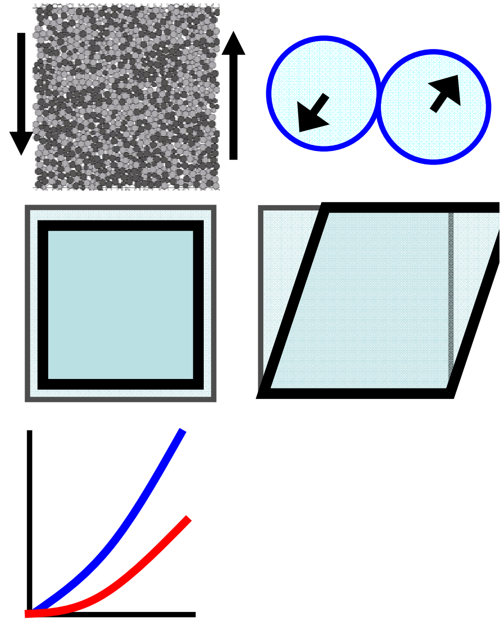
BP Tighe et al, PRL **105** 088303 (2010)

Scaling Model for Rheology near Jamming

Energy: $\sigma\,\dot{\gamma} \sim \langle \Delta v^2 \rangle$

Strain: $\gamma_{eff} = \Delta\phi + \dot{\gamma}/\Delta v$

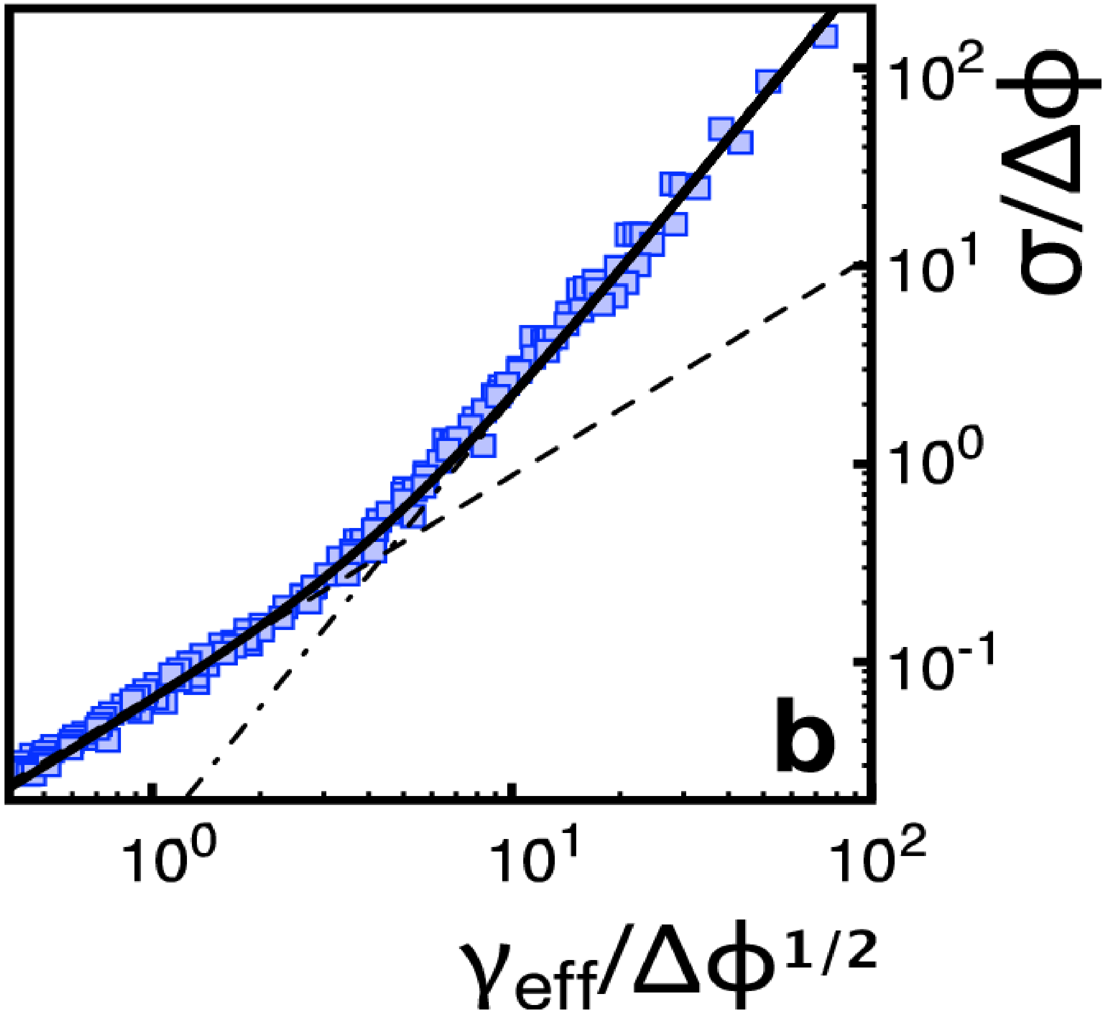
Stress: $\sigma = \left[(\Delta\phi)^{1/2} + |\gamma_{eff}| \right] \gamma_{eff}$



Does is work?

Test 1: $\sigma/\Delta\phi$ as function of $\gamma_{eff} / (\Delta\phi)^{1/2}$

Scaling Model for Rheology near Jamming



Test 1: $\sigma/\Delta\phi$ as function of $\gamma_{eff} / (\Delta\phi)^{1/2}$

Scaling Model for Rheology near Jamming

Energy: $\sigma \dot{\gamma} \sim \langle \Delta \mathbf{v}^2 \rangle$

Strain: $\gamma_{eff} = \Delta\phi + \dot{\gamma}/\Delta \mathbf{v}$

Stress: $\sigma = \left[(\Delta\phi)^{1/2} + |\gamma_{eff}| \right] \gamma_{eff}$

Test 2: Solve for Rheology

Scaling Model for Rheology near Jamming

Energy: $\sigma \dot{\gamma} \sim \langle \Delta v^2 \rangle$

Strain: $\gamma_{eff} = \quad \dot{\gamma}/\Delta v$

Stress: $\sigma = [\quad |\gamma_{eff}|] \; \gamma_{eff}$

$\Delta\phi$ very small

$\sigma \sim \gamma_{eff}^2 \sim (\dot{\gamma}/\Delta \mathbf{v})^2 \sim \dot{\gamma}^2, \; \sigma \dot{\gamma} \sim \dot{\gamma}, \; \sigma$

