Universal solutions for Boussinesq and non-Boussinesq plumes*

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Closed-form solutions describing the behaviour of rising buoyant axisymmetric turbulent plumes, emitted vertically from area sources in unconfined quiescent environments of uniform density, are proposed in a form that is universally applicable to Boussinesq and non-Boussinesq plumes. This work [1] thereby generalizes and consolidates results obtained separately for Boussinesq and non-Boussinesq plumes both based on the seminal plume model of Morton et al (1956) [2]. The universality we identify crucially relies on expressing the plume equations in terms of the vertical variation of the plume Richardson number and the dimensionless radius, the introduction of an effective entrainment radius, and the classical entrainment model for which the horizontal entrainment velocity is proportional to the local vertical velocity; this velocity being modified in the non-Boussinesq case by a factor of $\sqrt{\rho/\rho_a}$, where ρ and ρ_a are the densities of the plume and the ambient fluids, respectively.

Using the solutions developed, three characteristic heights are described which give insight into the differences in behaviour between Boussinesq and non-Boussinesq plumes. Sufficiently lazy plumes first contract and then expand giving rise to a neck in the plume envelope and based on our analysis we identify the length scale that characterizes the height over which non-Boussinesq effects are important for these lazy plumes.

[1] Van den Bremer, T.S. & Hunt, G.R., 2010. Universal solutions for Boussinesq and non-Boussinesq plumes. *J. Fluid Mech.* **644**, pp. 165 - 192.

[2] Morton, B.R., Taylor, G.I. & Turner, J.S., 1956. Turbulent gravitational convection from maintained and instantaneous sources. *Proc. Roy. Soc. Lond. A* **234**, pp. 1 - 23.

*The abstract is based on van den Bremer & Hunt (2010).