$\sigma \sim \dot{\gamma}^{1/2}$

Scaling Model for Rheology near Jamming

Energy: $\sigma \dot{\gamma} \sim \langle \Delta v^2 \rangle$

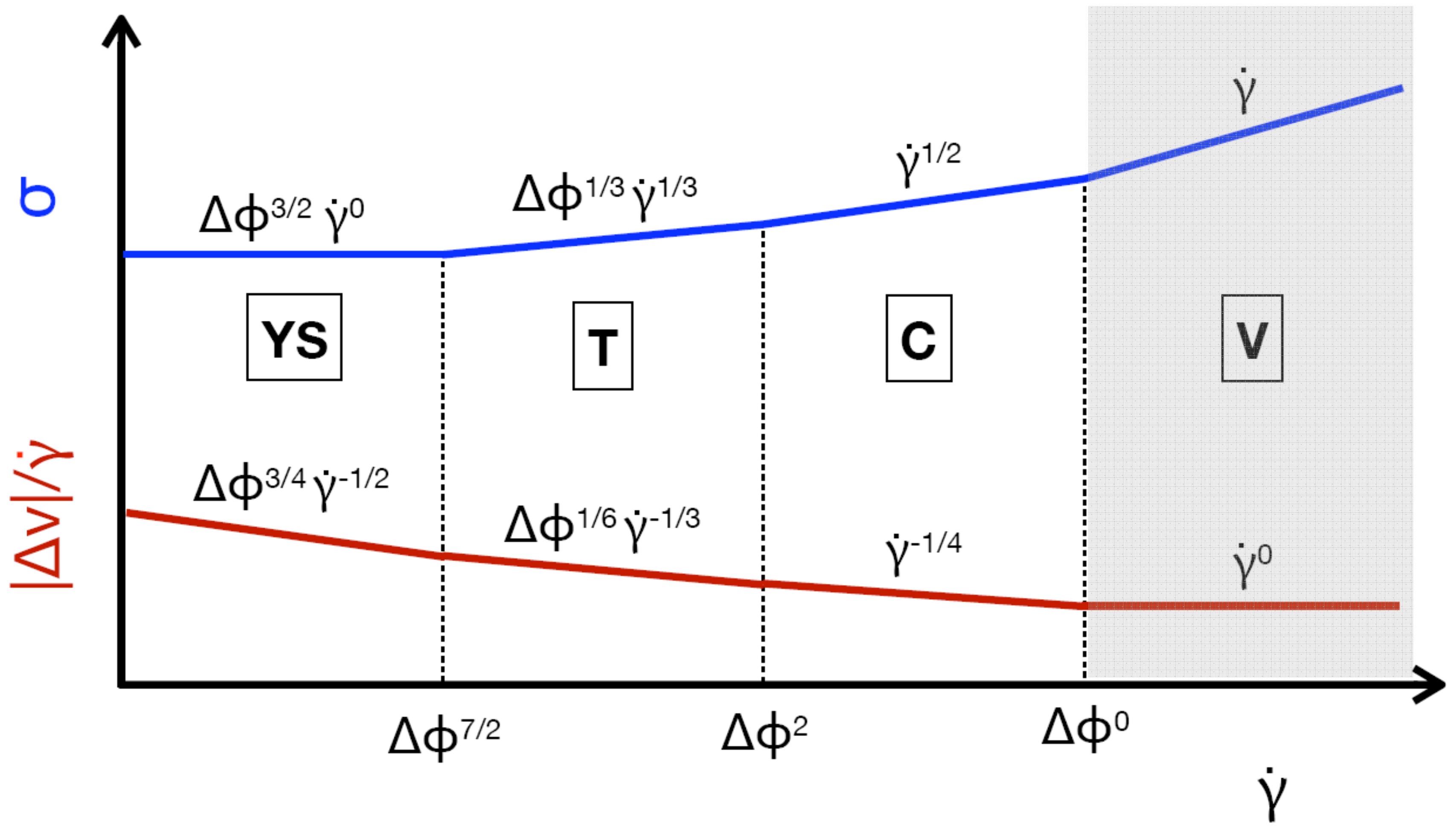
Strain: $\gamma_{eff} = \Delta\phi$

Stress: $\sigma = [(\Delta\phi)^{1/2} \quad] \gamma_{eff}$

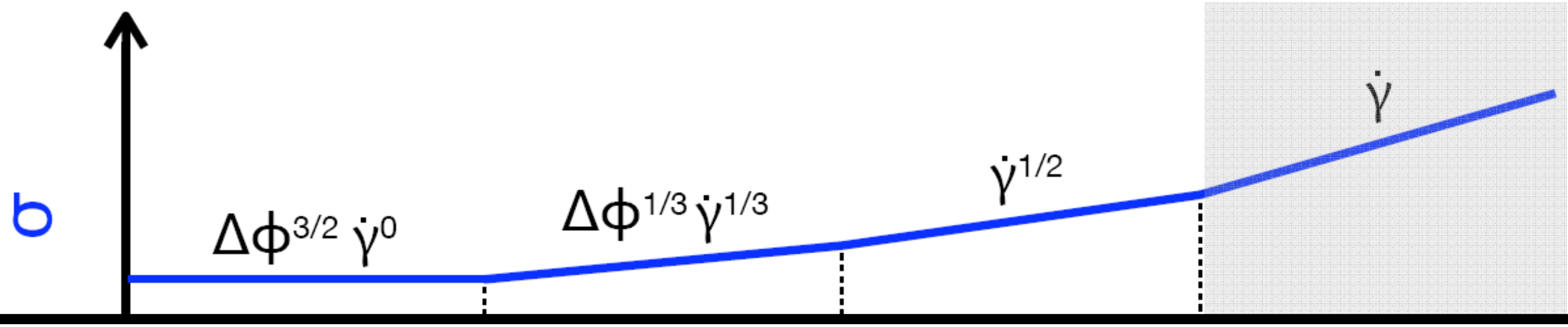
$\Delta\phi >>> \dot{\gamma}$

$\sigma \sim (\Delta\phi)^{3/2}$

Scaling Model for Rheology near Jamming



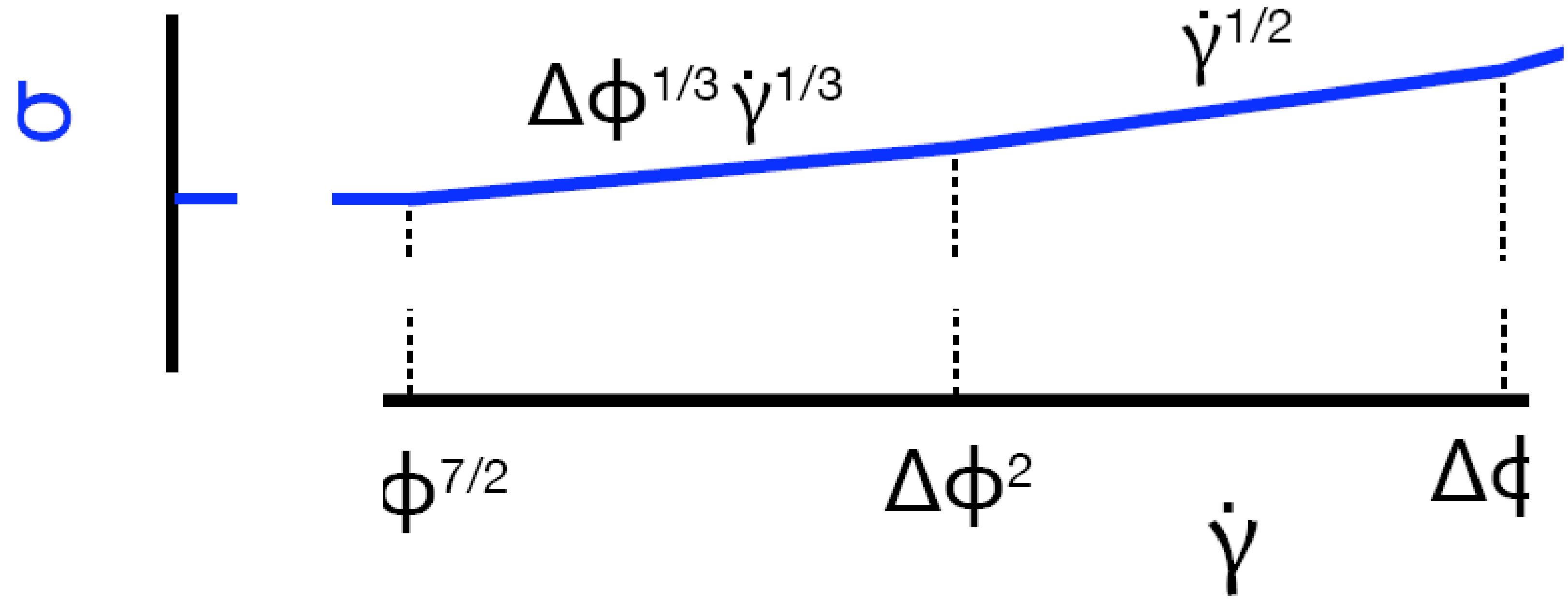
Scaling Model for Rheology near Jamming



- Not Herschel-Bulkley $(\sigma=\sigma_y+\dot{\gamma}^\beta)$

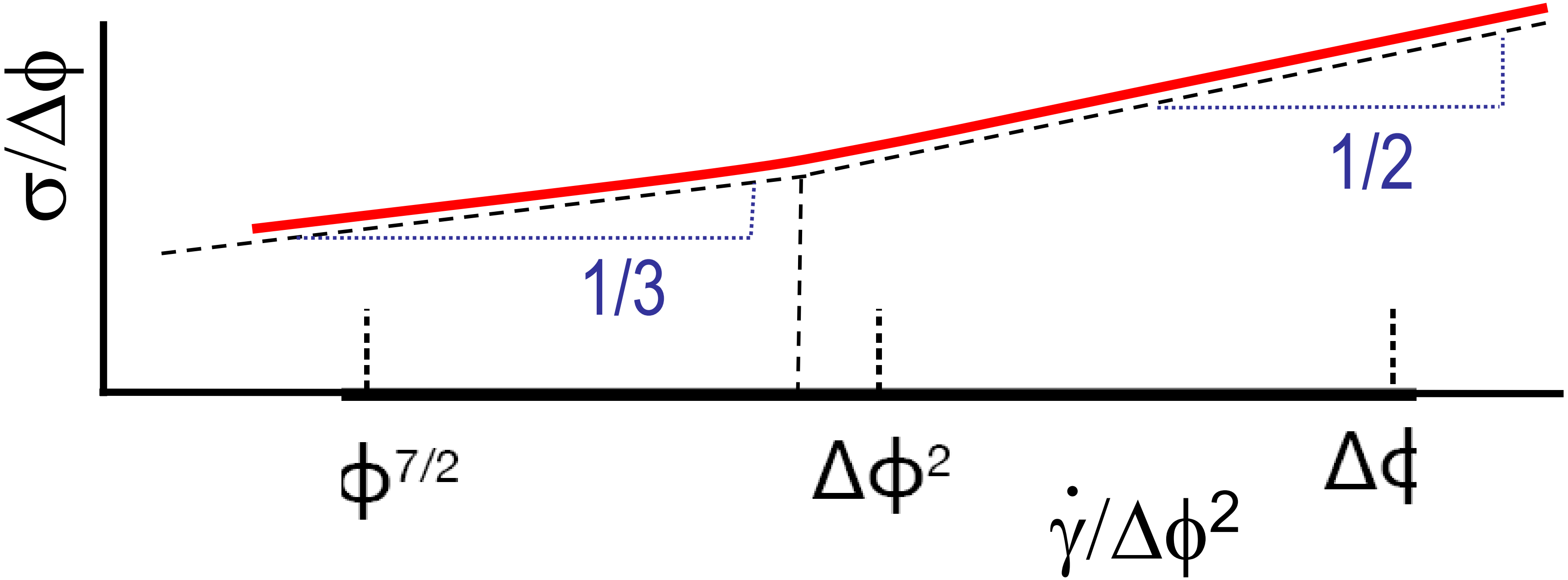
- Fit to HB: β=0.33, 0.36, 0.42, 0.54, 1

Scaling Model for Rheology near Jamming



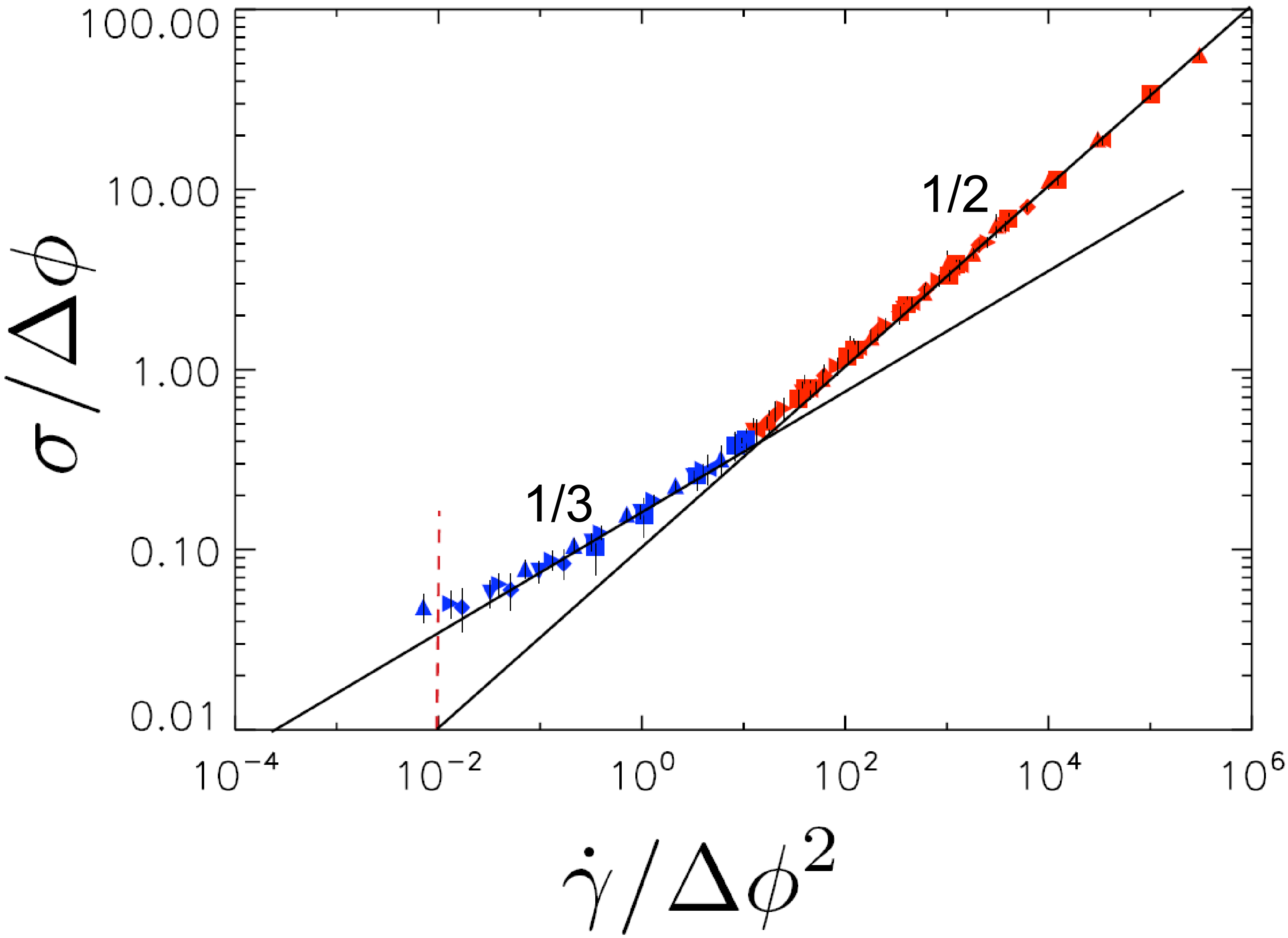
$\sigma/\Delta\phi$ as function of $\dot{\gamma}/(\Delta\phi)^2$

Scaling Model for Rheology near Jamming

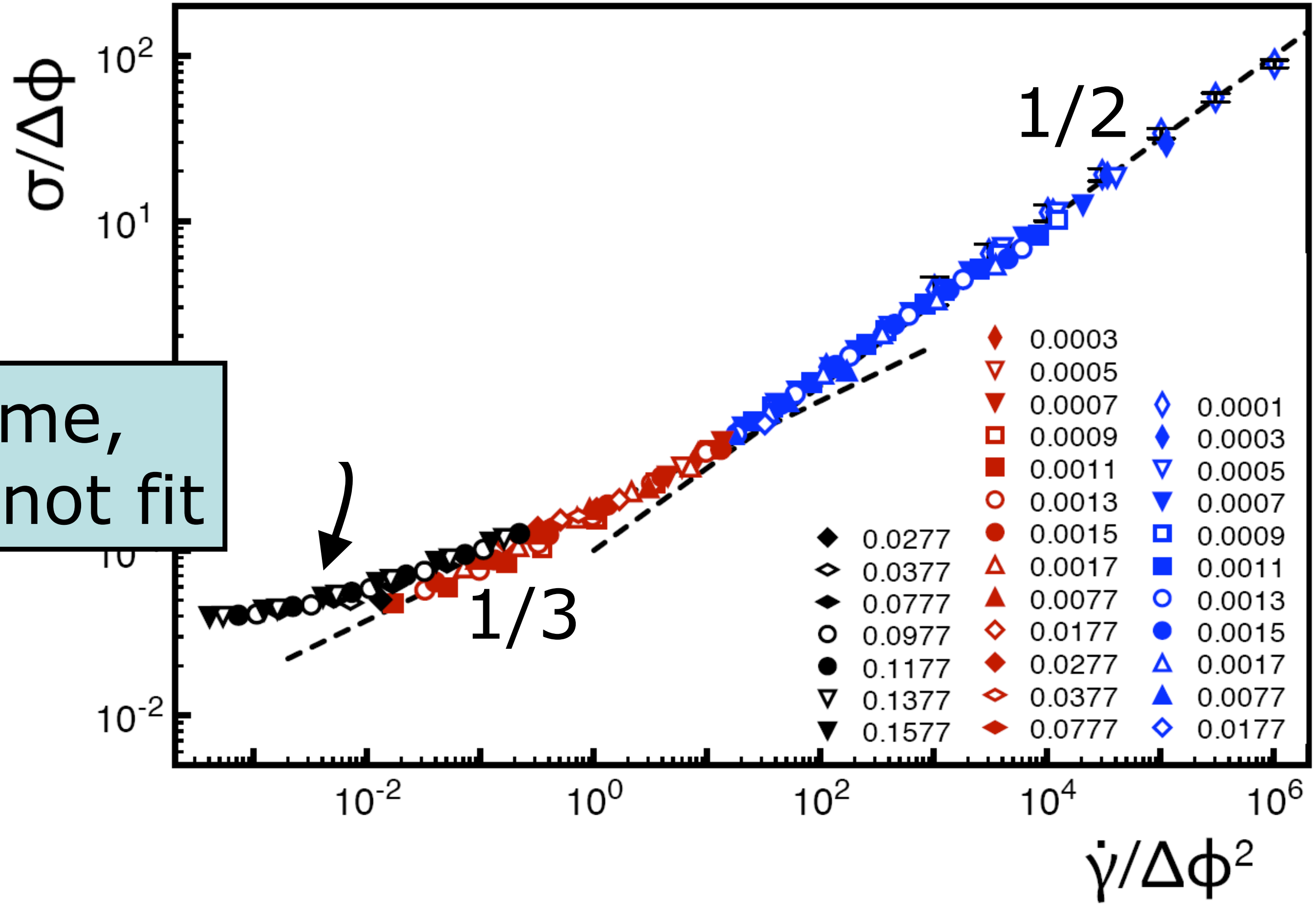


$\sigma/\Delta\phi$ as function of $\dot{\gamma}/(\Delta\phi)^2$

Scaling Model for Rheology Near Jamming



Scaling Model for Rheology near Jamming



YS regime, should not fit

$\sigma/\Delta\phi$ as function of $\dot{\gamma}/(\Delta\phi)^2$

Other Systems: Critical Regime

Disorder:	Local drag exponent →	Global drag exponent
OK	1	0.5
OK	$\alpha = 2/3$	$2\alpha/(\alpha+3) = 4/11 \approx 0.36$
OK	0	0

n optimally, we find that k syste
, but for $\beta = 0.36 \pm 0.05$, this s
nimized. We find that for $\alpha = ($

Other Systems: Critical Regime

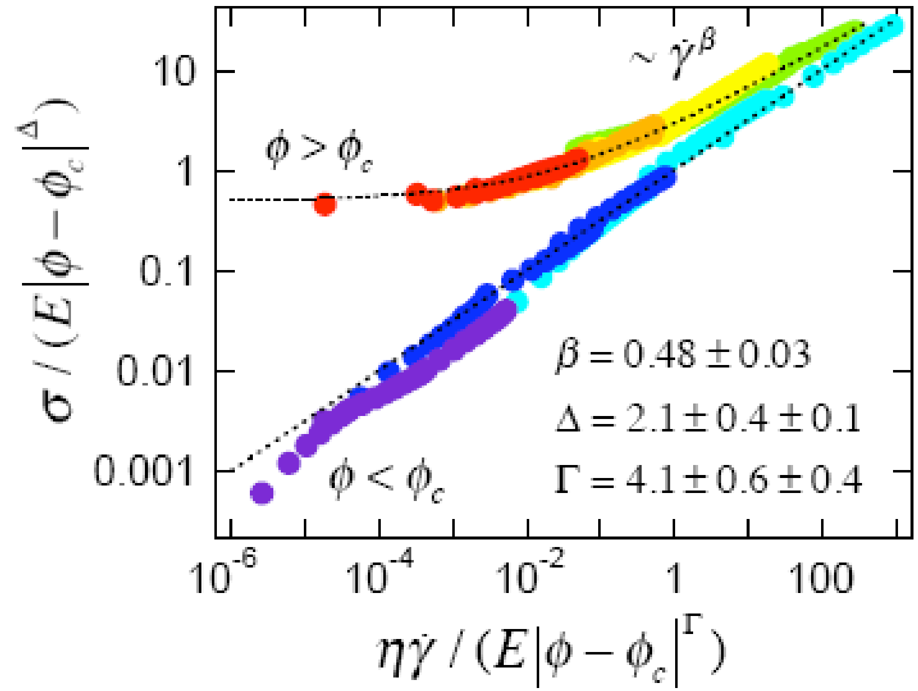
Disorder:	Local drag exponent →	Global drag exponent
OK	1	0.5
	2/3	0.36
	0	0

Elastic Interactions also Matter

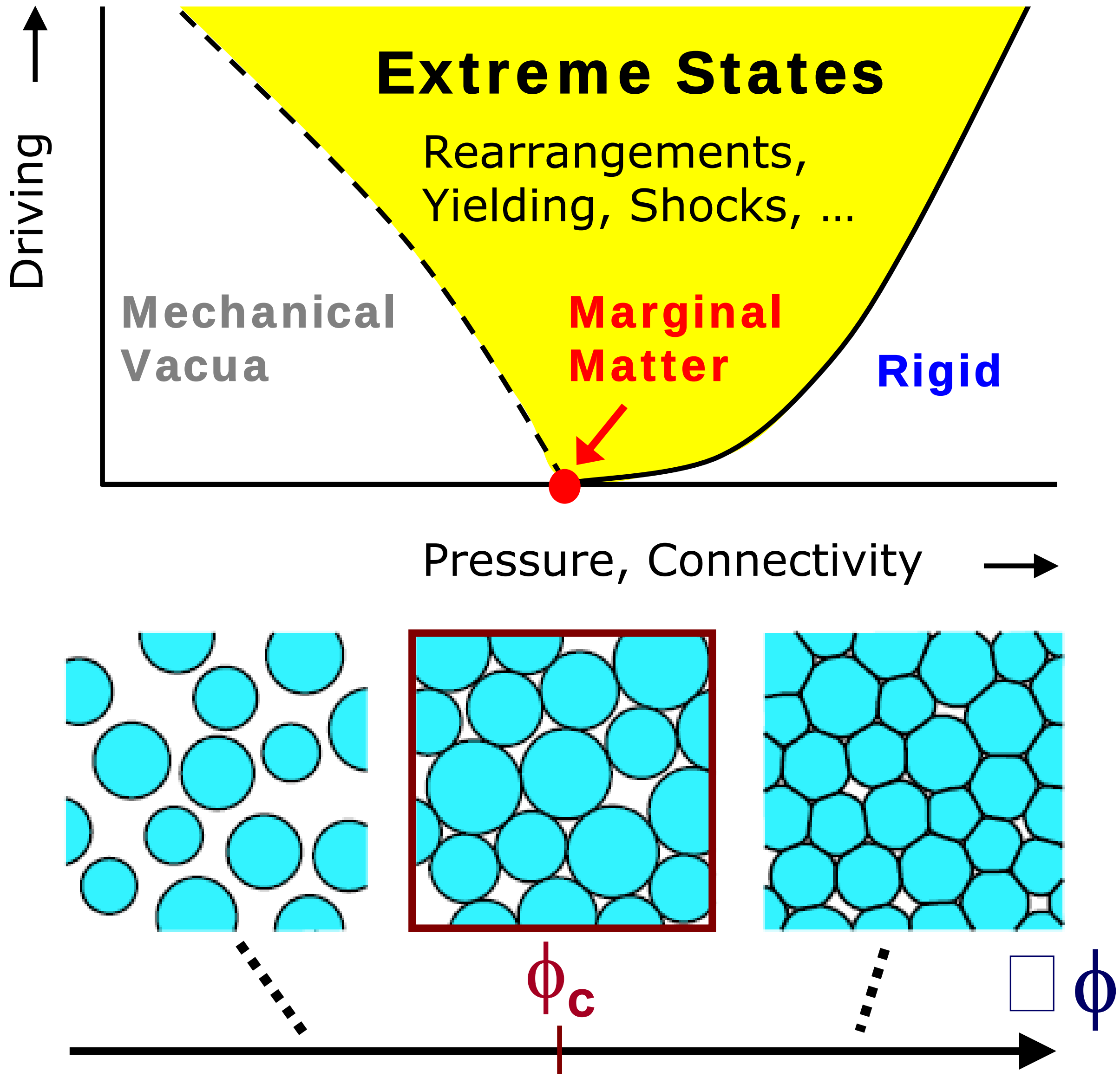
Nipa (Gollub, Durian, PRL 2010)

We: 1/2 , 2, 4

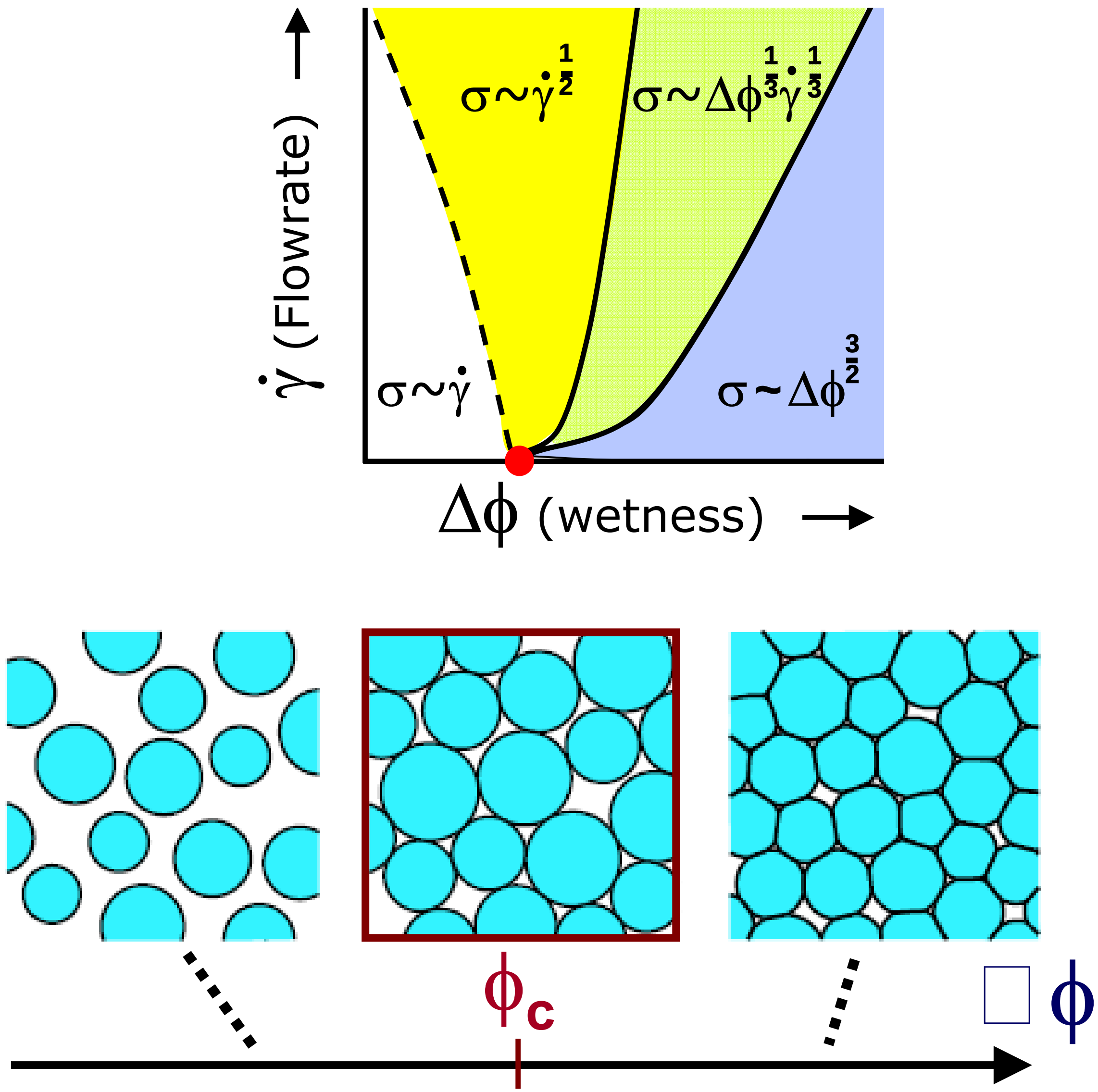
Data:



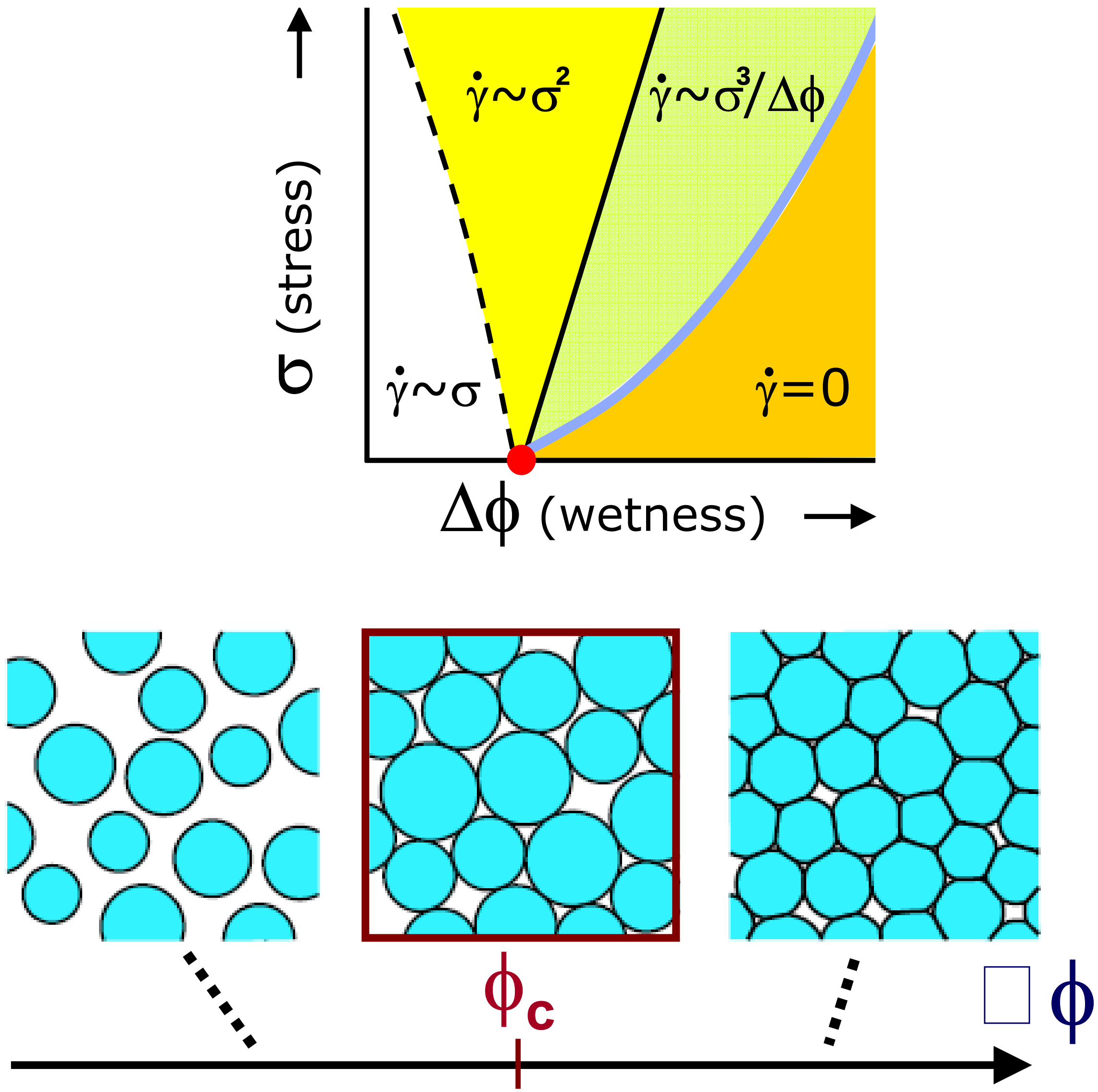
General Point



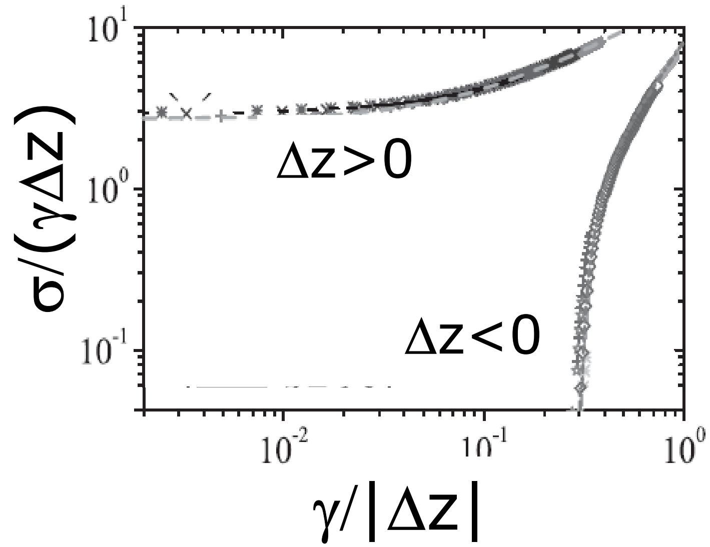
General Point

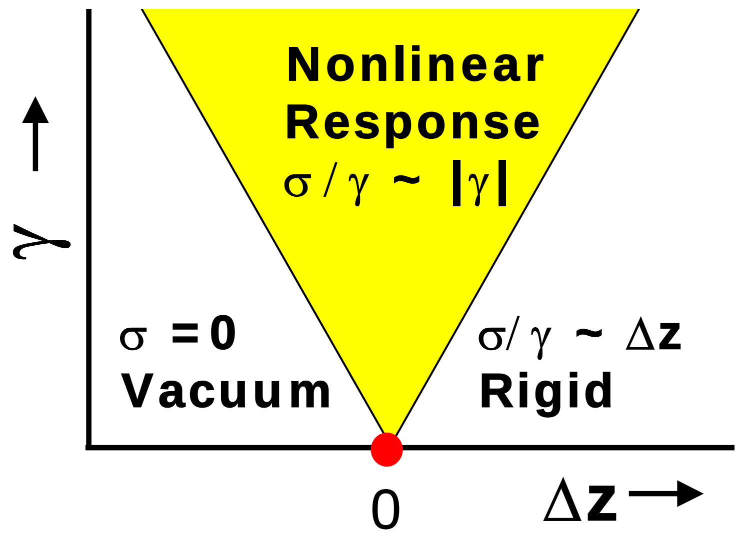


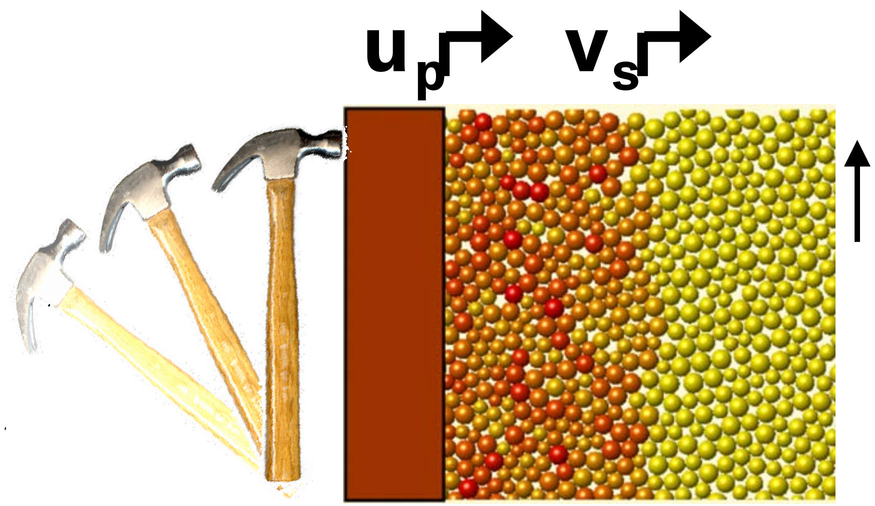
General Point

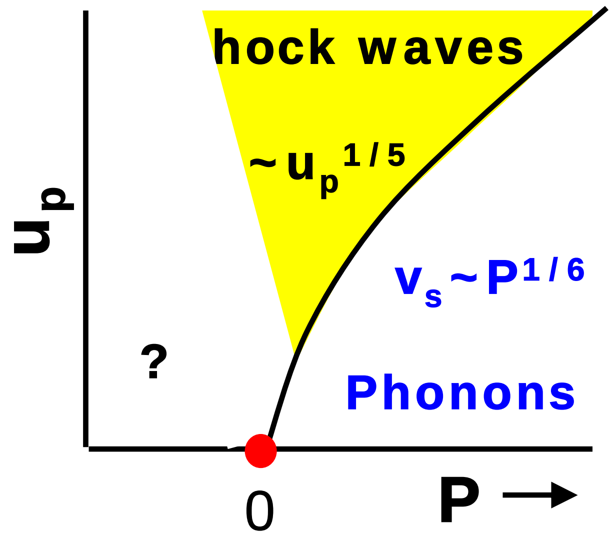


General Point

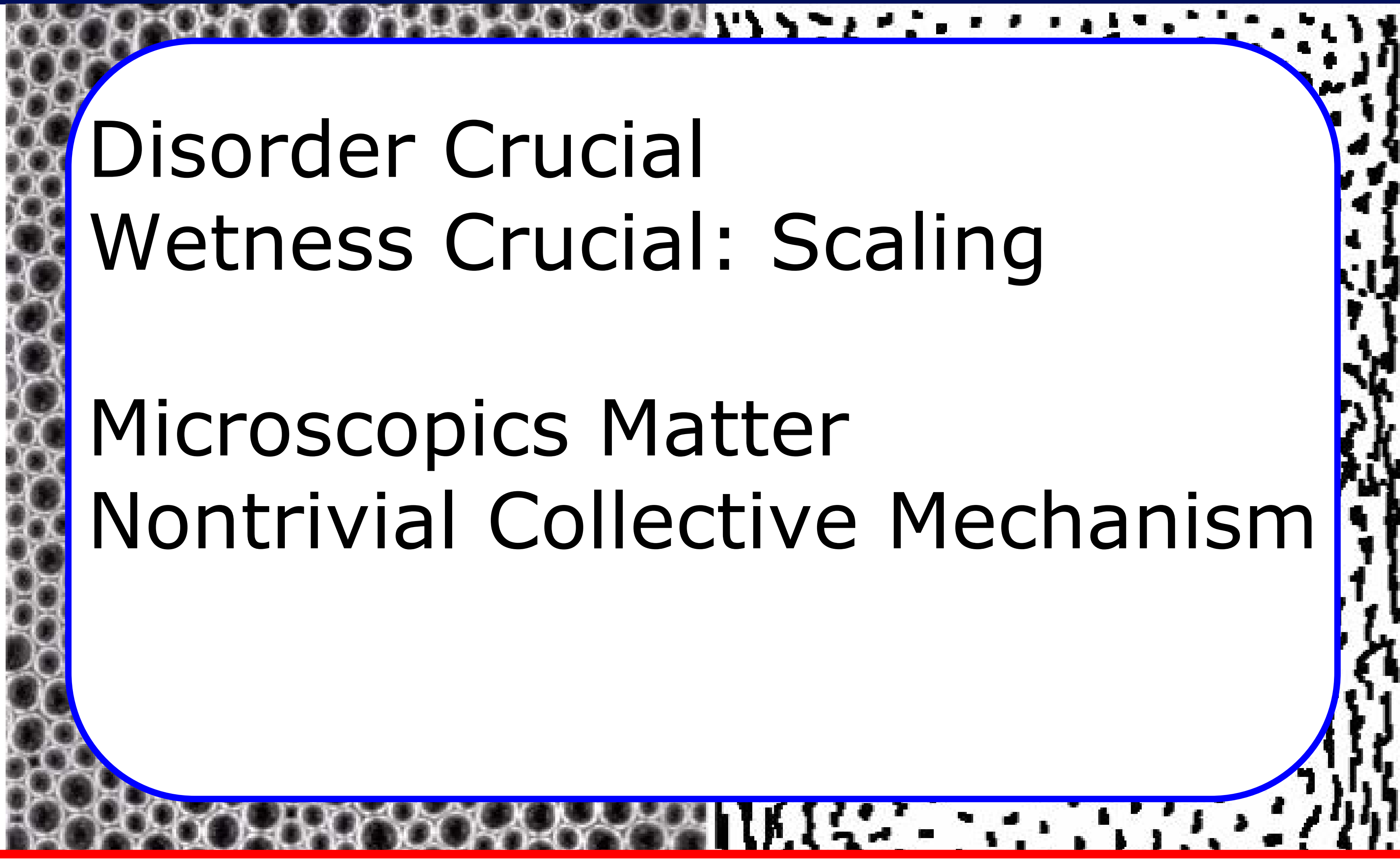








Foams from a Jamming Perspective



Disorder Crucial
Wetness Crucial: Scaling

Microscopics Matter
Nontrivial Collective Mechanism

Brian Tighe, Gijs Katgert, Matthias Mobius,
Erik Woldhuis, Zorana Zeravcic, Wim van Saarloos, Joris Remmers,
Ellak Somfai, Wouter Ellenbroek, Andrjez Latka, Daniel Brake, Alex Siemens, ……